Optimal life-cycle portfolios for heterogeneous workers*

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Abstract

Household portfolios include risky bonds, beyond stocks, and respond to permanent labour income shocks. This paper brings these features into a life-cycle setting, and shows that optimal stock investment is constant or increasing in age before retirement for realistic parameter *combinations.* The driver of such inversion in the life-cycle profile is the resolution of uncertainty regarding social security pension, which increases the investor's risk appetite. This occurs if a small positive contemporaneous correlation between permanent labour income shocks and stock returns is matched by a realistically high variance of such shocks and/or risk aversion. Absent this combination, the typical downward sloping profile obtains. Overlooking differences in optimal investment profiles across heterogeneous workers results in large welfare losses, in the order of 17-26% of lifetime consumption.

Keywords: Life-cycle portfolio choice, background risk, age rule, investor heterogeneity, stock market participation

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1 Introduction

Empirical studies point to differences in labour income risk borne by investors in order to account for the observed distribution of asset holdings. The volatility in the growth rate of proprietary income, as well as its correlation with common stock returns, affect portfolio composition in the early study of Heaton and Lucas (2000). More recent work emphasizes that it is persistent, rather than temporary, income shocks that matter (Angerer and Lam, 2009). Betermier et al. (2012) find that changes in wage volatility across industries explain changes in the portfolio share invested by households in risky assets. Another indicator of the sensitivity of equity investments to labour income risk rests on asset pricing models, whose ability to explain the cross sectional distribution of equity returns improves when human capital is considered. Importantly, it is heterogeneous industry-related human capital, rather than aggregate human capital, that appears to matter (Eiling 2012).

Against this background, this paper investigates the effect of heterogeneity in permanent shocks to labour income for optimal portfolio holdings over the life cycle. The consensus is that investors should reduce their stock investments as they approach retirement age under normal circumstances (Bodie, Merton and Samuelson 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The motive is that human capital relative to financial wealth is decreasing over the life cycle, and labour income provides a hedge against shocks to stock returns. We point out, though, that uncertainty concerning social security income falls as retirement approaches since labour income shocks are persistent. This makes financial risk bearing more attractive.

Our paper argues that the optimal portfolio share invested in stocks increases, or is constant, in age before retirement for reasonable parameter configurations. This result obtains in a standard life-cycle framework where the first pillar offers an exogenous replacement ratio and available assets include one riskless and two risky assets ("stocks" and "bonds"). The driver of this inversion of the standard life-cycle asset allocation profile is a positive contemporaneous correlation between permanent labour income shocks and innovations to stock returns, *when matched* by a relatively high variance of such shocks and/or a relatively high risk aversion. Importantly, this pattern obtains for realistic parameter values. Such parametric interactions are also able to generate non-participation in the stock market by the young - a robust empirical regularity that so far has been dealt with by resorting to various kinds of participation costs.[1]

[1] Our paper extends to the life-cycle framework the analysis carried out by Boyle and Guthrie (2005), who use the mean-variance model with two risky assets augmented to include human capital to show the role of the correlation between risky assets and labour

More precisely, when we simply introduce bonds as a second risky asset into the Cocco, Gomes and Maenhout (2005) model, we obtain minor variations with respect to known results. Early in the worker's life, the average asset allocation is tilted towards stocks, as labour income provides a hedge against financial risks, while it gradually shifts to bonds in the two decades before retirement because income profiles peak around age 45. Changing one parameter at a time also involves minor modifications *in profiles*, although portfolio *shares* are affected in known ways. On the contrary, a clearcut departure from previous results emerges when a moderately positive correlation between stock returns and permanent labour income innovations interacts with a slightly higher degree of risk aversion. The worker starts investing in risky stocks only around the age of 25, after accumulating a sufficient amount of financial wealth. Afterwards, the stock share increases over time to reach about 20% for the median investor at the age of 40, and remains virtually constant until retirement. The portfolio bond share is correspondingly decreased up to the age of 40, with no investment in the riskless asset at any age. Therefore, the interaction of a positive stock return-labour income correlation with a relatively high degree of risk aversion produces an opposite age pattern of stock investment with respect to standard calibrations of life-cycle models and popular target-date products. If we add to this picture a higher variance of permanent labour income shocks, a gradual decrease over time of the risky asset share applies to bonds instead of stocks and is accompanied by accumulation of the safe asset.

These results owe in part to the (small) positive correlation of income shocks with equity returns, implying that labour income becomes an imperfect substitute for stock investments inducing the investor to reduce the equity allocation (Viceira 2001). This explains higher bond investment at the beginning of the life cycle, when human capital is relative large, and possible non participation in the stock market by the young. At the same time, uncertainty over future pension income falls as retirement approaches, thereby increasing the investor's risk appetite. The interplay of these two effects determines the life-cycle invetment profile.

Our paper implies that multiple investment strategies ought to be offered to plan participants depending on their risk aversion and their specific labour income characteristics. We measure the welfare losses associated to offering a single "target-date fund" (TDF), mimicking those adopted by pension funds, with an initially high stock share which gradually falls in age while the bond share increases.[2] Such investment rule is very close to optimal for

income in solving the asset allocation puzzle.

[2] Target Date Retirement Funds (TDF) are a "safe harbor" investment default in

the benchmark parameters which were the focus of previous research. It generates very large welfare costs, in the order of 17-26% of lifetime consumption, when higher risk aversion is accompanied by realistically large income risk. We also consider two alternative rules of thumb. The first is an age rule, where the portfolio share allocated to risky assets decays deterministically with the worker's age, while the second one is an equally weighted portfolio of three financial assets. This echoes the "$1/N$ rule" of DeMiguel, Garlappi and Uppal (2008) that outperforms several investment strategies in ex post portfolio experiments. The latter strategy performs consistently better than the age rule in our ex ante experiment, and appears to be preferable to the TDF alternative in case the pension fund ignores workers' labour income profiles.

Bodie, Merton and Samuelson (1992) already specify exceptions to 'normal' circumstances, inducing workers to choose greater risk-taking with age. These are a very risky wage or a reduction in wage risk over the life cycle. Here, we analyse such cases using the realistic stochastic process for labour income proposed by Cocco et al. (2005) and argue that these can be quite 'normal' in practice: the variance of wage shocks need not be so high for the inversion to obtain, as long as such shocks are permanent as opposed to temporary and the asset menu includes bonds. Bodie and Treussard (2008) also suggest that the standard age rule may be far from optimal when wages are perfectly correlated with stock returns and risk aversion is relatively high, in which case a duration-matched portfolio of inflation-protected bonds may lead to higher welfare. We focus on the case when the correlation of permanent wage shocks with stock return innovations is low (0.2), broadly consistent with estimates obtained by a large part of the empirical literature.[3]

Dramatic investments in stocks when young may not be optimal if there is enough long-term cointegration between labour income and stock returns (Benzoni, Collin-Dufresne and Goldstein 2007)). Other explanations include the presence of housing wealth (Cocco 2004) and the sensitivity of the ex-

defined-contribution (DC) plans in the US since 2006. Vanguard life cycle fund with retirement date 2015 and 2045 respectively had stock allocations of 57% and 90% as of January 2012. Sweden's AP7 introduced in 2010 a new default arrangement that allocates 100 percent in equities until age 55 and then gradually moves into fixed income investments. Several developing countries adopt decreasing age-dependent default investment options (Giacomel and Rinaldi, 2008).

[3]For example, although this correlation is not significantly different from zero in Cocco, Gomes and Michaelides (2005) for households with any level of educational attainment, it ranges from 0.33 for households with no high-school education to 0.52 for college graduates in Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002). More detailed discussion of this point is provided in section 3.3 below.

pected labour income growth rate to the real short-term interest rate (Munk and Sorensen 2010). Here we resort to two observed features of household portfolios, namely their responsiveness to permanent income shocks and the presence of risky bonds in an otherwise benchmark model. A simple interaction between risk aversion (or background risk) and the correlation of permanent income shocks and stock returns, may even explain upward sloping age profiles and non-participation by the young. Importantly, these combination effects are specific to the three parameters we stress above, at least for realistic calibrations. For instance, when the replacement ratio falls,[4] simulations reveal that agents save more during their working life in anticipation of lower pension incomes, thus accumulating a higher level of financial wealth. This determines a lower optimal share of stocks at all ages and for all values of the labour income-stock return correlation, holding risk aversion fixed. However, it does not impact on the shape of life cycle profiles because income shocks, and therefore the resolution of uncertainty, are less relevant to pension income.

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlined the numerical solution procedure adopted. Simulation results are discussed in Section 3. The concluding Section 4 summarizes our main findings.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life. Though the maximum length of the life span is T periods, its effective lenght is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-1} p_{t_0+k}\right) \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma}\right]$$

where C_{it} is the level of consumption at time t, $\beta < 1$ is an utility discount factor, and γ is the constant relative risk aversion parameter.[5] In the

[4] Observed replacement ratios vary widely both within and across countries, ranging from 34.4% in UK to 95.7% in Greece (OECD 2007).

[5] As is well known, assuming power utility with relative risk aversion coefficient γ constrains the intertemporal elasticity of substitution to be equal to $1/\gamma$. Moreover, γ also

benchmark model specification we rule out utility derived from leaving a bequest, as in Cocco, Gomes and Maenhout (2005), which will be introduced in some extensions below. Moreover, we do not model labour supply decisions, whereby ignoring the insurance property of flexible work effort (allowing investors to compensate for bad financial returns with higher labour income), as in Gomes, Kotlikoff and Viceira (2008), and the opportunity to switch jobs as in Ruffino (2008).

2.1 Labour and retirement income

Available resources to finance consumption over the agent's life cycle derive from accumulated financial wealth and from the stream of labour income. At each date t during *working life*, the exogenous labour income Y_{it} is assumed to be governed by a deterministic age-dependent growth process $f(t, \mathbf{Z}_{it})$, and is hitted by both a permanent shock u_{it} and a transitory disturbance n_{it}, the latter being uncorrelated across investors. Formally, the logarithm of Y_{it} is represented by

$$\log Y_{it} = f(t, \mathbf{Z}_{it}) + u_{it} + n_{it} \qquad t_0 \leq t \leq t_0 + K \tag{1}$$

More specifically, $f(t, \mathbf{Z}_{it})$ denotes the deterministic trend component of permanent income, which depends on age t and on a vector of individual characteristics $\mathbf{Z}_{it}$, such as gender, marital status, houseohld composition and education. As in Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005), uncertainty of labour income is captured by the two stochastic processes, u_{it} and n_{it}, driving the permanent and the transitory component respectively. Consistently with the available empirical evidence, the permanent disturbance is assumed to follow a random walk process:

$$u_{it} = u_{it-1} + \varepsilon_{it} \tag{2}$$

where ε_{it} is distributed as $N(0, \sigma_\varepsilon^2)$ and is uncorrelated with the idiosyncratic temporary shock n_{it}, distributed as $N(0, \sigma_n^2)$. Finally, the permanent disturbance ε_{it} is made up of an aggregate component, common to all investors, $\xi_t \sim N(0, \sigma_\xi^2)$, and an idiosyncratic component $\omega_{it} \sim N(0, \sigma_\omega^2)$ uncorrelated across investors:

$$\varepsilon_{it} = \xi_t + \omega_{it} \tag{3}$$

As specified below, we allow for correlation between the aggregate permanent shock to labour income ξ_t and innovations to the risky asset returns.

governs the degree of relative "prudence" of the consumer, related to the curvature of her marginal utility and driving precautionary savings.

During *retirement*, income is certain and equal to a fixed proportion λ of the permanent component of income in the last working year:

$$\log Y_{it} = \log \lambda + f\left(t_{0+K}, \mathbf{Z}_{it_{0+K}}\right) + u_{it_{0+K}} \qquad t_0 + K < t \leq T \tag{4}$$

where the level of the replacement rate λ is meant to capture at least some of the features of Social Security systems. Other, less restrictive, modelling strategies are possible. For example, Campbell, Cocco, Gomes and Maenhout (2001) model a system of mandatory saving for retirement as a given fraction of the (stochastic) labour income that the investor must save for retirement and invest in the riskless asset, with no possibility of consuming it or borrowing against it;[6] at retirement, the value of the wealth so accumulated is transformed into a riskless annuity until death.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and in two risky assets, characterized as "*stocks*" and "*bonds*". The risky assets yield stochastic gross real returns R_t^s and R_t^b respectively. We maintain that the investment opportunities in the risky assets do not vary over time and model excess returns of stocks and bonds over the riskless asset as

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{5}$$
$$R_t^b - R^f = \mu^b + \nu_t^b \tag{6}$$

where μ^s and μ^b are the expected stock and bond premia, and ν_t^s and ν_t^b are normally distributed innovations, with mean zero and variances σ_s^2 and σ_b^2 respectively. We allow for the two disturbances being correlated, with correlation ρ_{sb}. Moreover, we let the innovation on the stock return be potentially correlated with the aggregate permanent disturbance to the labour income, and denote this correlation by ρ_{sY}. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides (2002) and Koijen, Nijman and Werker (2010). While both papers document market timing effects on asset allocations when parameters of the return distributions are known with certainty, there is still considerable debate as to the *ex-post* value of market timing (De Miguel, Garlappi and Uppal 2008) and return predictability in general (Goyal and Welch 2008) when such parameters are estimated by an asset manager.

[6] Koijen, Nijman and Werker (2011) argue that these mechanisms are suboptimal relative to alternative annuity designs, despite their diffusion across pension systems.

At the beginning of each period, financial resources available for consumption and saving are given by the sum of accumulated financial wealth W_{it} plus current labour income Y_{it}, that we call *cash on hand* $X_{it} = W_{it}+ Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R^P_{it} + Y_{it+1} \tag{7}$$

where R^P_{it} is the portfolio return

$$R^P_{it} = \alpha^s_{it}R^s_t + \alpha^b_{it}R^b_t + \left(1 - \alpha^s_{it} - \alpha^b_{it}\right) R^f \tag{8}$$

with α^s_{it}, α^b_{it} and $\left(1 - \alpha^s_{it} - \alpha^b_{it}\right)$ denoting the shares of the investor's portfolio invested in stocks, bonds and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then three financial assets are $F_{it} \geq 0$, $S_{it} \geq 0$ and $B_{it} \geq 0$ respectively for the riskless asset, stocks and bonds, and the portfolio shares are non negative in each period.

All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement. However, the results concerning asset allocation are qualitatively similar in unreported simulations based on the alternative assumption that retirees invest in the riskless asset only.

2.3 Solving the life-cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labour income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}^{T-1}_{t_0}, \left\{\alpha^s_{it}, \alpha^b_{it}\right\}^{T-1}_{t_0}} \left(\frac{C^{1-\gamma}_{it_0}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-1} p_{t_0+k} \right) \frac{C^{1-\gamma}_{it_0+j}}{1-\gamma} \right] \right) \tag{9}$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left(\alpha^s_{it}R^s_t + \alpha^b_{it}R^b_t + \left(1 - \alpha^s_{it} - \alpha^b_{it}\right) R^f\right) + Y_{it+1}$$

with the labour income and retirement processes specified above and short sales and borrowing constraints imposed.

Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning ot period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}u_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^s,\alpha_{it}^b\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta p_t E_t\left[V_{it+1}\left(X_{it_{+1},}u_{it+1}\right)\right]\right) \tag{10}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of the two state variables, the level of cash on hand at the beginning of time t, X_{it}, and the level of the stochastic permanent component of income at beginning of t, u_{it}.

In order to reduce the dimensionality of the original problem to one state variable we exploit the homogeneity of degree $(1-\gamma)$ of the utility function, and normalize the entire problem by the permanent component of income u_{it}. Thus, we can rewrite (10) as

$$V_{it}\left(X_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^s,\alpha_{it}^b\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta p_t E_t\left[V_{it+1}\left(X_{it+1}\right)\right]\right) \tag{11}$$

This problem has no closed form solution: hence the optimal values for consumption and portfolio shares at each point in time are obtained by means of numerical techniques. To this aim, we apply a backward induction procedure and obtain optimal consumption and portfolio rules in terms of the state variable starting form the last (possible) period of life T. In particular, the solution for period T is trivial, considering that, with no bequest motive, it is optimal to consume all available resources (i.e., $C_{iT} = X_{iT}$) implying that the value function at T coincides with the direct utility function over the cash on hand available at the beginning of the period:

$$V_{iT}\left(X_T\right) = \frac{X_{iT}^{1-\gamma}}{1-\gamma} \tag{12}$$

Then, going backwards, for every period $t = T-1, T-2, ..., t_0$, and for each possible value of the state variable (the initial level of cash on hand at t) the optimal rules for consumption and the assets' portfolio shares are obtained from the Bellman equation (10) using the grid search method.[7] From the Bellman equation, for each level of the state variable X_{it}, the value function at the beginning of time t, $V_{it}(X_{it})$, is obtained by picking the level of consumption and of portfolio shares that maximizes the sum of the utility from

[7] According to this method, the problem is solved over a grid of values covering the space of the state variables and the controls, to ensure that the solution found is a global optimum.

current consumption $U(C_{it})$ plus the discounted expected value from continuation, $\beta p_t E_t V_{it+1}(X_{it+1})$. The latter value is computed using $V_{it+1}(X_{it+1})$ obtained from the previous iteration. In particular, given $V_{it+1}(X_{it+1})$, the expectation term is evaluated in two steps. We use numerical integration perfomed by means of the standard Gaussian Hermite quadrature method to approximate the distribution of shocks to labour income and asset returns. Then, cubic spline interpolation is employed to evaluate the value function at points that do not lie on the state space grid.

3 Simulation results

The numerical solution method briefly outlined above yields, for each set of parameters chosen, the optimal policy functions for the level of consumption and the shares of the financial portfolio invested in the riskless and risky assets as functions of the level of cash on hand. Using those optimal rules, it is then possible to simulate the life-cycle consumption and asset allocation choices of a large number of agents. In this section, we describe results obtained from this procedure, focusing first on a benchmark case and then presenting extensions along various dimensions.

3.1 Calibration

Parameter calibration concerns the investor's preferences, the features of the labour income process during working life and retirement, and the moments of the risky asset returns. To obtain results for a benchmark case, we chose plausible sets of parameters referred to the US and based mainly on Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005).

The investor begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health Statistics.* As regards to preferences, we set the utility discount factor $\beta = 0.96$, and the coefficient of relative risk aversion $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005, Gomes, Kotlikoff and Viceira 2008), capturing an intermediate degree of risk aversion, though Cocco, Gomes and Maenhout (2005) choose a value as high as 10 in their benchmark setting.

The labour income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in

Cocco, Gomes and Maenhout (2005). The age-dependent trend is captured by a third-order polynomial in age, delivering the typical hump-shaped profile until retirement depicted as the dash-dotted line in Figure 1. After retirement, income is a constant proportion λ of the final (permanent) labour income, with $\lambda = 0.68$. The continuous line in the figure portrays the whole deterministic trend $f(t, \mathbf{Z}_{it})$, used in the simulations below, that allows also for other personal characteristics such as family size and marital status. In the benchmark case, the variances of the permanent and transitory shocks (ε_{it} and n_{it} respectively) are $\sigma_{\varepsilon}^2 = 0.0106$ and $\sigma_n^2 = 0.0738$; in some of the extensions below we let those parameters vary (to explore the effects of increasing labour income uncertainty) but keep the permanent-transitory ratio roughly constant at the 0.14 level. This choice is supported by the evidence in Cocco, Gomes and Maenhout (2005), showing that empirically the ratio is remarkably stable across occupational sectors despite widely different values for the labour income shock variances.

The riskless (constant) interest rate is set at 0.02, with expected stock and bond premia μ^s and μ^b fixed at 0.04 and 0.02 respectively. The standard deviations of the returns innovations are set at $\sigma_s = 0.157$ and $\sigma_b = 0.08$; in the benchmark case, we fix their correlation at a positive but relatively small value: $\rho_{sb} = 0.2$, calibrated on the historical annual correlation in the US and close to the choice of Gomes and Michaelides (2004). Finally, we initally impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$); we will assess below the impact on wealth accumulation and portfolio allocation of allowing for a moderately positive stock return-labour income shock correlation.

3.2 Benchmark results

In all simulations we look at the cross-sectional distribution of 10,000 agents' optimal choices over their life cycle. In the benchmark case, the typical life-cycle profiles for consumption, labour income and accumulated financial wealth are obtained over the working life and the retirement period, as in Cocco, Gomes and Maenhout (2005). Binding liquidity constraints make consumption closely track labour income until the 35-40 age range, when the consumption path becomes less steep and financial wealth is accumulated at a faster rate. After retirement at 65, wealth is gradually decumulated and consumption decreases to converge to retirement income in the last possible period of life.

Before presenting the age profile of optimal portfolio shares, Figure 2 displays the optimal policy rules for the risky asset shares α_{it}^s and α_{it}^b as functions of the level of cash on hand (the problem's state variable): in each panel, the optimal fraction of the portfolio invested in stocks and bonds is plotted against cash on hand for investors of four different ages (20, 30, 55 and 75). The basic intuition guiding the interpretation of the optimal policies, on which the following simulation results are based, is that labour income is viewed by the investor as an implicit holding of an asset (Bodie, Merton and Samuelson 1992). Although in our setting labour income is uncertain (its process being hit by both permanent and transitory shocks), as long as the correlation of asset returns' innovations and labour income disturbances is zero or sufficiently small, labour income is more similar to the risk-free than to the risky assets; therefore, when the present discounted value of the expected future labour income stream (i.e. human wealth) accounts for a sizeable portion of overall wealth, the investor is induced to tilt her portfolio towards the risky assets. The proportion of human out of total wealth is widely different across investors of different age and is one of the main determinants of their chosen portfolio composition.

Looking at Figure 2, in the case of an investor of age 75, the certain retirement income acts as a holding of the riskless asset and the relatively poor investor (with a small amount of accumulated wealth and current income) holds a financial portfolio entirely invested in stocks.[8] Wealthier investors hold a lower portfolio share in stocks (and increase their holdings of bonds), since for them the proportion of the overall wealth implicitly invested in the riskless asset (i.e. human wealth) is lower. At age 55, the investor still has a decade of relatively high expected labour income before retirement, and she will tend to balance this implicit holding of a low-risk asset with a financial portfolio more heavily invested in risky stocks (and less in bonds) than older investors: her optimal policies in Figure 2 are shifted outwards with respect to the 75-year-old agent for all levels of cash on hand.[9] The same intuition applies to earlier ages, for which the optimal stock and bond policies shift gradually outwards as younger investors are considered. The only exception to this pattern occurs for the very young investors (approximately in the 20-25 age range), for whom the labour income profile increases very steeply, making it optimal to hold portfolios more invested in stocks.

On the basis of such optimal investment policies, the portfolio shares of stocks, bonds and the riskless asset for 10,000 agents have been obtained by simulation over the whole investors' life cycle. Figure 3 shows the median portfolio shares for stocks (upper panels) and bonds (lower panels)

[8] The portfolio shares of the risky assets are not defined for extremely low values of cash on hand since the investor (af any age) has no savings in this case.

[9] The step-wise appearence of the policy rules is due to the choice of the grid in the numerical solution procedure. The use of a finer grid would deliver smoother policies, at the cost of additional computing time.

from the cross-sectional distribution, plotted against age. In order to assess the amount of heterogeneity in investors' portfolio choices, also the 5th and the 95th percentiles of the cross-sectional distributions of optimal shares are shown. Two assumptions on the amount of background risk faced by agents are considered: a "normal" variance scenario (left column), in which the variances of the permanent and transitory labour income shocks are set at the already mentioned benchmark levels ($\sigma_{\varepsilon}^2 = 0.0106$ and $\sigma_n^2 = 0.0738$), and a "high" variance scenario (right column), in which, while keeping the permanent to total variance ratio constant (0.14), the labour income shock variances are set at the larger values $\sigma_{\varepsilon}^2 = 0.042$ and $\sigma_n^2 = 0.30$, consistently with the evidence presented by Cocco, Gomes and Maenhout (2005) for US workers in the agricultural sector.

Figure 3 well summarizes a relatively standard set of results on the age profiles of stock and bond portfolio shares, mainly determined by the fact that over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through expected labour incomes varies, being large for young investors and declining as retirement approaches. In fact, in the "normal" labour income risk scenario, younger agents invest heavily in stocks until approximately the age of 40. Middle-age investors (between 40 and the retirement age of 65) gradually shift the composition of their portfolio away from stocks and into bonds, to reach median shares of around 55% and 45% respectively at the retirement date. After retirement, income becomes certain and the proportion of implicit holdings of the safe asset increases again; moreover, previously accumulated financial wealth is run down quickly to support a relatively stable consumption level. Consequently, the share of stocks starts increasing, at the expense of bonds, to compensate for it. Throughout both working life and retirement, holdings of the riskless asset are kept at a minimum, very often zero.

The effects of increasing labour income risk on optimal asset allocation over the working life are portrayed in the right column panels of Figure 3. A larger amount of background risk induces agents to increase precautionary savings, accumulating more financial wealth over time. Therefore, there is less need for investors to tilt their asset allocation towards the riskiest asset available: the optimal share of stocks in the portfolio is reduced at any age, and the bond share is correspondingly increased. The age profiles show that investors start decreasing the stock share very early in life, to reach a bottom level (slightly larger than 40% for the median of the distribution) around the age of 40; then, the share remains remarkably flat for investors between 40 and 65, to be gradually increased again during retirement. This age profile is mirrored exactly by the bond share, with no room for investment in the safe asset at any age.

Overall, the popular financial advice of holding a portfolio share of risky stocks equal to 100 minus the investor's age (so that $\alpha^s_{age} = (100-\text{ age})/100$), implying a gradual shift toward bonds over life, is not completely at variance with the optimally designed investment policies displayed in Figure 3, at least over the investors' working life. However, in our benchmark cases the decumulation of stocks is not linear, as the simple age-dependent rule would predict (with the stock share going down from from 80% at the age of 20 to 35% at retirement), but depends on the relative dynamics of the investor's human and financial wealth. This helps explaining also the behavior of portfolio shares when the agent's incentives to save and accumulate financial wealth are changed in various ways.

For example, if replacement ratio λ is reduced, investors, anticipating relatively lower incomes during retirement, choose to save more during their working life, thereby accumulating a higher level of financial wealth. This determines a lower optimal share of stocks (and a correspondingly larger bond share) at all ages (and in both labour income risk scenarios) and a declining time profile over the working life, as human capital decreases relative to financial wealth, confirming the patterns displayed in Figure 3. Also the introduction of a bequest motive, inducing even young investors to save more, enhances financial wealth accumulation, whereby reducing the optimal stock share in favor of less risky bonds; again, the age profile of the stock share is declining as the investor approaches retirement. During retirement, the presence of a bequest motive induces agents not to increase the portfolio share of the riskiest asset: even in the "normal" background risk scenario, the optimal share of stocks remains nearly flat at the bottom level attained at retirement age.

3.3 Inverted Life-Cycle Profiles

To evaluate the robustness of the life-cycle asset share profiles obtained above, we modify the benchmark setting in various ways. In this subsection, we focus on two important dimensions, i.e. the correlation between stock return innovations and the aggregate permanent shock to labour income (ρ_{sY}) and the degree of investors' risk aversion (γ), and their interactions.

First, we let the stock return innovations be positively correlated with the innovations in permanent labour income. The available empirical estimates of this correlation for the US differ widely. Cocco, Gomes and Michaelides (2005) report estimated values not significantly different from zero for households with any level of educational attainment, whereas Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002) find higher values, ranging from 0.33 for households with no high-school education to

0.52 for college graduates.[10] Moreover, Cocco, Gomes, and Maenhout (2005) provide estimates between -0.01 and 0.02, while Heaton and Lucas (2000) between -0.07 and 0.14,and Munk and Sørensen (2005) report a correlation of 0.17. Furthermore, as to the correlation between labour income risk and industry-specific equity risk, Davis and Willen (2000) report correlations ranging between -0.10 and 0.40. Since our calibration of the labour income process reflects the features of households with high-school education, we adopt an intermediate positive value of $\rho_{sY} = 0.2$. This choice results in a modest correlation between the growth rate of individual labour income and stock return innovations, in accordance with the empirical evidence.[11]

Figure 4 displays the optimal portfolio shares of stocks and bonds when $\rho_{sY} = 0.2$. Now, the positive correlation between labour income shocks and stock returns makes labour income closer to an implicit holding of stocks rather than of the other assets.[12] In the "normal" labour income variance scenario (left column), younger investors, for whom human capital is a substantial fraction of overall wealth, are therefore heavily exposed to stock market risk and will find it optimal to offset such risk by holding a relatively lower fraction of their financial portfolio in stocks if compared with the benchmark case in Figure 3. This effect decreases as workers move along the steepest part of their labour income path, determining a gradual increase in the portfolio share of stocks until around the age of 25. From that age on, the size of human capital decreases and the investor shifts her portfolio

[10] In Campbell and Viceira (2002) the correlation is estimated with a one-year lag and treated as a contemporaneous correlation in simulations.

[11] In fact, using (1), (2) and (3) we can express the correlation between the growth rate of individual labour income ($\Delta \log Y_{it}$) and the stock return innovation (ν_t^s) in terms of ρ_{sY} and the variances of the aggregate and idiosyncratic labour income shocks as:

$$corr(\Delta \log Y_{it}, \nu_t^s) = \frac{1}{\sqrt{1 + \frac{\sigma_\omega^2 + 2\sigma_n^2}{\sigma_\xi^2}}} \cdot \rho_{sY} < \rho_{sY}$$

Using our benchmark ("normal" labour income variance) value for $\sigma_n^2 = 0.0738$ and attributing all permanent disturbances to the aggregate component, so that $\sigma_\varepsilon^2 = \sigma_\xi^2 = 0.0106$ (σ_ω^2 being 0), we derive an upper bound for $corr(\Delta \log Y_{it}, \nu_t^s)$:

$$corr(\Delta \log Y_{it}, \nu_t^s) \leq 0.26 \cdot \rho_{sY}$$

Therefore, the value for ρ_{sY} used in our simulations (0.2) implies a modest value for $corr(\Delta \log Y_{it}, \nu_t^s)$ of (at most) 0.052. This value is only slightly changed in the "high" labour income variance scenario (0.054).

[12] Note that the positive correlation between stock and bond return innovations (0.2) makes also bond return positively correlated with permanent labour income innovations, but with a much smaller coefficient (0.04).

composition again towards safer bonds: this yields a hump-shaped profile for the optimal share of stocks during working life. The stock share reaches a bottom level of about 40% for the median investor at around the age of 40, and remains substantially flat until retirement. At 65 labour income becomes certain (and therefore uncorrelated with stock return innovations), and the investor sharply rebalances her portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the benchmark case shown in Figure 3. Again, throughout working life and retirement, the age profile of the bond share mirrors that of stocks, with no investment in the riskless asset. When background risk is increased (Figure 4, right column) financial wealth is accumulated more rapidly and the portfolio share of stocks is lower at any age. The hump-shaped pattern of the stock share disappears and, as in the benchmark case of $\rho_{sY} = 0$, investors start decreasing the stock share from the very beginning of their working life, reaching a bottom level of about 35% at the age of 30 and then rebalancing the portfolio towards shares at the retirement date. The standard age-dependent pattern of stock investment is therefore restored at least in the early part of the working life.

Sharp differences in optimal asset allocation over the life cycle emerge when a moderately positive correlation between stock returns and permanent labour income innovations interacts with a relatively higher degree of risk aversion. Figure 5 portrays the age profile of the portfolio shares of stock and bonds for investors with a risk aversion parameter $\gamma = 8$, keeping the stock return-labour income correlation at $\rho_{sY} = 0.2$. To focus on the relevance of the interaction between those two parameters in shaping optimal life-cycle asset allocation choices, we choose a value for γ that, though higher than in the benchmark case ($\gamma = 5$), is not extreme; for example, Cocco, Gomes and Michaelides (2005) set $\gamma = 10$ in their baseline calibration exercise, considering this value as the upper bound of the range of reasonable values. Setting γ to values larger than 8 would even strengthen the results presented below.

In the "normal" background risk scenario (Figure 5, left column), the more risk-averse (and prudent) investor saves more for precautionary reasons and starts investing in risky stocks a positive fraction of her financial portfolio only around the age of 25, after accumulating a sufficient amount of financial wealth. Afterwards, the stock share increases over time to reach about 20% for the median investor at the age of 40, and remains virtually constant until retirement, when the portfolio is rebalanced in favor of stocks to compensate for the now riskless nature of income streams. The portfolio bond share is correspondingly decreased up to the age of 40, and then kept constant by the median investor until rebalancing occurs at the retirement date. No room for investment in the riskless asset is detected at any age. Therefore, the interaction of a positive stock return-labour income correlation with a relatively high degree of risk aversion produces an opposite age pattern of stock investment with respect to standard calibrations of life-cycle models and popular financial advice. Now it is optimal for the investor not even enter the stock market when very young, and build up the stock share later during working life, when the ratio of human to financial wealth decreases due to larger savings and the hump-shaped labour income dynamics. This gradual rebalancing process towards less risky bonds stops quite early (around the age of 40), when the investor attains an optimal asset allocation which is kept constant for the rest of her working life.

This age pattern is broadly confirmed when a larger amount of labour income risk is considered, as shown in the right column of Figure 5, with one important difference. With more background risk, precautionary savings are larger and financial wealth is accumulated more rapidly: the investor enters the stock market at the very beginning of her working life with a modest share (about 15%), which is only slightly increased over time to reach 20% very early, around the age of 25; then, as in the "normal" labour income risk scenario, the stock share is kept constant until retirement. The difference concerns the age profile of the portfolio shares of bonds and of the riskless asset: in this case the bond share decreases throughout the entire investor's working life, as the individual invests an increasingly larger portfolio share into the riskless asset. At retirement, riskless asset holdings amount to about 25% of the financial portfolio for the median investor. Therefore, with high background risk, a standard age-dependent rule implying a gradual reduction over time of the risky asset share does apply to bonds instead of stocks and is accompanied by accumulation of the safe asset.

With our parameter configuration, this result obtains only when we realistically enlarge the available asset class menu to consider two risky assets alongside a safe one. In fact, when the investor faces a choice between only stocks and the riskless asset, the interaction between $\rho_{sY} = 0.2$ and $\gamma = 8$ yields a stock share profile decreasing with age during working life, consistent with standard age-dependent rules, followed by a portfolio rebalance in favor of stocks at the retirement date. Moreover, the investment patterns showed in Figure 5 are robust to several changes in the structure of asset returns. In particular, setting the correlation between stock and bond returns to zero (whereby eliminating the already small correlation between bond returns and permanent labour income innovations induced by $\rho_{sY} = 0.2$), or reducing the return on the safe asset with unchanged premia on stocks and bonds do not affect the asset allocation choices over the investor's life cycle.

The main features of the asset allocation patterns illustrated in Figure 5 are even more pronounced when the investor anticipates a lower level of (certain) income streams during retirement. Figure 6 displays optimal portfolio shares for stocks and bonds when the replacement ratio is reduced to $\lambda = 0.4$. Compared with the case of $\lambda = 0.68$ (Figure 5), the age profile of the stock share is virtually unaffected, the only difference being a smaller rebalancing towards stocks at the retirement date, due to the lower amount of human capital available during retirement. The bond share displays an age-dependent behavior, decreasing during the whole investor's working life. With lower retirement income, the share of the portfolio invested in the riskless asset is gradually increased even in the "normal" background risk scenario. Results very similar to the case of a lower replacement ratio obtain when a bequest motive is introduced: the age profile of the portfolio shares displayed in Figure 5 in confirmed during working life, a smaller rebalancing towards stocks (and away from bonds) occurs at the retirement age, and all portfolio shares are kept virtually unchanged during retirement.

3.3.1 Optimal portfolio shares heterogeneity

So far, we discussed simulation results in terms of the median optimal portfolio shares across the investors' population. However, in our framework the presence of idiosyncratic labour income shocks may generate substantial heterogeneity in the pattern of financial wealth accumulation over time, and consequently a potentially wide dispersion of the optimal portfolio shares across individuals of the same age but with different levels of accumulated wealth. The degree of heterogeneity in portfolio choices is an important feature of life-cycle asset allocation models for several reasons. First, it can help to rationalize observed investors' behavior, which is characterized by a high degree of heterogeneity both in stock market participation and in the distribution of portfolio shares conditional on age (Gomes and Michaelides 2005; Benzoni, Collin-Dufresne and Goldstein 2007). Second, the amount of heterogeneity of optimal asset allocation due to idiosyncratic labour income dynamics may be relevant to the design of pension funds' default investment options, to be offered to different classes of investors. Therefore, we now focus on the features of the distribution of optimal portfolio shares across the investors' population, looking at the 5th and 95th percentiles of the cross-sectional distributions conditional on age.

In the benchmark case displayed in Figure 3, with moderate risk aversion ($\gamma = 5$) and stock returns uncorrelated with labour income innovations ($\rho_{sY} = 0$), in the "normal" background risk scenario the distribution of optimal stock and bond shares is highly heterogeneous for both workers and retirees, with the exception of young workers who invest the entire portfolio in stocks to compensate for the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions, relating portfolio shares to the amount of resources available for investment (cash on hand), portrayed in Figure 2 for our benchmark case. Given the investor's age, a relatively steep policy function implies that even small differences in the level of accumulated wealth (the increasingly more important component of cash on hand) result in widely different asset allocation choices: this happens typically to young investors, who are in their initial stage of wealth accumulation. When the amount of background risk is increased (Figure 3, right column) larger savings and wealth accumulation push investors on the flatter portion of their policy functions, determining a gradually decreasing heterogeneity in optimal portfolio shares of shocks and bonds during their working life. After retirement, investors start decumulating financial wealth, moving along the steeper portion of their relevant policy functions: therefore the dispersion of optimal shares increases again.

The pattern of decreasing heterogeneity in portfolio shares during working life is enhanced when we introduce a positive correlation between labour income shocks and stock returns ($\rho_{sY} = 0.2$), keeping risk aversion at the moderate level $\gamma = 5$. Figure 7 shows the policy rules for selected investors' ages (20, 30, 55 and 75) and the dispersion of optimal stock and bond portfolio shares along the life cycle. In both labour income risk scenarios, the distribution of portfolio shares shrinks rapidly around the median value, the more so when background risk is relatively high, implying more rapid financial wealth accumulation. In the "normal" variance scenario portrayed in panel(a), the shape of the policy functions for 20-year old investors, who start working life with relatively small cash on hand, determines the already mentioned hump-shaped behavior of optimal portfolio shares. From the age of 30 onwards, the policy functions are very close and flat, delivering more similar asset allocation choices throughout the remaining part of working life. After retirement, the different position and shape of the policy rules (as shown for the 75-year old investor) determine an increase in the dispersion of portfolio shares for both stocks and bonds around their median values. This pattern is even more pronounced in panel (b), when the larger background risk induces investors to save more and accumulate financial wealth more rapidly.

Figure 8 shows the policy rules and the quantiles of the optimal portfolio share distributions for stock, bonds and the riskless asset when a higher degree of risk aversion ($\gamma = 8$) interacts with the positive labour shock-stock return correlation ($\rho_{sY} = 0.2$). Already in the "normal" labour shock variance scenario in panel (a), the shape of the policy functions changes dramatically. As regards to stocks, the policy rules for workers of any age are extremely close and display a positive slope only for very small values of cash on hand; thereafter, they take a very flat shape. As a consequence, the optimal stock share - conditional on wealth - increases for all investors in the early part of their working life to remain constant from the age of 40 until retirement, with no dispersion of stock allocation choices across the investors' population. This behavior is mirrored by the policy rules and portfolio shares for bonds, with one remarkable difference: during working life, policy functions for bonds become more steeply downward sloping from relatively high levels of cash on hands. Wealthier investors sharply decrease their optimal bond portfolio share over time in favor of the riskless asset: the resulting distribution of bond shares until retirement is therefore strongly skewed towards lower values. The features of the policy rules discussed above are even more evident when the background risk is larger, as shown in panel (b). The policy functions for stocks do not vary through working age and become flat from very low values of cash on hand: therefore, apart from very young investors, no dispersion in optimal stock shares is obtained throughout working life. Also the policy functions for bonds almost coincide for workers of different age, but their downward-sloping shape now starts from intermediate values of cash on hand. As a consequence, the reduction of the optimal bond portfolio share starts earlier and continues throughout the entire investors' working life, with a broadly constant (and roughly symmetric) dispersion of bond shares across the population. Such degree of heterogeneity is mirrored by the dispersion of the portfolio shares of the riskless asset, which is now accumulated over time also by young workers and even at relatively low levels of cash on hand.

3.3.2 Household portfolios and labour income risk: empirical regularities

The key implication of our model is that optimal investment profiles are sensitive to parametric *combinations*, giving rise to heterogeneity in optimal asset holdings within age groups. Heterogeneity in portfolio shares should thus be explained by combinations of age, volatility of permanent labour income shocks and their correlation with assets returns. Relatively low (high) risk aversion and zero (positive) correlations should lead to high (zero or low) equity portfolio shares when young that decrease (increase or stay constant) as retirement approaches. To the best of our knowledge, no empirical research is addressing this possibility by interacting volatility, correlation and - where possible - risk aversion. This may explain why there is little consensus as to the sign of this relationship, on top of the identification problem documented in Ameriks and Zeldes (2004).[13]

As regards to *non-participation* in the equity market, Haliassos and Michaelides (2003) already pointed out the relevance of permanent rather than transitory income shocks. They also realized that a positive correlation was essential, but dismissed it as a plausible explanation on two grounds. First, early estimates attributed higher correlation to more educated groups and entrepreneurs, that are not the typical non-participants in the equity market. Recent investigations by Angerer and Lam (2009) find instead positive correlation between stock returns and labour income in occupations such as craftsman, operatives, managers and administrators, farm labourers, private household workers and armed forces. As far as educational attainment is considered, correlation is positive for certificates below a college degree. The second reason for dismissing income shocks as a source of non-partcipation was the absence of an alternative risky asset with positive risk premium: this pushed up to 0.5 the correlation needed to achieve non-participation. In our model it is sufficient to have a small, positive correlation between permanent shocks to income and stock returns (0.2), which translates in a correlation between total labour income and equity returns even lower than 0.057. This is because risky bonds are better substitutes to equities than cash.

A feature of our simulated profiles that appears at odds with observed profiles is optimal investments during retirement. We therefore allow for a bequest motive in Figure 9, that smoothes out the post-retirement portfolio profiles: comparison with the corresponding Figure 8 reveals that the pre-retirement patterns, that are the focus of our investigation, are unaffected.

3.4 Welfare costs of suboptimal asset allocations

Optimal asset allocation strategies tailor portfolio shares over the investor's life cycle to the characteristics of her labour income. In several instances, the optimal strategies differ substantially from simple investment rules suggested by pension funds and from popular financial advice, broadly sharing the

[13] A downward sloping age profile for equities, both in raw data and in regression analysis, appears in Bodie and Crane (1997) who investigate the asset allocation behavior across stocks, cash and fixed income. The cross-sectional survey is restricted to TIAA-CREF participants, who are predominantly employees of colleges and universities. On the contrary, the regression coefficient of equity holdings as a share of liquid wealth on age is not statistically different from zero in the large Survey of Consumer Finance (Heaton and Lucas 2000). Ameriks and Zeldes (2004) find that equity ownership of TIAA-CREF participants has a hump-shape pattern with age, while equity shares conditional on participation are nearly constant across age groups. The inclusion of age and cohort effects leads to equity portfolio shares that increase strongly with age. In Guiso, Jappelli and Terlizzese (1996), age has again a hump-shaped effect on risky asset holdings.

common feature of a decreasing age profile of investment in the riskier assets. In order to provide a quantitative assessment of the welfare loss associated with adopting such simpler rules instead of the optimal life-cycle strategy, we consider three alternative asset allocation patterns. The first is an "age rule", whereby the risky portfolio share is set at 100 minus the investor's age and equally allocated between stocks and bonds.[14] The second alternative (denoted as "target-date fund (TDF) rule") is designed to come closer to actual strategic asset allocation patterns adopted by Target-Date Funds. As shown in Figure 10, the stock portfolio share is set at 90% until the age of 40, is gradually decreased over the remaining working life up to 50% at the retirement age (65), and is further reduced in the early retirement period to reach a bottom of 30% at the age of 72. Over the same life span, the share of bonds increases from 10% to 40% at 65 and further up to 45% at 72; finally, the riskless asset is accumulated only in the final stage of the working life, to reach a share of 10% at 65 and 25% at the age of 72.[15] The third alternative strategy fixes portfolio shares at 1/3 for each financial asset in our model: this mirrors the $1/N$ rule of DeMiguel, Garlappi and Uppal (2008), that systematically outperforms several optimal asset allocation strategies in *ex post* portfolio experiments.

The metric used to perform welfare comparisons is the standard consumption-equivalent variation as in Cocco, Gomes and Maenhout (2005) and Winter, Schlafmann and Rodepeter (2012): for each suboptimal asset allocation rule we compute the percentage increase in consumption required by the investor to obtain the same level of expected utility warranted by the optimal life-cycle strategy.[16] Table 1 shows the welfare losses associated with the three

[14] In a two-asset framework, including only a riskless asset and stocks, several variants of the above "age rule" are adopted in the literature. For instance, Cocco, Gomes and Maenhout (2005) consider a rule whereby the portfolio share of stocks is 100% until the age of 40 and decreases linearly thereafter, to reach 50% at the age of 60. In Bodie and Treussard (2007), the investor starts the process of saving for retirement 40 years before the target retirement date, setting the initial share of stocks at 80% and letting it fall to 40% at the target date.

[15] Vanguard (2010) describes a broadly similar age profile for the strategic asset allocation of target-date funds, but with a richer asset class menu including US and international stocks, US nominal investment-grade bonds, Treasury inflation-protected securities and cash.

[16] The consumption-equivalent variation is obtained by simulating consumption and wealth accumulation choices of 10,000 agents following the optimal asset allocation strategy and each of the alternative (suboptimal) investment rules, and deriving the associated expected discounted life-time utility levels. From the average expected discounted utility across individuals, the constant consumption stream needed to compensate investors (in each period and state) for the adoption of suboptimal strategies is computed using the $CRRA$ utility function.

suboptimal asset allocation rules for several combinations of inverstors' risk aversion, background risk, and correlation between innovations to labour income and stock returns.

In the benchmark case of moderate risk aversion ($\gamma = 5$), uncorrelated labour income-stock return innovations ($\rho_{sy} = 0$) and a "normal" level of labour income shock variance, the "age rule" and the 1/3 strategies entail losses in the range of 1-2% of life-time consumption, whereas investors following the TDF rule lose only 0.3%. In this case, individuals following the optimal investment strategy (as shown in Figure 3, left column), after a first period of liquidity-constrained working life, start wealth accumulation with a high share of stocks until the age of 40, and turn gradually to bonds as their retirement date approaches. Among the alternative investment strategies, the TDF rule imposes an age profile of investment in stocks and bonds which is closer to the optimal pattern than the other two alternatives, resulting in a more limited welfare loss. Broadly similar results emerge when the labour income-stock return correlation is set to the slightly higher value of 0.2.

Still considering $\gamma = 5$, a larger amount of background risk, captured by the "high" labour income shock variance, entails more sizeable welfare losses for all alternative investment strategies, the TDF rule now yielding the worst performance with a welfare loss in the 3-6% range for both values of the labour income-stock return correlation. In fact, more background risk increases precautionary savings and wealth accumulation, and determines a quick reduction of the optimal portfolio share invested in stocks, which reaches a bottom level before the age of 40 (see the left columns of Figures 3 and 4), whereas according to the TDF rule the stock share is kept at 90% until that age, and only slowly reduced thereafter.

The worst performance of the TDF investment strategy occurs when a large amount of background risk is combined with a relatively higher degree of risk aversion ($\gamma = 8$), a case in which the optimal investment strategy displays an inverted age profile for stocks, with investors entering the stock market early in life with a modest share that is only slightly increased over time to reach 20% from the age of 25 to retirement, whereas the bond share decreases throughout the entire investor's working life (see Figure 5, right column). Under the alternative TDF rule, the high risk and expected return on her financial portfolio (with a 90% stock share over the first two decades of working life) induce investors to increase savings and wealth accumulation: consumption is therefore substantially lower than optimal over the first half of the working life, determining a sizeable decrease in expected utility that is not compensated by higher than optimal consumption levels over the remaining part of the working life and during retirement. Such excessive saving and wealth accumulation under the TDF rule yield a remarkably large welfare loss, in the range of 17-26% of lifetime consumption, whereas the other suboptimal investment strategies determine more limited welfare losses (1-2%).

Overall, the results of our welfare analysis show that investment strategies that overlook labour income characteristics of pension plan participants may entail substantial losses. In particular, the equally weighted (1/3) portfolio rule performs consistently better than the "age rule", showing lower welfare losses for most parameter combinations.[17] Importantly, the magnitude of welfare losses is never larger than 2% of life-time consumption. In this respect, a $1/N$ strategy challenges the choice of TDF as default investment rule.

4 Conclusions

The persistence of labour income shocks implies that a young person faces large uncertainty concerning future income and social security pension, especially in the presence of a high variance of permanent income shocks. As retirement age approaches such uncertainty resolves, making the worker more willing to take on equity market risk. Permanent shocks to labour income risk are thus able to generate, in conjunction with minor changes in other parameters, optimal equity portfolio shares that increase as retirement approaches and non-participation by young workers in the equity market. They also imply high heterogeneity in portfolio shares conditional on age, as a function of past work histories. Thus the simple life-cycle model with risky bonds is potentially able to account for several empirical regularities that so far appeared at odds with it.

Our analysis also questions the use of a one-size-fits-all default investment strategy for pension funds. A Target Date Fund investment rule, that is close to optimal when labour income risk and risk aversion are relatively low, determines deviations from the optimal life-cycle consumption resulting in large welfare losses for investors with relatively high risk aversion and background risk.

Our model considers workers as being able to know with certainty the parameters characterizing the labour income process, even at the beginning of their career, as well as the process generating financial returns, even forty years in advance. Accounting for parameter uncertainty would reduce the attractiveness of equities relative to other assets as in Barberis (2000), the

[17] For both alternatives, welfare losses fall as risk aversion increases, since high risk aversion implies reduced optimal exposure to the stock market, and risky assets in general.

more so the further away is retirement age. We leave this important extension for future work.

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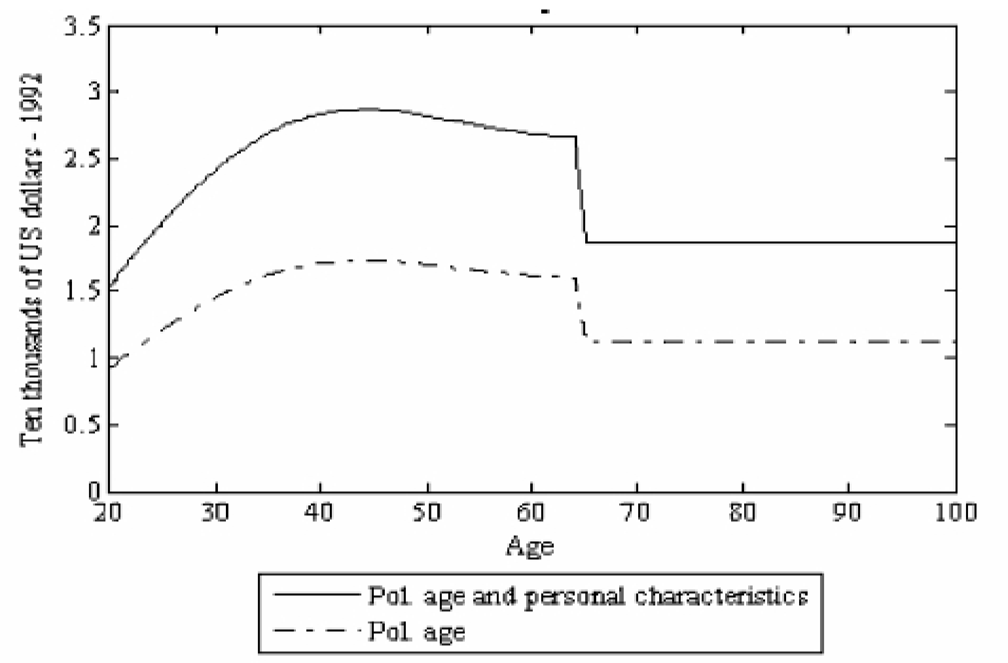
Table 1. Welfare losses from suboptimal life-cycle asset allocation strategies
(percentage of life-time consumption)

	"normal" labour income shock variance			"high" labour income shock variance		
	"age-rule"	"TDF rule"	$1/N$	"age-rule"	"TDF rule"	$1/N$
Risk aversion $\gamma = 5$						
$\rho_{sy} = 0$	1.8	0.3	1.2	2.7	3.4	2.0
$\rho_{sy} = 0.2$	1.3	0.6	0.8	2.2	6.2	1.5
Risk aversion $\gamma = 8$						
$\rho_{sy} = 0$	1.6	1.4	1.1	1.1	17.4	1.2
$\rho_{sy} = 0.2$	1.2	5.4	0.8	1.5	25.9	1.7

Life-time welfare losses are expressed as the percentage increases in the constant consumption level that would ensure the same expected discounted utility as with optimal asset allocation strategies. "Normal" labour income shock variance: $\sigma_{\varepsilon}^2 = 0.0106$ and $\sigma_n^2 = 0.0738$; "high" labour income shock variance: $\sigma_{\varepsilon}^2 = 0.0418$ and $\sigma_n^2 = 0.296$. Other relevant parameters: replacement ratio $\lambda = 0.68$, correlation between stock and bond returns $\rho_{sb} = 0.2$.

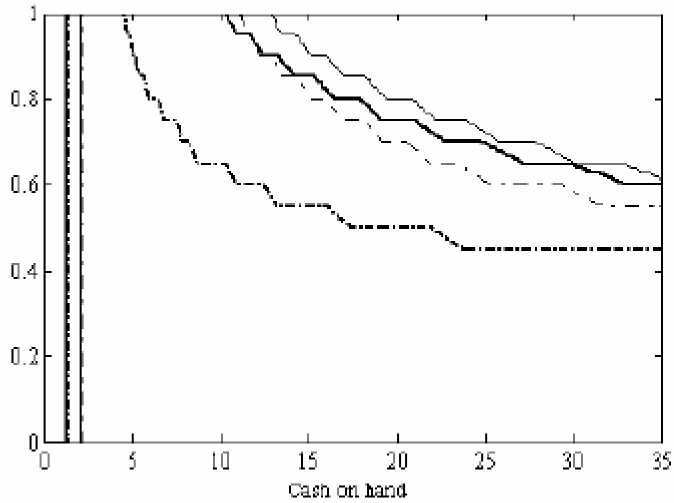
Figure 1. Labour income process



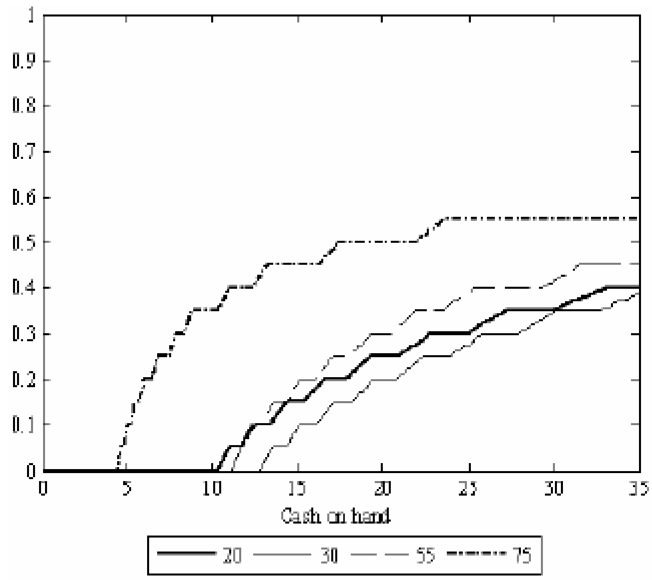
The figure reports the fitted polynomial in age (dash-dotted line) and in age and personal characteristics (continuous line) derived using the calibration in Cocco et al. (2005) for households with high school education.

Figure 2. Optimal policy rules for stocks and bonds in the benchmark case

Stocks

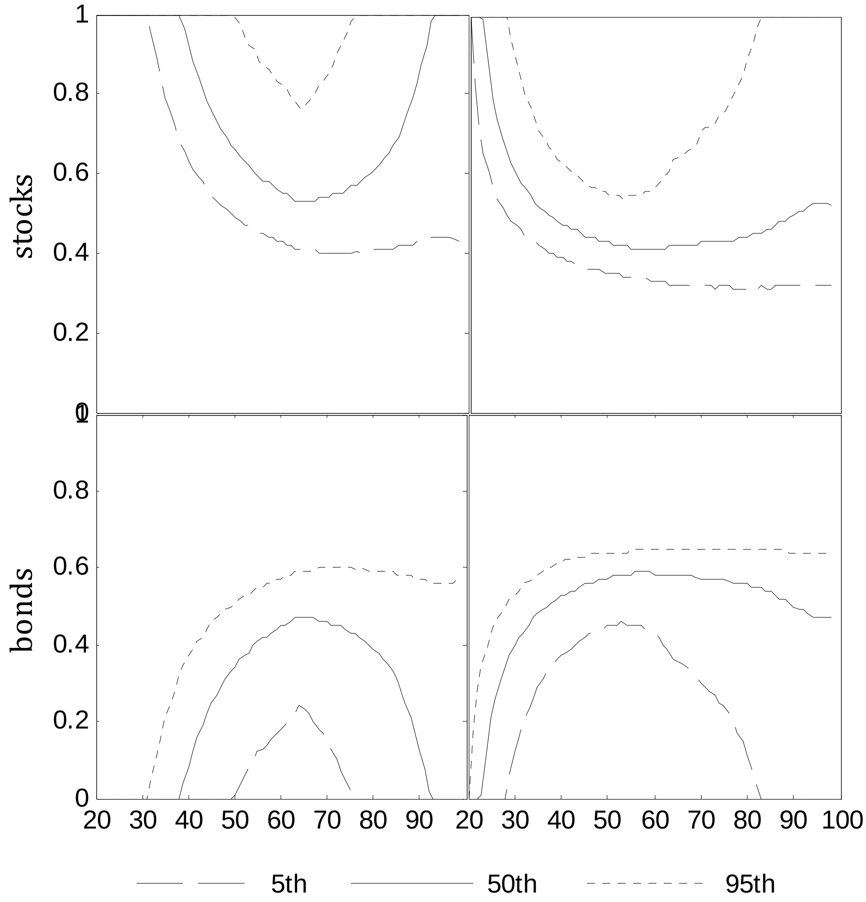


Bonds



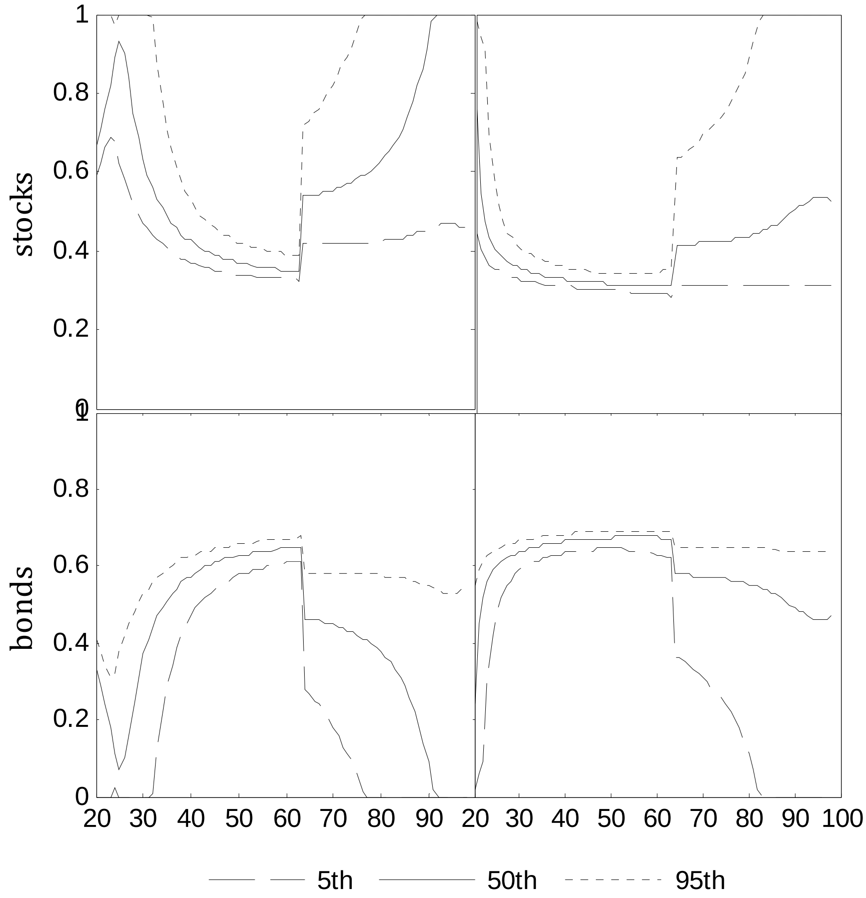
The figure shows the portfolio rules for stocks and bonds as a function of normalized cash on hand for individuals of age 20, 30, 55 and 75.

Figure 3. Portfolio shares of stocks and bonds in the benchmark case



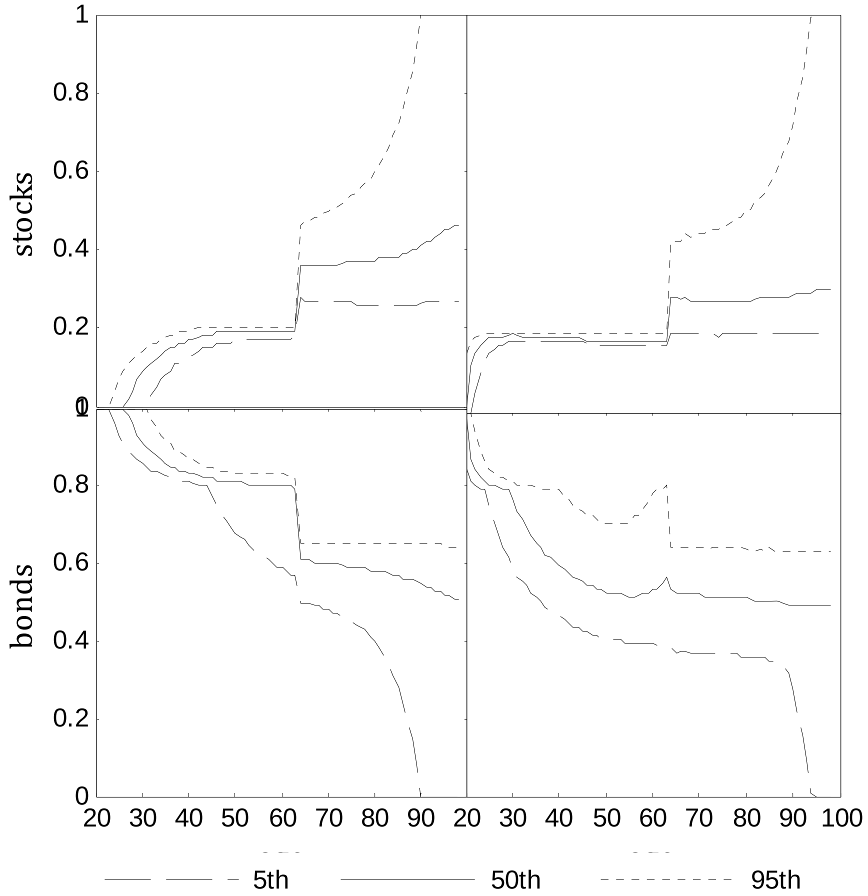
The figure displays the 5th , 50th and 95th percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: "normal" labour income shock variance ($\sigma_\varepsilon^2 =$ 0.0106 and $\sigma_n^2 =$ 0.0738). Right column: "high" labour income shock variance ($\sigma_\varepsilon^2 =$ 0.0418 and $\sigma_n^2 =$ 0.296). Other relevant parameters: risk aversion γ=5, replacement ratio λ=0.68, correlation between shocks to labour income and stock returns ρ_{sY}=0, correlation between stock and bond returns ρ_{sb}=0.2.

Figure 4. Portfolio shares of stocks and bonds with positive labour income-stock returns correlation (ρ_{sY}=0.2)



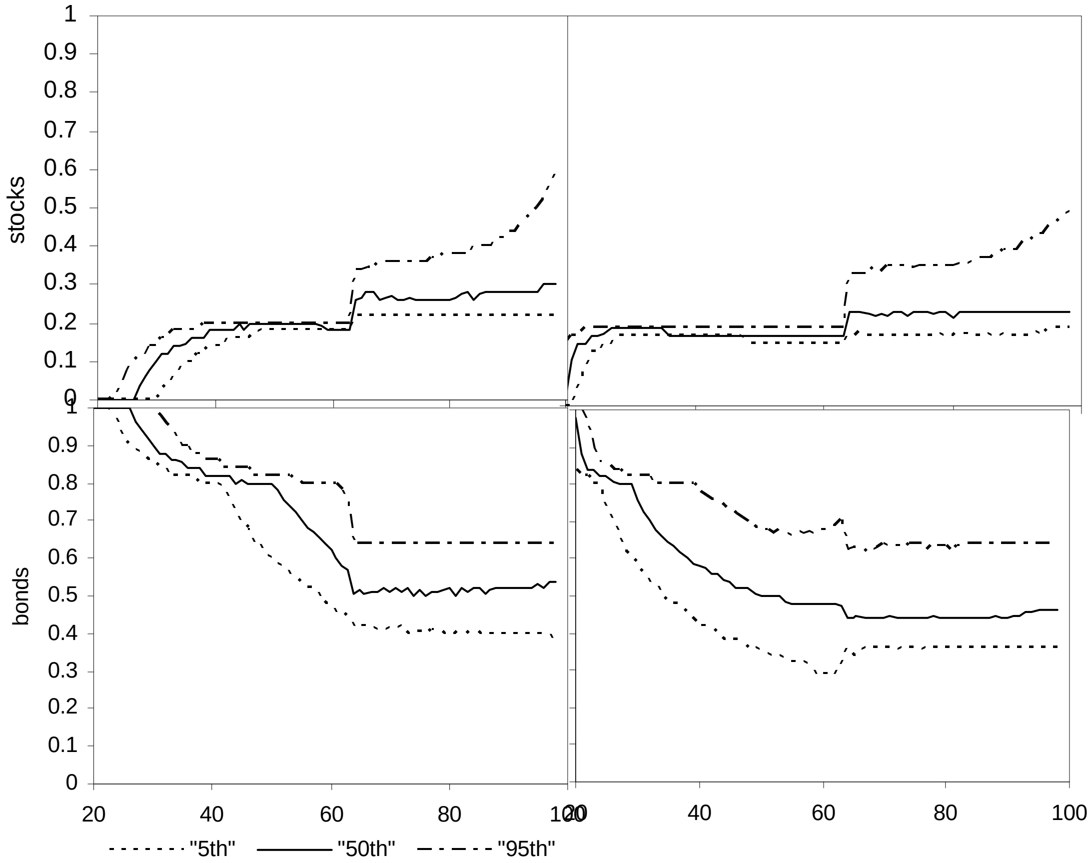
The figure displays the 5th , 50th and 95th percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: "normal" labour income shock variance ($\sigma_\varepsilon{}^2$ = 0.0106 and $\sigma_n{}^2$ = 0.0738). Right column: "high" labour income shock variance ($\sigma_\varepsilon{}^2$ = 0.0418 and $\sigma_n{}^2$ = 0.296). Other relevant parameters: risk aversion γ=5, replacement ratio λ=0.68, correlation between shocks to labour income and stock returns ρ_{sY}=0.2, correlation between stock and bond returns ρ_{sb}=0.2.

Figure 5. Portfolio shares of stocks and bonds with positive labour income-stock returns correlation (ρ_{sY}=0.2) and high risk aversion (γ=8)



The figure reports the 5th , 50th and 95th percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: "normal" labour income shock variance ($\sigma_\varepsilon{}^2$ = 0.0106 and $\sigma_n{}^2$ = 0.0738). Right column: "high" labour income shock variance ($\sigma_\varepsilon{}^2$ = 0.0418 and $\sigma_n{}^2$ = 0.296). Other relevant parameters: risk aversion γ=8, replacement ratio λ=0.68, correlation between shocks to labour income and stock returns ρ_{sY}=0.2, correlation between stock and bond returns ρ_{sb}=0.2.

Figure 6. Portfolio shares of stocks and bonds with positive labour income-stock returns correlation (ρ_{sY}=0.2), high risk aversion (γ=8) and low replacement ratio (λ=0.40)

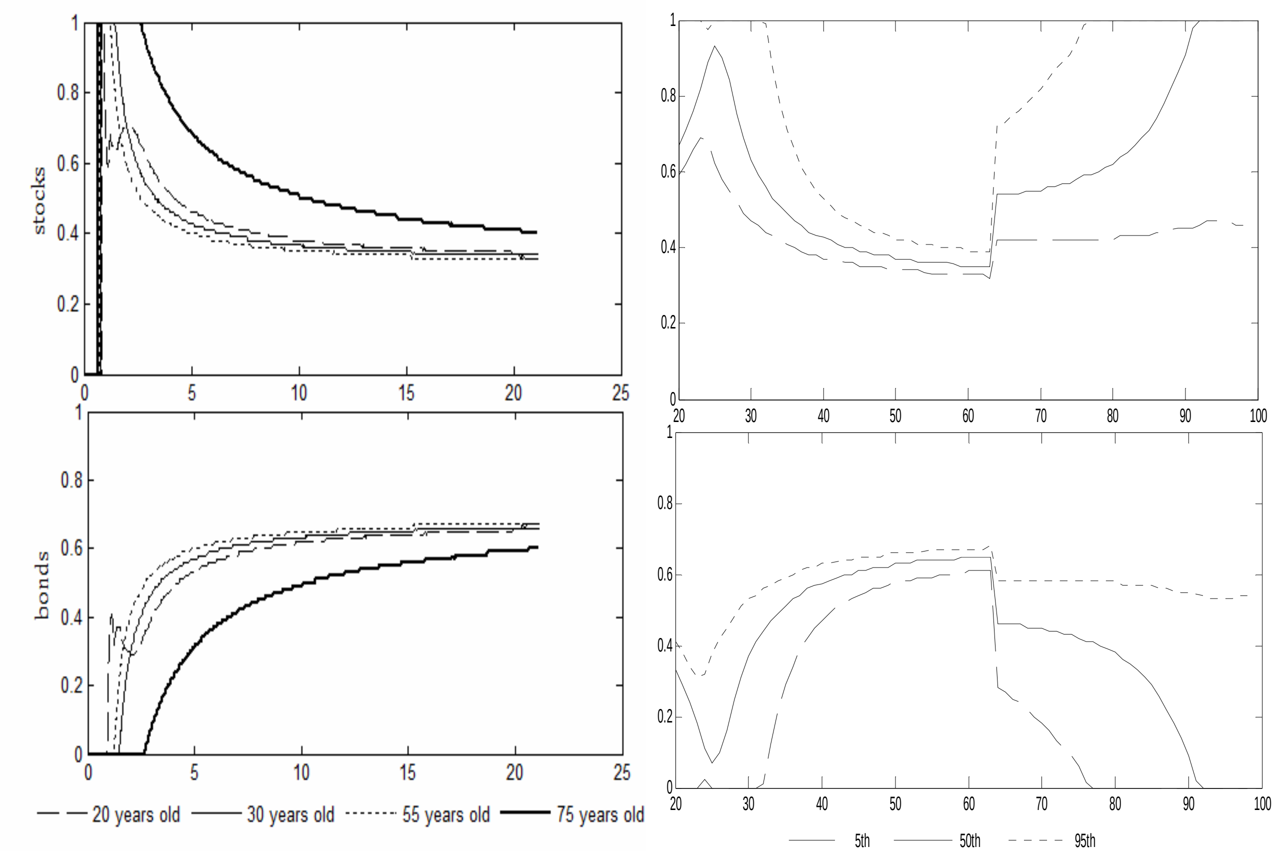


The figure reports the 5th , 50th and 95th percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: "normal" labour income shock variance ($\sigma_\varepsilon{}^2 =$ 0.0106 and $\sigma_n{}^2 =$ 0.0738). Right column: "high" labour income shock variance ($\sigma_\varepsilon{}^2 =$ 0.0418 and $\sigma_n{}^2 =$ 0.296). Other relevant parameters: risk aversion γ=8, replacement ratio λ=0.40, correlation between shocks to labour income and stock returns ρ_{sY}=0.2, correlation between stock and bond returns ρ_{sb}=0.2.

Figure 7. Optimal policy rules and portfolio shares heterogeneity with positive labour income-stock returns correlation (ρ_{sY}=0.2) and moderate risk aversion (γ=5)

(a) “normal” labour income shock variance

Policy rules Portfolio shares



(b) “high” labour income shock variance

Policy rules Portfolio shares

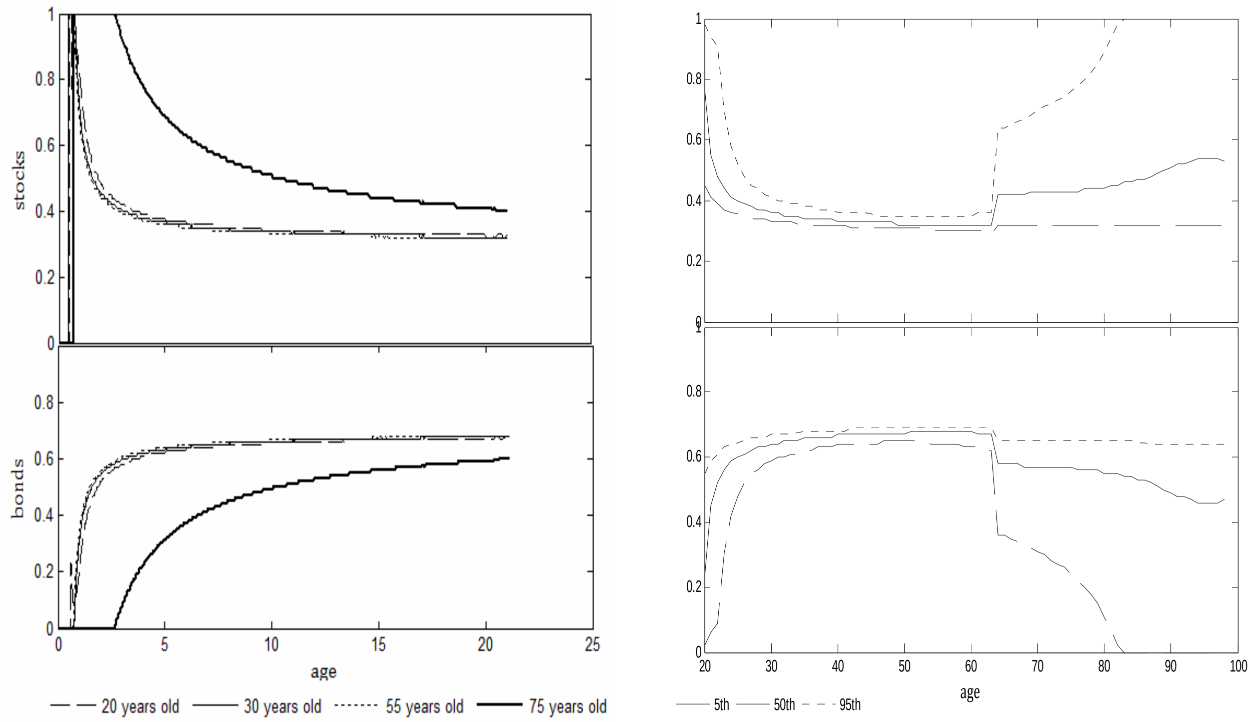
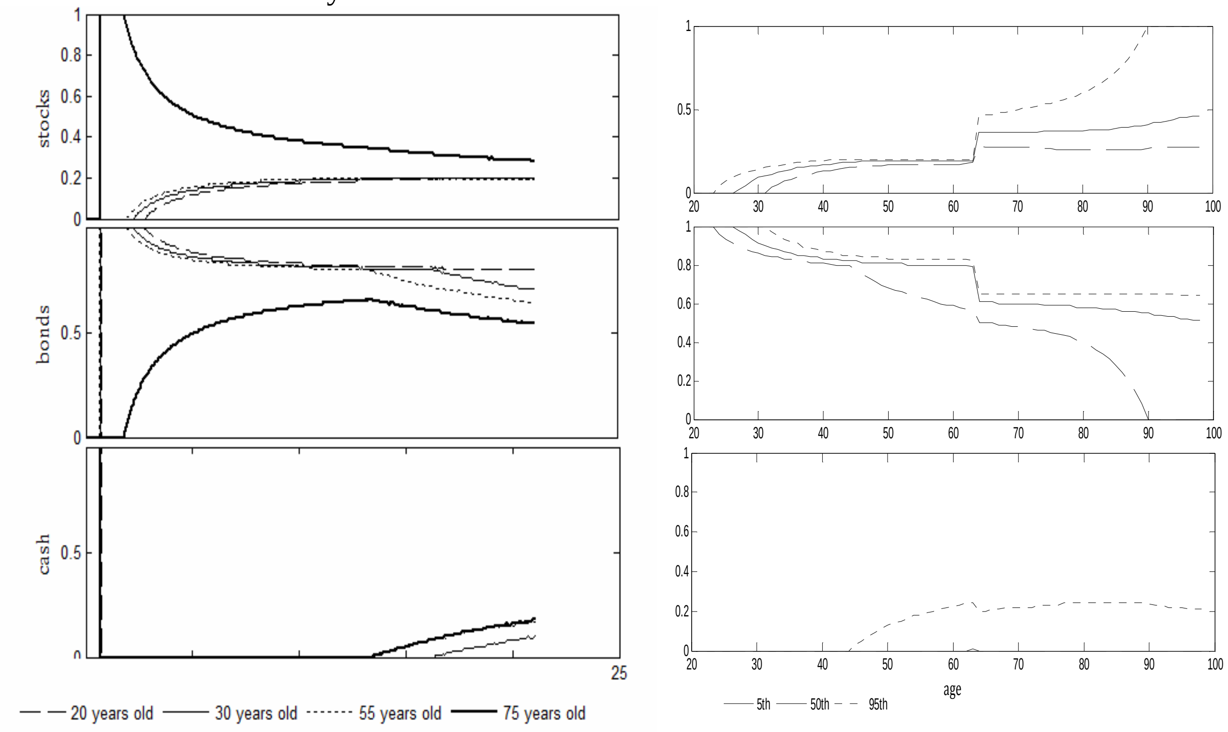


Figure 8. Optimal policy rules and portfolio shares heterogeneity with positive labour income-stock returns correlation (ρ_{sY}=0.2) and high risk aversion (γ=8)

(a) "normal" labour income shock variance

Policy rules | Portfolio shares



(b) "high" labour income shock variance

Policy rules | Portfolio shares

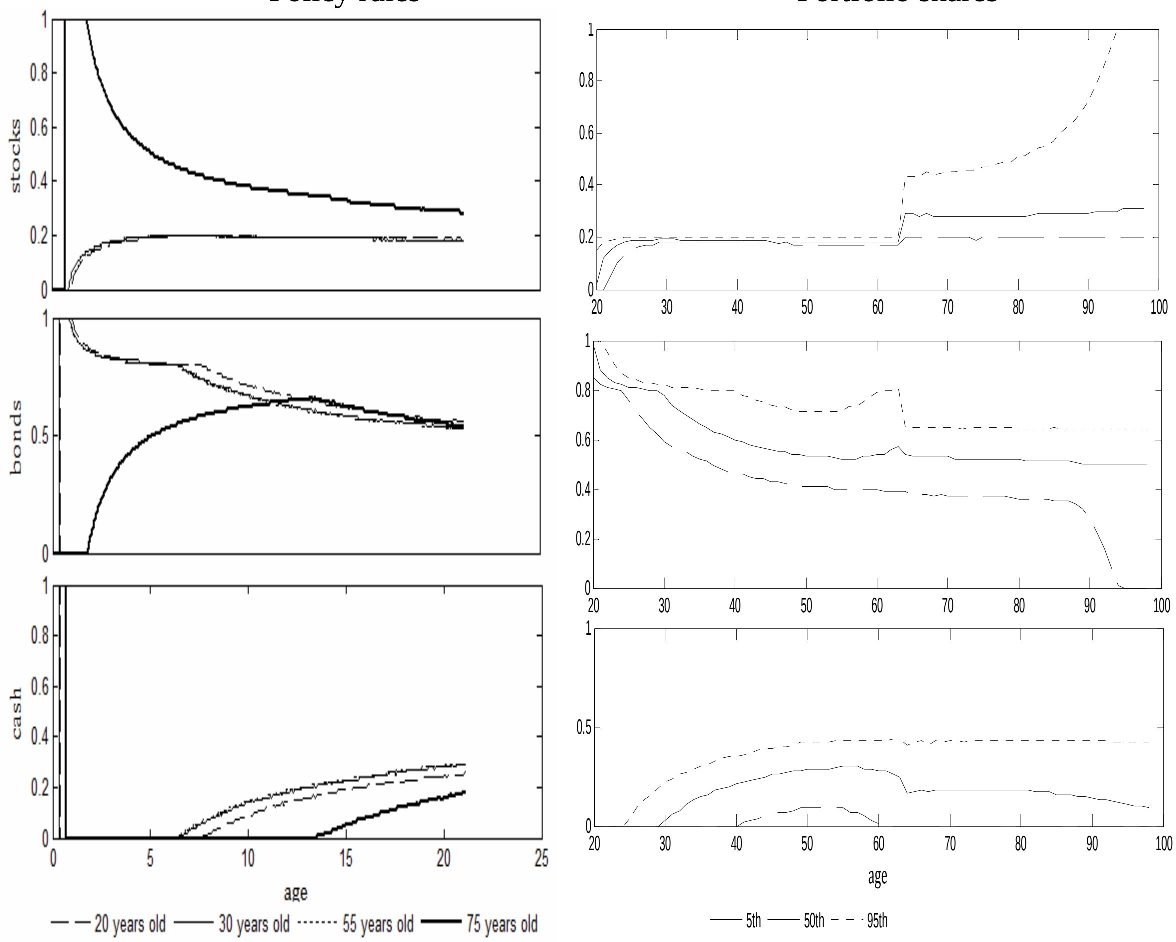


Figure 9. Optimal portfolio shares with positive labour income-stock returns correlation (ρ_{sY}=0.2) and moderate risk aversion (γ=8) in the presence of a bequest motive

(a) “normal” labour income shock variance

(b) “high” labour income shock variance

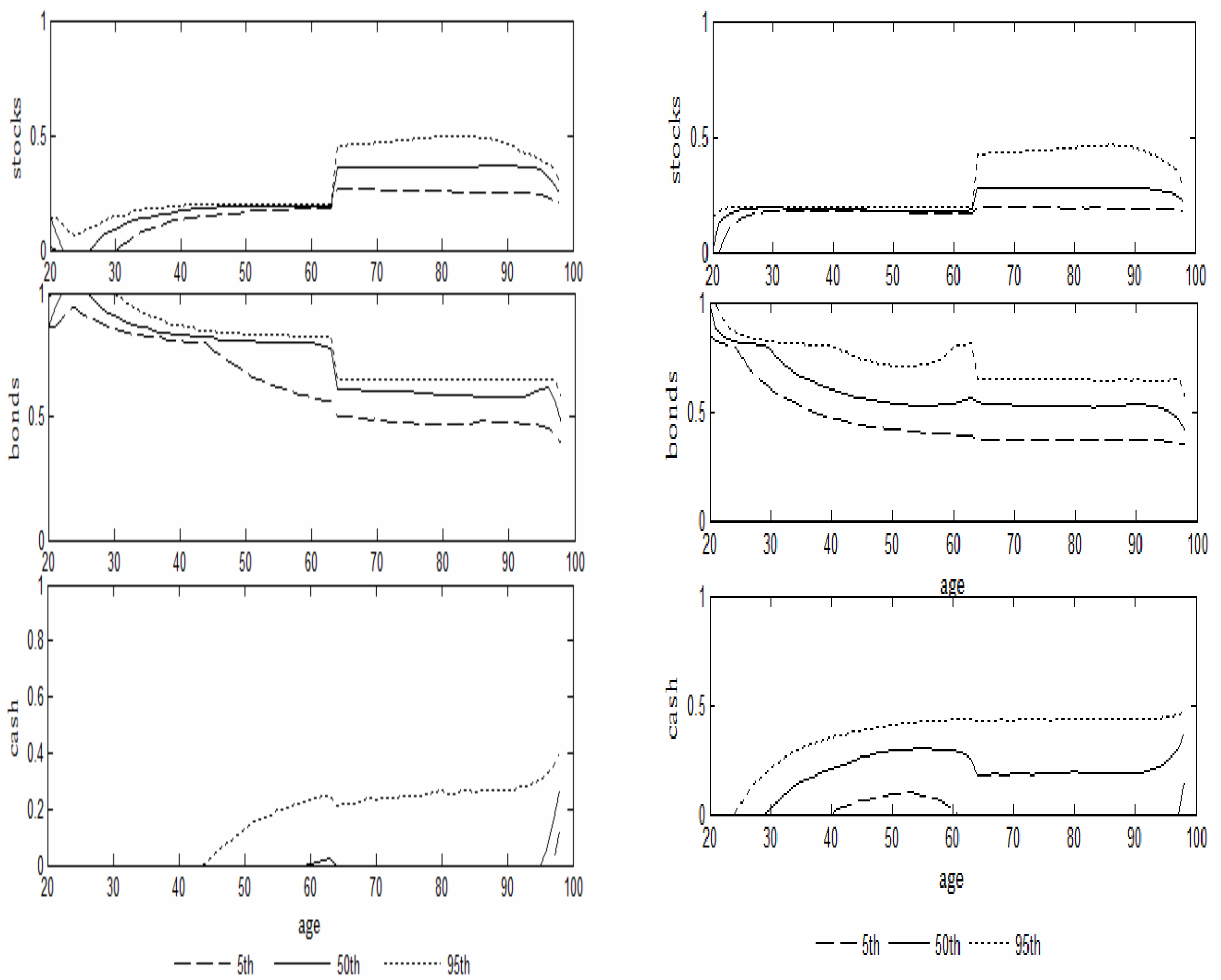


Figure 10. Portfolio shares for stocks, bonds and the riskless asset from a typical Target-Date Fund asset allocation strategy.