A General Equilibrium Exploration of Minsky's Financial Instability Hypothesis*

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Abstract

The worst and longest depressions have tended to occur after periods of prolonged, and reasonably stable, prosperity. This results in part from agents rationally updating their expectations during good times and hence becoming more optimistic about future economic prospects. Investors then increase their leverage and shift their portfolios towards projects that would previously have been considered too risky. So, when a downturn does eventually occur, the financial crisis, and the extent of default, become more severe. Whereas a general appreciation of this syndrome dates back to Minsky [1992, Jerome Levy Economics Institute, WP 74] and even beyond, to Irving Fisher [1933, Econometrica 1, 337-357], we model it formally. Endogenous default introduces a pecuniary externality, since investors do not factor in the impact of their decision to take risk and default on the borrowing cost. We explore the relative advantages of alternative regulations in reducing financial fragility, and suggest a novel criterion for improvement of aggregate welfare.

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1 Introduction

The second theorem of Minsky's Financial Instability Hypothesis, (Minsky (1992)), states that over periods of prolonged prosperity and optimism about future economic prospects, financial institutions invest more in riskier assets, which can make the economic system more vulnerable in the case that default materializes.

In this paper, we focus on the role that expectations formation about future states of the economy -in the sense of investment profitability and growth- plays in the borrowing decision of investors, in their portfolio choice, and eventually in the extent of default in the economy.[1] We thus examine the effect of leverage, as a path-dependent process, on financial stability, by linking learning to risk-taking behaviour. In particular, we consider investors that face a multiperiod portfolio problem. In each period, they use their own capital, augmented by accumulated profits, in combination with short-term borrowing to invest in projects, which mature in one period as well. The absence of any maturity mismatch is unimportant for our results, as we shall be abstracting from any fire sales externality, which should be regarded complementary as to our modeling. Credit is raised from a competitive credit market. Given endogenous default, the borrowing rate, which depositors will require, will be endogenously determined by expectations of future repayment on loans. This will depend on depositors' information and understanding of the portfolio choices of investors.

We assume two types of projects and two states of the world that can materialise in every period, one of which is good, whereas the other is bad. Although both projects are risky, their payoffs differ and one is riskier that the other under any probability distribution, which assigns positive probability to both states. We assume that the outcomes are perfectly correlated in the sense that both do well in the good state and poorly in the bad state, and that both projects are in perfectly elastic supply, thus abstracting from the general equilibrium effects of projects' origination and supply.[2] Investors choose their portfolio of projects, at each point in time, according to their expectations about the

[1] We refer to the agents in our model as investors, which can be any financial institution or asset management company in the broad sense, engaging into borrowing in order to invest in projects/assets

[2] Endogenising project supply can allow the study of asset price bubbles together with variations in the leverage cycle. Adam and Marcet (2010) and Branch and Evans (2011) use a different learning model to explain bubbles and crashes in asset prices. We leave this interesting extension for future work.

future realization of payoffs and the borrowing rates that they may face. Projects' payoffs do not change over time. What changes is the perceived belief about the likelihood of good realizations. Following Cogley and Sargent (2008), we assume that agents have incomplete information about the true probability measure. They learn it over time by observing the past realizations and updating their priors.[3] Although their beliefs will converge to the true probability measure in the limit, they will fluctuate in finite time, resulting in fluctuations in portfolios and leverage. We show that financial institutions will start investing in the riskier project after a number of good past realizations, since their expectations are boosted and the risk/return profile from one's individual perspective improves thereby.

Our second contribution to an otherwise canonical portfolio problem is the modeling of endogenous default and of a separate market for credit. The interest rate charged on loans depends on the expectations about future repayment. Creditors hold beliefs about the debtors' portfolios and update their expectations about the future realization of payoffs and subsequent defaults, subject to their information set. We thus connect the credit spread to risk taking via the introduction of endogenous default. Low credit spreads allow investors to borrow more. However, if they keep increasing their risk taking, the credit spreads will increase. Risk taking is penalized ex-post via penalties for default, and ex-ante via higher borrowing rates. Nevertheless, expectations, in our framework, vary over time and optimism can build after periods of good news. Riskier projects become more attractive for investors, since the expected penalty for default decreases. Moreover, expectations about the possibility of default go down and so creditors are willing to offer low borrowing rates even though debtors invest more in the riskier project.

In a sense, there always going to be cycles of optimism and pessimism, both in finance and elsewhere, and there is not much that we can, or perhaps should, do to stop that. But there are several reasons why banking and finance involve externalities that cause particular amplification to a potential debt-deflation spiral. The pioneer of such a view was Fisher (1933), who suggested a Debt-Deflation theory of Great Depressions. His analysis was based on two fundamental principles,

[3] One might ask why enough time has not elapsed since the innovation of fractional banking to allow the learning process to converge to the true probability measure. The answer is that the learning process is itself flawed. As Reinhart and Rogoff (2008) show, the young think that 'This time it is different', and the old have retired and disappeared.

over-indebtedness and deflation. He argued that over-indebtedness can result in deflation in future periods and that can cause liquidation of collateralised debt. This theory brings financial intermediation to the center of attention. A number of papers have built on the debt-deflation theory of financial amplification to analyse the effect of collateral constraints on borrowing, production and eventually financial stability. In a seminal paper, Bernanke and Gertler (1989) modeled a collateral-driven credit constraint, which introduced an external finance premium, and analysed interactions between the balance sheet of financial institutions and the real economy. Debt deflation dynamics may also give rise to an important externality, that of fire sales -to institutions that can, at times, realize less value from such assets- which can act as an amplification mechanism in financial crises. Financially distressed institutions liquidate their assets to meet their debt obligations, and in doing so, they reduce the value of their own and other institutions' portfolios, which exacerbates the fire sale discounts, and worsens further their debt position.

We do not pursue such an analysis here, partly because it has been so thoroughly examined elsewhere.[4] Indeed, there are other approaches to financial crises that we do not cover. In particular, we do not model bank run externalities arising from coordination problems (Diamond and Dybvig (1983)), network externalities (Bhattacharya and Gale (1987)) , Rochet and Tirole (1996)), or market freezes due to portfolio opaqueness and unduly pessimistic beliefs (Stiglitz and Weiss (1981), Dubey et al. (2005)). Instead, the externality that we do address is that investors do not incorporate the impact of their portfolio and default decisions on the borrowing rates, since they are price takers in the credit market. By investing more in the riskier assets, investors take on more downside risk. Thus, they likely to default more, and as a result their borrowing rates go up, which makes them default even more, and so on. A social planner understands the impact of risk-taking on interest rates and the deadweight loss associated with default, and does not switch to the riskier asset as fast

[4]Mendoza (2006) and Mendoza and Smith (2006) analyse the role of fire sales in Sudden Stops in emerging markets within a Real Business Cycle model. Geanakoplos (2003) and Fostel and Geanakoplos (2008) show how the arrival of bad news about the future economic prospects results in a reduction in the price of assets used as collateral and leads to a drying up of liquidity and fire-sales externalities. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) show how the borrowing capacity of agents, i.e. funding liquidity, and the pricing of assets, i.e. market liquidity, interact and how an idiosyncratic liquidity shock can lead to fire-sales and the unraveling of the whole market. Other papers, which model fire-sales due to adverse productivity or funding shocks to capture debt-deflationary effects on asset prices, leading to loss spirals and financial instability, include Shleifer and Vishny (1992), Kiyotaki and Moore (1997), Kyle and Xiong (2001), Morris and Shin (2004), Acharya et al. (2011), Diamond and Rajan (2011). Adrian and Shin (2009) examine the importance of this channel empirically for financial institutions.

as individual investors do after a series of good realizations, which invoke higher optimism.

We consider three policy responses to tackle the issue of excessive risk-taking accompanied by high leverage; stricter penalties for default, lower leverage ratios, and finally our novel criterion, which is to limit the ratio of riskier minus safer portfolio holdings over total borrowing. Only the latter is successful in both reducing the amount of default and in increasing aggregate welfare, as it addresses risk taking during optimistic times directly. Compared to the use of a crude leverage ratio, our suggestion regarding optimal regulation resembles a combination of leverage ratios on all assets, which is responsive to their relative riskiness in the cross-section. Hence, it is close to the regulatory proposal of Geanakoplos (2010), to introduce higher margin/haircut requirements on bank generated asset holdings during good times. We discuss the difference in more detail in section 5.

Finally, we discuss some empirical implications of our paper. We argue that the underlying reason why commonly used return-based risk measures, such as VIX or the TED spread[5], failed to capture the build-up of risk before the 2007-2008 crisis is that they were biased by optimistic expectations, as are the credit spreads in our model. Boz and Mendoza (2010) consider a learning model in which agents update their expectations about the leverage constraints they will face, as exogenous multiples of asset values which will prevail in the future. They examine the interaction between their borrowing constraints and the mispricing of risk. A sequence of periods characterised by lax borrowing constraints induces optimistic expectations about the continuation of such regimes, and leads to the underpricing of risk, high leverage, and over inflated collateral values. A sharp collapse then follows after the realization (exogenously) of a tighter constraint. We believe, in contrast, that our quantity-based measure, capturing the shift in portfolios holdings towards riskier projects in optimistic times, is likely to be more effective in conceptualizing endogenous credit cycles, assuming that projects' relative riskiness can still be correctly evaluated when expectations change, although each of them may look safer.

[5]VIX is the CBOE Volatility Index, created by the Chicago Board Options Exchange as a measure of equity market volatility. The computation of the value of VIX is based on the implied volatility of eight option series on the S&P 100 index. The TED spread is the difference between the interest rates on interbank loans and on short-term U.S. government debt.

The rest of the paper proceeds as follows. Section 2 presents the model. In section 3 we present an analytical solution which explores the implications for divergences across private and socially optimal risk-taking. In section 4, we numerically examine and extend our arguments. In section 5, we compare the relative quantitative performance of alternative regulatory regimes. Section 6 discusses the empirical implications and concludes.

2 The Model

Consider a multi-period economy with investors $i \in I$. At any date $t = 0, \ldots, T$, the economy can be in one of two states, denoted by u ("up"/good state) and d ("down"/bad state) respectively. For example, the "up" state at time t is denoted by $s_t = s_{t-1}u$. The set of all states is $s_t \in S = \{0, u, d, \ldots, uu, ud, du, dd, \ldots, s_t u, s_t d, \ldots\}$. The probability that a good state occurs at any point in time is denoted by θ, which is chosen by nature. For simplicity we assume that $\theta \in \{\theta_1, \theta_2\}$ with $1 > \theta_1 > \theta_2 > 0$. However, agents do not know this probability and try to infer it by observing past realizations of good and bad states. Agents have priors $\Pr(\theta = \theta_1)$ and $Pr(\theta = \theta_2)$ that the true probability is θ_1 or θ_2 respectively. Their subjective belief in state s_t of a good state occurring at t+1 is denoted by π_{s_t} and that of the bad $1 - \pi_{s_t}$. These probabilities depend on the whole history of realizations up to t. In other words, $\pi_{s_t} = Pr_{s_t}(s_{t+1} = s_t u | s_0, \ldots, s_t)$. Given our notation, state s_t completely summarizes the history of realizations up to t. Thus, $\pi_{s_t} = Pr_t(s_{t+1} = s_t u | s_0, \ldots, s_t) = Pr_{s_t}(s_{t+1} = s_t u | s_t)$. We assume that past realizations of the states of the world are observable by all agents, thus there is no information asymmetry on top of the imperfect information structure.

Consequently, agents' subjective belief is $\pi_{s_t} = Pr_{s_t}(\theta = \theta_1 | s_t) \cdot \theta_1 + Pr_{s_t}(\theta = \theta_2 | s_t) \cdot \theta_2$. Agents are Bayesian updaters and try to learn from past realizations the true probability θ. Their conditional probability given past realizations is:

$$\begin{aligned}
Pr_t(\theta=\theta_1\,|s_t\,) &= \frac{Pr_t(s_t\,|\theta=\theta_1\,)\cdot Pr(\theta=\theta_1)}{Pr(s_t)} \\
&= \frac{Pr_t(s_t\,|\theta=\theta_1\,)\cdot Pr(\theta=\theta_1)}{Pr_t(s_t\,|\theta=\theta_1\,)\cdot Pr_t(\theta=\theta_1)+Pr_t(s_t\,|\theta=\theta_2\,)\cdot Pr(\theta=\theta_2)} \\
&= \frac{\theta_1^n(1-\theta_1)^{t-n}\cdot Pr(\theta=\theta_1)}{\theta_1^n(1-\theta_1)^{t-n}\cdot Pr(\theta=\theta_1)+\theta_2^n(1-\theta_2)^{t-n}\cdot Pr(\theta=\theta_2)},
\end{aligned}$$

where n is the number of good realization up to time t. Then,

$$\begin{aligned}
\pi_{s_t} = & \frac{\theta_1^n(1-\theta_1)^{t-n}\cdot Pr(\theta=\theta_1)}{\theta_1^n(1-\theta_1)^{t-n}\cdot Pr(\theta=\theta_1)+\theta_2^n(1-\theta_2)^{t-n}\cdot Pr(\theta=\theta_2)}\theta_1 \\
& + \frac{\theta_2^n(1-\theta_2)^{t-n}\cdot Pr(\theta=\theta_2)}{\theta_1^n(1-\theta_1)^{t-n}\cdot Pr(\theta=\theta_1)+\theta_2^n(1-\theta_2)^{t-n}\cdot Pr(\theta=\theta_2)}\theta_2. \qquad (1)
\end{aligned}$$

As the number of good realizations increases, the subjective probability of the good state realizing in the following period increases as well, i.e. given that $s_t = s_{t-1}u$, then $\pi_{s_t} > \pi_{s_{t-1}}$. Assume that the priors are the same, that is $Pr(\theta=\theta_1) = Pr(\theta=\theta_2)$.

To prove our claim that agents become more optimistic after they observe good outcomes in the past, we just need to show that $Pr_{s_t}(\theta=\theta_1\,|s_t\,) > Pr_{s_{t-1}}(\theta=\theta_1\,|s_{t-1}\,)$ and $Pr_{s_t}(\theta=\theta_2\,|s_t\,) < Pr_{s_{t-1}}(\theta=\theta_2\,|s_{t-1}\,)$ given that $s_t = s_{t-1}u$.

Proof.

$$\begin{aligned}
& Pr_{s_t}(\theta=\theta_1\,|s_t\,) > Pr_{s_{t-1}}(\theta=\theta_1\,|s_{t-1}\,) \\
& \Rightarrow \frac{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)}}{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)}+\theta_2^{n+1}(1-\theta_2)^{t+1-(n+1)}} > \frac{\theta_1^n(1-\theta_1)^{t-n}}{\theta_1^n(1-\theta_1)^{t-n}+\theta_2^n(1-\theta_2)^{t-n}} \\
& \Rightarrow \left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^n \left(\frac{1-\theta_2}{1-\theta_1}\right)^t > \left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^{n+1} \left(\frac{1-\theta_2}{1-\theta_1}\right)^{t+1} \\
& \Rightarrow 1 > \frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\frac{1-\theta_2}{1-\theta_1} \Rightarrow 1 > \frac{\theta_2}{\theta_1}.
\end{aligned}$$

□

At each date $t \in \{0, 1, \ldots, T-1\}$ investor i faces two investment opportunities; a safer project, denoted by L (standing for "*low*" risk), and a riskier one, denoted by H (standing for "*high*" risk). Both projects are in perfectly elastic supply, with their prices normalized to 1, and expire in one period. The safer project yields a payoff X_u^L in the good state and X_d^L in the bad. Equivalently, the payoffs for the riskier project are X_u^H and X_d^H. We assume that $X_u^H > X_u^L > 1 > X_d^L > X_d^H > 0$, such that the riskier project is more profitable if the good state realizes. Ex-post payoffs are independent of the history of past realizations.

Each investor $i \in I$ has the following payoff/utility function in state s_t:

$$\tilde{U}_{s_t}^i = \tilde{\Pi}_{s_t}^i - \gamma^i \cdot \left(\tilde{\Pi}_{s_t}^i\right)^2, \tag{2}$$

where γ^i is the risk aversion coefficient of i and $\tilde{\Pi}_{s_t}^i$ are the distributed profits in s_t.

The amount of funds available for investment by investors is equal to their equity capital, plus funds borrowed from credit markets, plus the profits from the previous period's investment that are not distributed as profits and consumed. We consider a general portfolio problem under which investors decide how much of the available funds to invest in the safer project and how much in the riskier one. We denote by $w_{s_t,j}^i$ the portfolio holdings of investor i in project $j \in \{L,H\}$ at s_t. For example, the riskier project's holdings in the second period after a good state realization at $t = 1$ are denoted by $w_{u,H}^i$. The interest rate for borrowing from the credit market is denoted by r_{s_t} at s_t.

We allow for default in the credit market. The amount repaid is an endogenous decision by investor i, who weighs the benefits from defaulting against a deadweight loss. The latter is assumed to be a linear function of the amount that the investor chooses not to deliver.[6] Denoting by $1 - v_{s_t}^i$ the percentage default on one unit of borrowed funds, the deadweight loss is equal to $\lambda_{s_t}(1 - v_{s_t}^i)(1 + r_{s_{t-1}})$, where λ_{s_t} is the (potentially) state-contingent default penalty, $v_{s_t}^i$ the percentage repayment on one unit of owned debt and $r_{s_{t-1}}$ the interest rate set at the node preceding state s_t. We assume risk-neutral creditors who break even in expectation and their valuation of the debt is independent of the position they take. Thus, the interest rate will be inversely related to their expectation about future

[6] In the event of default, investors can extract a private benefit, which is pinned down by the exogenously set non-pecuniary default penalty, given the linearity of the disutility of default. As the marginal penalty for default increases, the private benefit investors can extract decreases and they have a lower incentive to default. Shubik and Wilson (1977) and Dubey et al. (2005) are canonical models of such choice processes.

percentage delivery. The amount of funds that investor i chooses to borrow is denoted by $w^i_{s_t}$ and his initial capital at $t = 0$ by $\bar{w}^i_0$. Without loss of generality, investor i is not endowed with additional capital for $t = 1, \ldots, T$.

Geanakoplos (2003) develops a theory of the leverage cycle where he considers a General Equilibrium model with collateralised borrowing and agents who differ in their beliefs about the realization of uncertainty. Bad news result in wealth losses for the most optimistic agents in the economy, who are the natural buyers of the long-term assets, and as a result their valuation is driven down by the more pessimistic agents who would rather sell than buy the assets. A fire-sale externality, in the broad sense, leads to lower leverage, since the assets are pledged as collateral for borrowing, and their price drops even further creating a funding/market liquidity spiral. Herein, we follow an alternative approach to modeling default using non-pecuniary default penalties. The reason is that we focus on the incentives of investors for risk-taking as expectations improve. This approach of modeling default enables the examination of leverage at the level of the investor/institution, rather than on the level of specific assets in the portfolio. Hence, we abstract from margins, spirals and fire-sales, which have been extensively studied in the literature (for example, Gromb and Vayanos (2002), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009)). Including such issues should result in even more severe crashes.

We now turn to the formal representation of the investor's i optimization problem. Investor i tries to maximize his lifetime expected utility by choosing the amount he invests in the safer and riskier projects at each point in time $\left(w^i_{s_t,j},\ \mathrm{j} \in \{\mathrm{L}, \mathrm{H}\}\right)$, the amount that he borrows from the credit market $\left(w^i_{s_t}\right)$, the percentage repayment on past loans $\left(v^i_{s_t}\right)$, and the amount of realized profits that he reinvests $\left(T^i_{s_t}\right)$, i.e.,

$$\max_{w^i_{s_t,j}, v^i_{s_{t+1}}, \Pi^i_{s_{t+1}}, T^i_{s_{t+1}}} \sum_t \mathbb{E}_{s_t} \left[\tilde{U}^i_{s_{t+1}} - \lambda_{s_{t+1}} \max\left[(1 - \tilde{v}^i_{s_{t+1}}) \tilde{w}_{s_t} (1 + \tilde{r}_{s_t}), 0 \right] \right],$$

where $\mathbb{E}_{s_t}$ is the expectations operator in state s_t, under the probability measure π_{s_t}, when the investment decision is made, and $\tilde{U}^i_{s_{t+1}}$ is given by equation 2. Note that investors do not derive any utility at $t = 0$, thus all the funds go to investment, i.e., $w^i_{0,L} + w^i_{0,H} \leq f\left(T^i_0\right) + w^i_0$ and $f\left(T^i_0\right) = \bar{w}^i_0$.

$f(\cdot)$ is a concave function which captures a cost of retaining high levels of realized earnings and guarantees boundedness of the solution for probabilities for the good state approaching one. Every investor i optimizes the payoff function above subject to the following budget constraints:

$$\Pi^i_{s_{t+1}} + T^i_{s_{t+1}} \leq w^i_{s_t,L} X^L_{s_{t+1}} + w^i_{s_t,H} X^H_{s_{t+1}} - w^i_{s_t} v^i_{s_{t+1}} (1 + r_{s_t})$$

i.e., distributed + retained profits $\leq$ safer and riskier investments' payoff - loan repayment in s_{t+1}.

$$w^i_{s_{t+1},L} + w^i_{s_{t+1},H} \leq f\left(T^i_{s_{t+1}}\right) + w^i_{s_{t+1}}$$

i.e., investment in the safer and the riskier projects $\leq$ reinvested profits + leverage in s_t.

The second type of agents in our economy are the suppliers of credit, $c \in C$, who want to maximize expected utility as well. At every time t, they are endowed with capital and they face the decision how much to lend and how much to consume. For simplicity we assume that they are risk-neutral, which means that the interest rate they are willing to accept depends only on the underlying expected risk and not on the level of credit they extend. Creditors have the same expectations as investors. Their consumption in state s_t is denoted by $c^c_{s_t}$, whereas the credit extension by $w^c_{s_t}$ and the capital endowment by $\bar{w}^c_{s_t}$. They face the following optimisation problem:

$$\max_{c^c_{s_t}, w^c_{s_t}} \sum_t \mathbb{E}_{s_t} \tilde{c}^c_{s_t}$$

$$s.t.\ c^c_0 \leq \bar{w}^c_0 - w^c_0$$

i.e., consumption $\leq$ initial endowment - credit extension at t=0.

$$c^c_{s_t} \leq \bar{w}^c_{s_t} + v^i_{s_t}(1 + r_{s_{t-1}}) w^c_{s_{t-1}} - w^c_{s_t}$$

i.e., consumption $\leq$ endowment + loan repayment - credit extension in s_t.

Optimizing with respect to credit extension, we get the following expression that connects the interest rate with the expected delivery on the loan.

$$\mathbb{E}_{s_t}\left[v^i_{s_{t+1}}\right] \cdot (1 + r_{s_t}) = 1 \tag{3}$$

For example, $1 + r_u = \dfrac{1}{\pi_u v^i_{uu} + (1 - \pi_u) v^i_{ud}}$. One can observe the reverse relationship between the interest rate and expected percentage delivery. When the latter increases, the interest rate charged falls. This provides some intuition for the seemingly counterintuitive result that when expectations are optimistic, investors increase their leverage, paying lower than otherwise expected interest rates, though at the end their percentage repayment is lower in a bad state and default is higher. The result follows from the fact that the perceived probability that a good state realizes is higher, since expectations are optimistic. Thus, overall expected delivery is higher, though loss given default is higher as well.

Equilibrium is reached when creditors and investors optimize given their constraints and the credit and projects' markets clear. Interest rates are determined endogenously by equation 3 and are taken as fixed by agents. Credit market clear when supply of credit $\sum_c w^c_{s_t}$ is equal to the aggregate demand $\sum_i w^i_{s_t}$. Condition 3 is necessary for credit markets to clear. The above modeling has assumed a perfectly elastic supply of projects. Equilibrium purchases are determined by investors' demand at a given price of 1 for each project. The analysis of equilibrium and our main result that leverage, investment in the riskier project and realised default all increase when expectations become more optimistic would not have changed had we assumed an upward sloping supply curve. One can find endowments of projects that support the price of 1 in equilibrium. Thus, endogenising asset prices as well would have allowed the joint investigation of changes in leverage and asset prices during a boom or a bust. We consider this an interesting extension for future work.

The variables determined in equilibrium and taken by agents as fixed are, thus, given by $\eta = \{r_0, r_u, r_d\}$. The choices by agents i and c are given by $\square^i = \left\{w^i_{s_t,j}, w^i_{s_t}, v^i_{s_{t+1}}, \Pi^i_{s_{t+1}}, T^i_{s_{t+1}}\right\}$ and $\square^c = \left\{c^c_{s_t}, w^c_{s_t}\right\}$, respectively. We say that $\left(\eta, (\square^i)_{i\in I}, (\square^c)_{c\in C}\right)$ is an equilibrium of the economy $\mathbb{E} = \left(\left(c^i, \omega^i, \bar{w}^i\right)_{i\in I}, \left(\bar{w}^c\right)_{c\in C}; \lambda, \theta_1, \theta_2\right)$ if and only if:

i. $(\square^i) \in Argmax_{\square^i \in B(\eta)} \ \mathbb{E}_{s_t}\tilde{U}^i$

ii. $(\square^c) \in Argmax_{\square^c \in B(\eta)} \ \mathbb{E}_{s_t}\tilde{U}^c$

iii. $\mathbb{E}_{s_t}\left[v^i_{s_{t+1}}\right] \cdot (1 + r_{s_t}) = 1$

iv. $\sum_c w^c_{s_t} = \sum_i w^i_{s_t}$

v. Total demand for project j, $\sum_i w_{s_t,j}$, is equal to the supply of projects, which is trivially satisfied due to perfectly elastic supply

vi. Creditors expectations are rational, i.e. they anticipate correctly the delivery $v^i_{s_t}$ by $i \in I$

Conditions (i) and (ii) state that all agents optimize; (iii) and (iv) say that credit markets clear; (v) says that projects' markets clear, and (vi) that creditors are correct about their expectations of loan delivery or default.

3 Optimism, Risk-taking and Externalities

In this section we present an analytical solution for this model, the intuition for which is Minsky's financial instability hypothesis. In section 3.1 we solve for the endogenous variables of the model and derive propositions about the effect that an increasing probability of a good outcome has on them. For tractability, we assume that the initial capital of investors is zero and that investors do not reinvest any of the realized profits at any time $t > 0$, but rather consume it before they invest in new assets. Thus, their portfolio is fully debt financed. Under these assumptions, solving for equilibrium reduces to a static problem for which we can derive a closed form solution.

A closed form solution is important to show explicitly the externality inherent in our model and to enable us to perform welfare analysis. Default twists investors utilities by adding a component which depends on downside risk and represents the disutility of default. Investing in the riskier project increases downside risk. However, consumption in the case of default is pinned down by the default penalty and does not depend on the probability of a bad realization. This can be easily seen from the first order condition with respect to the repayment rate, $v^i_{s_t}$, which is $\lambda = 1 - 2\gamma^i c^i_{s_t}$, i.e. investors equate the marginal loss from defaulting to the marginal benefit of an additional unit of consumption.[7] As expectations improve the (total) expected disutility from default decreases, while the private benefit in the event of default remains fixed. There is a probability threshold after which investors start investing in the riskier asset and shift consumption to the good state of the world.

[7] We have set $\lambda_{s_t} = \lambda$ for all s_t. We discuss time-dependent default penalties as a policy response in section 5.1.

Increased risk-taking results in higher default, which puts upwards pressure on the borrowing rates and the amount owed. Investors do not take into consideration the impact that their portfolio decisions have on the optimizing decisions of creditors and the resulting borrowing rates. This is a type of pecuniary externality along the lines of Stiglitz (1982). We show in section 3.2 that a social planner, who incorporates this externality in her decisions, ceases to invest in the riskier asset. In particular, borrowing rates go down as does default, and the welfare of investors increases in the social planner's solution. Creditors' welfare is unaffected, since they are risk-neutral and break even in expectation. Hence, we get a Pareto improvement.

3.1 Optimism and Risk-taking

Under the assumptions made above, the investors' problem reduces to maximizing

$$\max_{c_u, c_d, w, v} \mathbb{U} = \mathbb{E}c - \gamma \mathbb{E}c^2 - \lambda(1-\pi)\max\left[(1-v), 0\right] wR,$$

subject to the budget constraints

$$c_u \leq a \cdot wX_u^L + (1-a) \cdot wX_u^H - wR = w\left[a(X_u^L - X_u^H) + X_u^H - R\right] \quad (\mu_u),$$

$$c_d \leq a \cdot wX_d^L + (1-a) \cdot wX_d^H - vwR = w\left[a(X_d^L - X_d^H) + X_d^H - vR\right] \quad (\mu_d)$$

and the short-sale constraints $0 \leq a \leq 1$, where a stands for the percentage of borrowed funds invested in the safer asset, L, and w is the amount of borrowing. Thus, agents cannot go short on any of the two projects. The good and the bad state occur with probabilities π and $1-\pi$ respectively. If the bad state occurs, investors choose to repay a fraction v of the amount owed, wR, where $R (= 1 + r)$ is the gross borrowing rate. μ_u and μ_d are the Lagrange multipliers associated with the budget constraints. Finally, the credit market clears when

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{4}$$

The Lagrangian is

$$\begin{aligned}\mathbb{L} =& \mathbb{E}c - \gamma \mathbb{E}c^2 - \lambda(1-\pi)\max\left[(1-v),0\right] wR - \mu_u\left[c_u - w\left[a(X_u^L - X_u^H) + X_u^H - R\right]\right] \\ &- \mu_d\left[c_d - w\left[a(X_d^L - X_d^H) + X_d^H - vR\right]\right] - \phi\left[a-1\right] + \psi \cdot a.\end{aligned}$$

The Lagrange multipliers μ_u and μ_d are positive due to concave utility, thus the budget constraints hold with equality in equilibrium.

Investors are price takers and do not factor in the impact of their decision to default on R. They take R as given. Thus, when they optimize they will take $\frac{\partial R}{\partial v} = 0$. However, their decision to default does have a price effect, since from equation 4, $\frac{\partial R}{\partial v} = -(1-\pi)R^2$.
Optimizing with respect to the delivery rate, v, we get

$$(1-\pi)\lambda = \mu_d. \tag{5}$$

Moreover, consumption in the event of default satisfies $\lambda = (1 - 2\gamma c_d)$.

Optimizing with respect to borrowing, w, we get

$$\mu_u\left[a(X_u^L - X_u^H) + X_u^H - R\right] + \mu_d\left[a(X_d^L - X_d^H) + X_d^H - vR\right] - \lambda(1-\pi)(1-v)R = 0. \tag{6}$$

Combining the last equation together with equation 5 yields

$$\frac{\mu_u}{\mu_d} = -\frac{a(X_d^L - X_d^H) + X_d^H - R}{a(X_u^L - X_u^H) + X_u^H - R}. \tag{7}$$

The complementary slackness conditions $\phi\left[a-1\right] = 0$ and $\psi \cdot a = 0$ yield the following three (candidate) equilibrium solutions for percentage investment in the safer asset:

1. The portfolio consists of both assets, i.e., $0 < a < 1$. This implies $\phi = 0$ and $\psi = 0$.

2. The portfolio consists solely of the riskier asset, i.e., $a = 0$. This implies $\phi = 0$ and $\psi > 0$.

3. The portfolio consists solely of the safer asset, i.e., $a = 1$. This implies $\phi > 0$ and $\psi = 0$.

We consider these three cases in turn and evaluate the range of exogenous parameters such that each of them holds. We denote by π^H, derived below, the probability threshold after which investors choose only the riskier asset (case 2). The region for which there is investment for both assets is $\pi \in \left[\pi^*, \pi^H\right]$ (case 1). Finally, π^L stands for the probability threshold after which it is profitable to invest in the safer asset. Thus, the portfolio consists solely of the safer asset for the region of $\pi \in \left[\pi^L, \pi^*\right)$ (case 3). Figure 1 presents the regions corresponding the three cases for a.

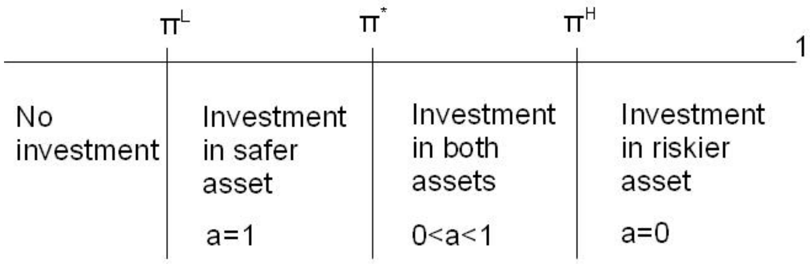

Figure 1: Probability regions and investment in assets

We start with the case that investors choose both assets, i.e., $0 < a < 1$, since we can easily derive a closed form solution to evaluate the corner solutions, which correspond to the other two cases.

The first order condition with respect to the allocation, a, of borrowed funds yields

$$\frac{\mu_u}{\mu_d} = -\frac{X_d^L - X_d^H}{X_u^L - X_u^H}. \tag{8}$$

Combining equations 7 and 8 we can calculate the equilibrium borrowing rate, which is given by

$$R = \frac{X_u^H \cdot X_d^L - X_d^H \cdot X_u^L}{X_u^H - X_d^H - (X_u^L - X_d^L)} \tag{9}$$

It is easy to see from the market clearing condition 4 that, as the probability of a good outcome increases, the repayment rate decreases, i.e.,

$$\frac{\partial v}{\partial \pi} = \frac{\partial \frac{1}{1-\pi}\left(\frac{1}{R} - \pi\right)}{\partial \pi} = \frac{1}{(1-\pi)^2}\left(\frac{1}{R} - 1\right) < 0, \tag{10}$$

since $R > 1$; otherwise creditors would not break even and there would be no trade. We, thus, focus on values for asset payoffs that yield a gross rate greater than 1 in equation 9, i.e., $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$.

Next we need to show that the percentage of borrowed funds, a, invested in the riskier project increases with the probability of a good outcome. Substituting the budget constraints into the first order conditions with respect to consumption,

$$\mu_u = \pi(1 - 2\gamma c_u) \tag{11}$$

and

$$\mu_d = (1-\pi)(1 - 2\gamma c_d), \tag{12}$$

we can solve for a and w, which yields

$$a = \frac{(X_u^H - R)\left(1 - \frac{\mu_d}{1-\pi}\right) - (X_d^H - vR)\left(1 - \frac{\mu_u}{\pi}\right)}{(X_d^L - X_d^H)\left(1 - \frac{\mu_u}{\pi}\right) - (X_u^L - X_u^H)\left(1 - \frac{\mu_d}{1-\pi}\right)} \tag{13}$$

and

$$w = \frac{1}{2\gamma} \frac{1}{a(X_u^L - X_u^H) + X_u^H - R}\left(1 - \frac{\mu_u}{\pi}\right). \tag{14}$$

Equations 4, 5, 8, 9, 13, 14, together with 11 and 12 give the closed form solution for all endogenous variables.

Lemma 1: *As the probability of a good realization increases, investors reallocate their portfolio towards the riskier asset.*
See Appendix I for the proof.

The lemma above holds for any π. To verify that there exists probability regions such that investors choose both assets, the safer or only the riskier one, we need to prove that π^* and π^H are below one (and greater than zero) and that $\pi^* < \pi^H$. The following proposition establishes these two facts.

Proposition 1: *There are thresholds for the probability of the good state π^* and π^H, such that investors choose both assets for $\pi \in [\pi^*, \pi^H\ )$ and only the risker asset for $\pi \in [\pi^H, 1]$.*
See Appendix I for the proof.

For proposition 1 we took the limit of equation 34 in Appendix I as $\pi \to 0$ to prove that there is a threshold π^* such that $a(\pi^*) = 1$. However, for $\pi < \pi^*$ investors invest only in the safer asset, thus $a = 1$ and $\phi > 0$. We, thus, need to show that there exist π^L greater than zero and lower than π^* such that the investor chooses the safer asset for $\pi \in [\pi^L, \pi^*\ )$. We first establish that the equilibrium variables w and R (and hence consumption) are continuous at π^*.

For $\pi < \pi^*$ the solution for the amount of borrowing, the borrowing rate and the percentage default are given by

$$\frac{X_u^L - R}{X_d^L - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{R - X_d^L}{X_u^L - R}\lambda}{1-\lambda}, \tag{15}$$

$$\frac{\pi}{1-\pi}\frac{1 - 2\gamma w(X_u^L - R)}{\lambda} = \frac{R - X_d^L}{X_u^L - R}, \tag{16}$$

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{17}$$

We evaluate these conditions as π approaches π^* from the left (denoted by π^{*-}) and compare them with those as π goes to π^* from the right (denoted by π^{*+}). In the latter region, investors invest in both assets, but as $\pi \to \pi^{*+}$, $a \to 1$. The equivalent of equations 15 and 16 as $\pi \to \pi^{*+}$ are

$$\frac{X_u^L - R}{X_d^L - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{X_d^L - X_d^H}{X_u^H - X_u^L}\lambda}{1-\lambda} \tag{18}$$

and

$$\frac{\pi}{1-\pi}\frac{1 - 2\gamma w(X_u^L - R)}{\lambda} = \frac{X_d^H - X_d^L}{X_u^L - X_u^H}. \tag{19}$$

Evaluating the first order conditions 7 and 8 as $\pi \to \pi^{*+}$ we get that $\dfrac{X_d^L - X_d^H}{X_u^H - X_u^L}$ approaches $\dfrac{R(\pi^{*+}) - X_d^L}{X_u^L - R(\pi^{*+})}$. From equations 15 and 18 we get that $R(\pi^{*-}) = R(\pi^{*+})$. Also, $v(\pi^{*-}) = v(\pi^{*+})$ and $w(\pi^{*-}) = w(\pi^{*+})$ from equations 16 and 19. Hence, there is no discontinuity in equilibrium variables when π

crosses the threshold π^* and investors start investing in the riskier project as well.

Investors optimize at π^*. In addition, it is easy to show that utility is increasing at π for $\pi \in (0, \pi^*)$. The proof follows the same steps as in proposition 1. We, thus, need to show that there exists a probability $0 < \pi^L < \pi^*$ such that for $\pi < \pi^L$ the individual rationality of investors violates the participation constraint of creditors. Proposition 2 establishes this result.

Proposition 2: *There exists a probability threshold π^L greater than zero and lower that π^*, such that investors choose only the safer asset for $\pi \in [\pi^L, \pi^*)$.*
See Appendix I for the proof.

Corollary 1: *The rate of default, $1 - v$, is falling as investors become more optimistic for $\pi \in (\pi^L, \pi^*)$, then gradually increases for $\pi \in (\pi^*, \pi^H)$, and finally starts falling again for $\pi \in (\pi^H, 1)$.*
See Appendix I for the proof.

In the analysis above, the perceived probability of a good realization could change continuously. However, agents are Bayesian learners and they update their beliefs in discrete intervals as new information about payoff realizations arrives. The perceived probability of a good realization will exhibit jumps as new information arrives. Moreover, investors can accumulate profits over time, which affects their risk-taking behaviour and the resulting borrowing rates at each point in time. We examine these issues in section 4 where we present a calibrated example of the dynamic model. The results presented in this section hold in the more elaborate framework as well.

The following section discusses the externality induced by optimism and risk-taking in the static framework presented above. The nature of the externality provides intuition about potential regulatory interventions, which are discussed in section 5 for the case of the calibrated example.

3.2 Social Planner's Solution

The ability to default twists investors' preferences and allows them to take on more downside risk. As shown in the previous section, they may do so by starting to invest in the riskier project when

$\pi > \pi^*$. Consequently, they will default more. This introduces a pecuniary externality along the lines of Stiglitz (1982) and Korinek (2011). Investors are price takers and do not take into account the effect that their default decision has on the equilibrium borrowing rate. Once they start investing in the riskier project, the borrowing rate can stop being a decreasing function of the probability of a good realization (corollary 1). Investors increase their downside risk by investing more in the riskier project and are charged with a higher rate than otherwise. Investors do not factor in the effect of their default on borrowing rates, and thus, on aggregate default and the deadweight loss/disutility associated with it.[8]

We consider a social planner who takes into account the effect of her decisions on the borrowing rate and aggregate default. The social planner objective is to maximize the utility of both investors and creditors by choosing the level of investment, the allocation between the safer and the riskier assets, the rate of default, as well as the borrowing rate, which in the context of the social planner's equilibrium should be thought as the promised return to creditors. The social planner is otherwise constrained in her decisions by the same payoff structure, the same penalties for default and the borrowing contract she can write with creditors, which cannot be state-contingent. The social planner will try to minimize the deadweight loss from default and she will return the whole investment payoff to creditors in the event of a bad realization. The consumption of investors in the bad state will, thus, be zero. Note that the social planner still defaults on creditors in the bad stare, since the payoffs of both assets L and H are less than one and creditors demand an interest rate higher than one to break even.

The social planner sets the return to creditors equal to $R^{sp} = \dfrac{1-(1-\pi)\left[a^{sp}\left(X_d^L - X_d^H\right) + X_d^H\right]}{\pi}$, such that they break even for any level of investment allocation, a^{sp}. This constraint should always hold with equality and we substitute it directly into the optimization problem. Due to risk-neutral creditors and homogeneous risk-averse investors, aggregation is easy and the social planner maxi-

[8]The pecuniary externality does not affect creditors' welfare in equilibrium, since they are risk-neutral and break even in expectation. This will not be the case with risk-averse creditors.

mizes the utility of the representative investor, i.e.,[9]

$$\max_{c_u^{sp},w^{sp},a^{sp}} \mathbb{U}^{sp} = \pi c_u^{sp} - \gamma\pi c_u^{sp2} - \lambda(1-\pi)w^{sp}\left[\frac{1-\left[a^{sp}\left(X_d^L - X_d^H\right)+X_d^H\right]}{\pi}\right],$$

subject to the budget constraints

$$\begin{aligned} & c_u^{sp} \leq a^{sp}\cdot w^{sp}X_u^L + (1-a^{sp})\cdot w^{sp}X_u^H - w^{sp}R^{sp} \\ \Rightarrow & c_u^{sp} \leq w^{sp}\left[\frac{a^{sp}\left[\pi(X_u^L - X_u^H)+(1-\pi)(X_d^L - X_d^H)\right]+\pi X_u^H + (1-\pi)X_d^H - 1}{\pi}\right] \quad (\mu_u^{sp}), \end{aligned}$$

and the short-sale constraints $0 \leq a^{sp} \leq 1$.

The Lagrangian which the social planner maximizes is

$$\begin{aligned} \mathbb{L}^{sp} = & \pi c_u^{sp} - \gamma\pi c_u^{sp2} - \lambda(1-\pi)w^{sp}\left[\frac{1-\left[a^{sp}\left(X_d^L - X_d^H\right)+X_d^H\right]}{\pi}\right] \\ & -\mu_u^{sp}\left[c_u^{sp} - w^{sp}\left[\frac{a^{sp}\left[\pi(X_u^L - X_u^H)+(1-\pi)(X_d^L - X_d^H)\right]+\pi X_u^H + (1-\pi)X_d^H - 1}{\pi}\right]\right] \\ & -\phi^{sp}\left[a^{sp}-1\right] + \psi^{sp}\cdot a^{sp}. \end{aligned}$$

The first order condition with respect to w^{sp} is

$$\mu_u^{sp}\left[a^{sp}\left[\pi(X_u^L - X_u^H)+(1-\pi)(X_d^L - X_d^H)\right]+\pi X_u^H + (1-\pi)X_d^H - 1\right] = \lambda(1-\pi)\left[1-a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right]$$

$$\Rightarrow \mu_u^{sp} = \frac{\lambda(1-\pi)\left[1-a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right]}{a^{sp}\left[\pi(X_u^L - X_u^H)+(1-\pi)(X_d^L - X_d^H)\right]+\pi X_u^H + (1-\pi)X_d^H - 1} \tag{20}$$

With respect to investment allocation, a^{sp}, the optimizing condition is

$$\mu_u^{sp}\frac{\pi\left(X_u^L - X_u^H\right)+(1-\pi)\left(X_d^L - X_d^H\right)}{\pi} + \lambda(1-\pi)\frac{X_d^L - X_d^H}{\pi} - \frac{\phi^{sp}}{w^{sp}} + \frac{\psi^{sp}}{w^{sp}} = 0. \tag{21}$$

Lemma 2: *The social planner chooses to invest:*

[9] The superscript *sp* is used to distinguish the equilibrium values from the ones in the competitive equilibrium.

- *in both assets if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$,
- *in the safer asset if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$,
- *in the riskier asset if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$.

See Appendix I for the proof.

We turn to the solution under the assumption that $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$, i.e., $a^{sp} = 1$. The analysis is equivalent for $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$. For $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$, the equilibrium allocation, a^{sp}, is indeterminate. Thus, we will not consider this special case of measure zero.

For $a^{sp} = 1$, the equilibrium level of investment w^{sp} and the promised return to creditors are

$$w^{sp} = \frac{1}{2\gamma} \frac{1}{X_u^L - R^{sp}} \left(1 - \frac{\mu_u^{sp}}{\pi}\right) \tag{22}$$

$$R^{sp} = \frac{1 - (1-\pi) X_d^L}{\pi} \tag{23}$$

where

$$\mu_u^{sp} = \frac{\lambda(1-\pi)\left(1 - X_d^L\right)}{\pi X_u^L + (1-\pi) X_d^L - 1} \tag{24}$$

Proposition 3: *Assume that* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$. *There exist probability thresholds* $\bar{\pi}^L$, π^* *and* π^{**} *such that:*

- *Both competitive investors and the social planner invest only in the safer asset for* $\pi \in \left[\bar{\pi}^L, \pi^*\right]$.
- *Competitive investors gradually switch their investment towards the riskier asset for* $\pi > \pi^*$, *while the social planner continues investing in the safer one.*

See Appendix I for the proof.

The assumption about asset payoffs is made to satisfy the creditors' participation constraint in the competitive equilibrium, i.e., $R > 1$ (see equation 9), so that we can compare the solution in the competitive equilibrium with the social planner's solution for the whole range of $\pi's$. R^{sp}, which is given by equation 23, satisfies the participation constraint for $\pi \leq 1$. The following lemmas establish the change in equilibrium variables in the social planner's solution.

Lemma 3: *Investors consume more in the good state and less in the bad one in the social planner's solution compared to the competitive solution for* $\pi \in \left(\pi^{*}, \pi^{H}\right)$.
See Appendix I for the proof.

Lemma 4: *The amount of borrowing increases, whereas the borrowing and default rates decrease in the social planner's solution compared to the competitive equilibrium for* $\pi \in \left(\pi^{*}, \pi^{H}\right)$.
See Appendix I for the proof.

We now prove one of our main claims; that the competitive equilibrium is Pareto inefficient, thus there is scope for policy intervention. We consider the cases for which $\pi > \pi^{*}$, i.e. $a < 1$. We restrict the percentage default, v, and set it as an exogenous variable. Our objective is to evaluate the change in investors' utility for small changes in the percentage repayment, v. We maintain the equilibrium values for consumption in the good and the bad state, and we allow the investment allocation, a, and the level of borrowing, w, to vary in order to neutralize the effect of v on consumption. Thus, the only effect will be on the disutility of default.

Proposition 4: *In a competitive equilibrium, investors shift their portfolio towards the riskier asset once expectations improve sufficiently. This creates an externality, since investors are price takers and do not factor in the impact of their portfolio choices on default and credit spreads in equilibrium. An exogenous restriction on the equilibrium level of default can result in a Pareto improvement.*
See Appendix I for the proof.

The proposition above showed that an exogenous decrease in the level of default can result in a Pareto improvement given that creditors are risk-neutral and break even in expectation. We proved this by keeping the level of consumption constant and varying the level and the allocation of investment. However, in the social planner's solution the level of consumption will be different from the competitive equilibrium levels as shown in lemma 3. Figure 2 compares the equilibrium utility of competitive investors to the one resulting in the social planners solution. We have parameterized the model by setting $X_u^L = 1.4$, $X_d^L = 0.8$, $X_u^H = 2.1$, $X_u^L = 0.2$, $\lambda = 0.9$ and $\gamma = 0.035$. The parameters are such that the social planner chooses $a^{sp} = 1$ (lemma 2). Utility in both the competitive and the social planner's equilibria is increasing in the probability of the good state. However, the social planner's solution strictly dominates the competitive solution even for $\pi < \pi^*$ where there is only investment in the safer asset for both. The reason is that the social planner takes into consideration her impact on the deadweight loss and minimizes it. We also present the utility when the social planner chooses the riskier asset to demonstrate that it is Pareto dominated.

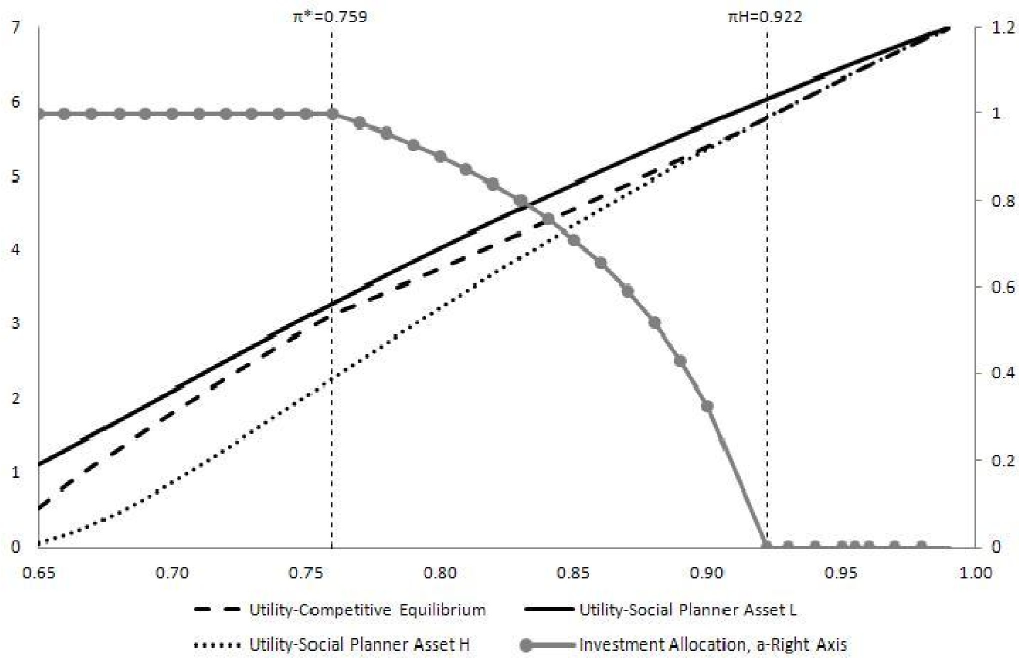

Figure 2: Utilities in the competitive and social planner's solutions versus the probability of a good realization

4 Quantitative analysis

The result of corollary 1 that the rate of default increases, while investors invest in both assets and become more optimistic, holds also within a dynamic framework, where investors accumulate reserves and increase their equity position over a path of good realizations. On one hand, the higher own funds engaged into future investment should reduce the incentive to take on additional risk and default more. On the other hand, the marginal benefit in the event of default is fixed by the exogenous default penalty and does not depend on the initial equity position. Thus, a marginal shift in risk depends crucially on the shadow value of a higher net payoff in the good state of the world, which is decreasing in the interest rate charged. The interest rate on borrowed funds can be calculated by the intertemporal first order conditions. The intertemporal budget constraints are

$$\Pi^i_{s_{t+1}} + T^i_{s_{t+1}} \leq w^i_{s_t,L} X^L_{s_{t+1}} + w^i_{s_t,H} X^H_{s_{t+1}} - w^i_{s_t} v^i_{s_{t+1}} (1 + r_{s_t}) \quad (\mu^i_{1,s_{t+1}})$$

and

$$w^i_{s_{t+1},L} + w^i_{s_{t+1},H} \leq f\left(T^i_{s_{t+1}}\right) + w^i_{s_{t+1}} \quad (\mu^i_{2,s_{t+1}}),$$

where $\mu^i_{1,t+1}$ and $\mu^i_{2,t+1}$ are the Lagrange multipliers associated with the two constraints.

Consider that, already at t, expectations are such that investors hold a portfolio of both assets (see proposition 1) and that a good state has realized at $t+1$. The first order conditions with respect to safe and risky investments, and borrowing are

$$-\mu^i_{2,s_t} + \mu^i_{1,s_t u} X^L_u + \mu^i_{1,s_t d} X^L_d = 0, \tag{25}$$

$$-\mu^i_{2,s_t} + \mu^i_{1,s_t u} X^H_u + \mu^i_{1,s_t d} X^H_d = 0 \tag{26}$$

and

$$\mu^i_{2,s_t} - \mu^i_{1,s_t u}(1 + r_{s_t}) - \mu^i_{1,s_t d} v^i_{s_t d}(1 + r_{s_t}) - (1 - \pi_{s_t})(1 - v^i_{s_t d})\lambda_{s_{t+1}}(1 + r_{s_t}) = 0,$$

or

$$\mu^i_{2,s_t} - \mu^i_{1,s_t u}(1 + r_{s_t}) - \mu^i_{1,s_t d}(1 + r_{s_t}) = 0, \tag{27}$$

given that

$$(1 - \pi_{s_t})\lambda_{s_{t+1}} = \mu^i_{1,s_t d}. \tag{28}$$

Equations 25 and 26 can be used to compute the Lagrange multiplier in state $s_t u$,

$$\mu^i_{1,s_t u} = \frac{X^L_d - X^H_d}{X^H_u - X^L_u}(1-\pi_{s_t})\lambda_{s_{t+1}}. \tag{29}$$

The borrowing rate and the Lagrange multiplier at s_t are given by the following expressions respectively.

$$r_{s_t} = \frac{X^H_u \cdot X^L_d - X^H_d \cdot X^L_u}{X^H_u - X^H_d - (X^L_u - X^L_d)} - 1 \tag{30}$$

and

$$\mu^i_{2,s_t} = \left[\frac{X^L_d - X^H_d}{X^H_u - X^L_u} X^L_u + X^L_d\right](1-\pi_{s_t})\lambda_{s_{t+1}}. \tag{31}$$

Equation 30 suggests that the rate of default is higher when investors update their expectations upwards and keep investing in both assets. Moreover, the shadow value of an additional unit in state $s_t u$ is fixed by exogenous variables from equation 29. Thus, investors can undertake more risk without being penalized with a higher interest rate and can keep increasing their borrowing. However, the expected disutility from defaulting, which is given by $(1-\pi_{s_t})(1-v^i_{s_t d})\lambda_{s_{t+1}} w^i_{s_t}(1+r_{s_t}) = \lambda_{s_{t+1}} w^i_{s_t} r_{s_t}$, is increasing in the amount of borrowed funds and limits the level of leverage that investors can undertake.

In order to illustrate the interaction between Minsky's hypothesis and endogenous leverage, which is core of greater optimism leading to more risk-taking, borrowing, and default, we simulate a simplified version of the model outlined. In particular, we consider a three-period economy, $t = 0,1,2$, where a good or a bad state can realize at $t = 1$ and $t = 2$. Thus, the state space is given by $S = \{0,u,d,uu,ud,du,dd\}$. We parametrize the model such that investors are on the verge of investing in the riskier asset at $t = 0$. We assume that investors have an initial belief that the good state will realize in the intermediate period with probability $\pi_0 = 0.82$. Expectations are updated according to Bayes rule given the state realization. Given a good realization at t=1, the (subjective) probability of a good outcome increases to $\pi_u = 0.87$, while it falls to $\pi_d = 0.59$ after a bad realization.[10] The safer project's payoff in the good state is $X^L_g = 1.41$, while the riskier project pay out $X^H_g = 1.85$. In the bad state their payoffs are $X^L_b = 0.8$ and $X^H_b = 0.42$ respectively. We

[10]The values that support these probabilities are $\theta_1 = 0.898$, $\theta_2 = 0.286$ and the prior $Pr(\theta = \theta_1) = 0.879$.

have assumed that banks are symmetric; thus they have the same risk-aversion and initial wealth. Finally, the default penalty is constant at every point in time. Table 1 presents the chosen values of the exogenous variables.

Table 1: Exogenous variables

Probability of good outcome at $t=0$	π_0=0.82	Riskier's project payoff in good state	X_g^H=1.85
Probability of good outcome at $s_t=u$	π_u=0.87	Riskier's project payoff in good state	X_b^H=0.42
Probability of good outcome at $s_t=d$	π_d=0.59	Risk-aversion coefficient	γ^i=0.035
Safer's project payoff in good state	X_g^L=1.41	Default penalty	λ=0.94
Safer's project payoff in bad state	X_b^L=0.80	Initial banking capital	$\bar{w}_0^i$=0.3

We show the equilibrium values for the endogenous variables in table 4 in Appendix II and here discuss the most important outcomes.

The main result presented in section 3 was that banks reallocate their portfolios towards the riskier asset once expectations become more optimistic, and this holds in the dynamic version of the model as well. However, we are now able to examine the effects on leverage, interest rates and most importantly default, which is (or should be) at the heart of any financial instability analysis. We follow the Goodhart-Tsomocos definition of financial instability as states characterized by high default and low banking profits-welfare (see Goodhart et al. (2006)).

In the initial period investors are on the verge of investing in the riskier project. As expected, they invest only in the safer one in the intermediate bad state, since they update their expectations downwards. However, once expectations improve (the economy moves to the good state in the intermediate period) investors switch heavily to the riskier project. In this simulation, the portfolio weight on it becomes almost twelve times the weight on the safer project (Table 2).

The increased holdings of the riskier asset in state $s_t=u$ are mainly financed by an increase in borrowed funds. Holdings of the safer project, on the contrary, decrease. In particular, as shown in table 3, borrowing almost increases by a factor of two once good news materializes. Although, when the good state is realized in the intermediate period, investors enjoy high profits, they choose to borrow even more and switch to riskier investments. An increase in borrowing is facilitated by the fact that

Table 2: Portfolio weight of the riskier project

Portfolio weight on the risky project at $t=0$	$w^i_{0,H}$=0.22%
Portfolio weight on the risky project after bad news	$w^i_{d,H}$=0
Portfolio weight on the risky project after good news	$w^i_{u,H}$=92.30%
Risky-to-safe project ratio of weights at t=0	$w^i_{0,H}/w^i_{0,L}$=0.22%
Risky-to-safe asset ratio of weights after bad news	$w^i_{d,H}/w^i_{d,L}$=0
Risky-to-safe asset ratio of weights after good news	$w^i_{u,H}/w^i_{u,L}$=11.98

the interest rate does not change. Although the bank borrows more and undertakes riskier projects, the interest rate remains the same, since the higher rate of default is balanced by a lower expected probability of default occurring, an expectation shared by both investors and creditors.

The expected percentage default remains the same, but inevitably the loss given default is much higher if and when the bad state realises at t=2. The percentage default and loss given default are higher when prosperity prevailed in the past, than in the case where a bad outcome had materialized earlier. In particular, we find that percentage default in state ud is 60.07% compared to 30.22% in state dd and 43.33% in state d. After a round of bad news borrowing goes down, since the prospects of the economy have deteriorated. This is captured in a higher interest rate charged. Investors default less in state dd than in state d. The interest rate at d is already higher due to bad expectations, thus by defaulting less they are facing a lower cost of borrowing. However, to do so they invest only in the safer asset. Naturally, loss given default is much lower. The most important result of our analysis is that loss given default in state ud is substantially higher than in any other state as shown in table 3. Optimism allowed investors to borrow more and undertake much riskier projects, which eventually can result in a catastrophic scenario. This is not the case when bad news occurs in the intermediate period and expectations were not boosted upwards.

Table 3: Interest rates, leverage and default

Increase in borrowing after good news	139%	Decrease in borrowing after bad news	17.22%
Interest rate change after good news	0%	Interest rate change after bad news	69.02%
Expected default at $s_t=0$	7.64%	Realized default at $s_t=d$	43.33%
Expected default at $s_t=u$	7.64%	Realized default at $s_t=ud$	60.07%
Expected default at $s_t=d$	12.26%	Realized default at $s_t=d$	30.22%
Loss given default at $s_t=d$	2.70	Loss given default at $s_t=ud$	8.94
Loss given default at $s_t=dd$	1.64		

5 Policy Responses

The driving force behind an increase in borrowing and increased risk taking is the optimism that comes after the realization of good news. The expectations formation mechanism is exogenous in our model and is implemented through Bayesian updating. Agents have imperfect information about the real world probability of a good state occurring and they try to infer it by observing past realizations. They are Bayesian learners. There is also no additional asymmetry of information. Every agent knows and observes the same information. Thus, regulation cannot control optimism in the markets. Agents are rational and none have more information than others. Regulation cannot affect optimism, but it can control its consequences. Simplified as it is, our model can be used to evaluate regulatory policies to control borrowing and mitigate excessive risk-taking and default. A first type of policy is to enforce more severe default penalties for investors, while a second is to control their leverage ratios in the good state of the world. We use the equilibrium outcomes calculated in section 4 to evaluate the effect of policy interventions on equilibrium variables and welfare.

The social planner's solution (see section 3.2) suggests that there can exist policy interventions which increase the welfare of investors, while not affecting creditors' welfare. The competitive equilibrium is Pareto suboptimal. We have shown in lemmas 3 and 4 that consumption is higher in the good and lower in the bad state in the social planner's solution, while the amount of borrowing is higher than in the competitive equilibrium. These equilibrium variables do not move in the same way as in the social planner's solution when stricter default penalties and leverage ratios are imposed in the competitive equilibrium (sections 5.1 and 5.2 respectively). Although they are successful in reducing borrowing and loss given default, they result in lower welfare for investors, while not changing the welfare of the creditors. We, thus, construct a different regulatory ratio, which better captures the social planner's solution. This policy ratio is equal to riskier minus safer levels of investment per unit of borrowing and captures the relative risk-taking for leveraged investors. Imposing a stricter regulatory threshold for this ratio results in lower risk-taking and higher welfare. Figure 3 present the change in welfare for higher default penalties, stricter leverage requirements and stricter values for the regulatory ratio we propose.

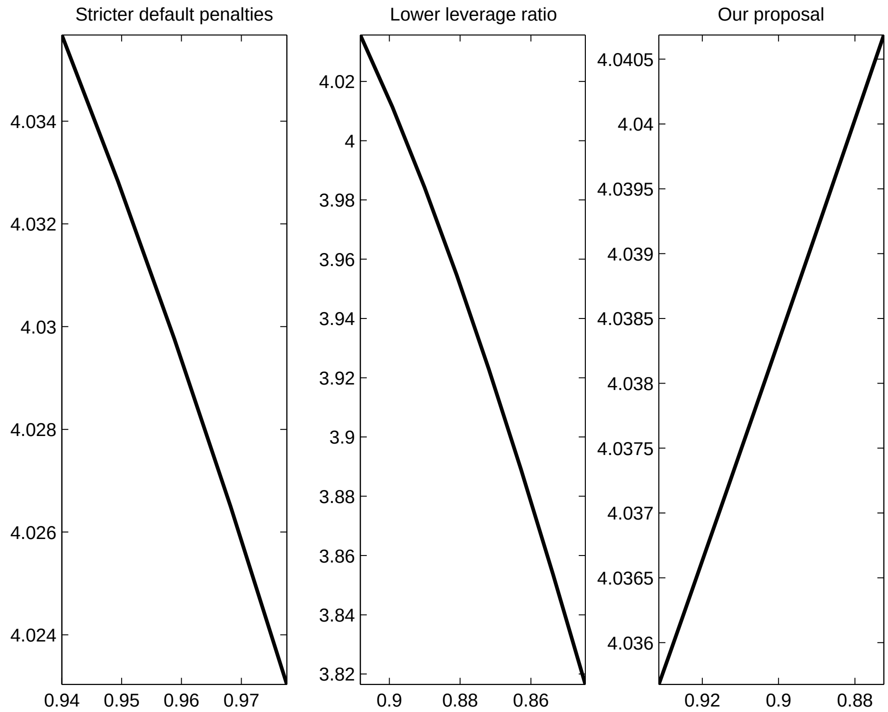

Figure 3: Regulation and Welfare

5.1 Stricter Default Penalties

Limited liability and the possibility of default allow investors to take on downside risk as expectations become more optimistic. Risky investment is mainly funded through an increase in borrowing. However, investors are not penalized ex-ante with a higher interest rate as equation 30 suggests. The expected rate of default does not change with the level of investment in the riskier asset. As a result, the loss given default and the deadweight loss from defaulting are higher ex-post. We showed in section 3.2 that investors do not factor this in their decisions.

A way to correct for this externality could be to make default more costly once expectation improve by setting a higher default penalty in state *ud*.[11] The first order effect would be a reduction in borrowing. However, the interest rate and the expected rate of default would not change, given that

[11]Given limited liability, direct ways for the legal system or regulation to affect these penalties are lengthy bankruptcy processes or time-consuming investigations to discover fraudulent behaviour, in which case default penalties are higher due to legal sanctions. Regulation can affect this component of default penalties and make default more costly during good times. An alternative way to impose penalties in a more quantifiable way is through renumeration reforms involving deferred managerial compensation, which would allow clawback of accrued past bonuses in the case of bad outcomes and default.

the first order conditions 25, 26 and 27 have the same functional form when default penalties are increased. The Lagrange multipliers in states *uu*, *ud* and *u* would all increase (equations 29, 28 and 31) suggesting lower consumption in states *uu* and *ud*, and lower borrowing in *u* (figures 4 and 5 in Appendix II).

An adverse effect of stricter default penalties is that investors switch their portfolio holdings towards the riskier asset despite the fact that borrowing goes down (figure 4). The reason is that the private benefit they can extract in the event of default is lower, thus they are inclined to take on more downside risk given that they are not penalized ex-ante with a higher interest rate. Hence, harsher penalties are effective in reducing borrowing, but not at mitigating excessive risk-taking. Overall, they result in lower welfare (figure 3).

5.2 Leverage Requirements

Another policy response would be to restrict leverage once the good state of the world occurs in the intermediate period. A leverage requirement can take the form of a maximum ratio of borrowing over the total investment in projects, i.e.,

$$w^i_{u,L} + w^i_{u,h} \geq \frac{1}{l} \cdot w^i_u, \tag{32}$$

where l is the leverage requirement set by regulation. The leverage constraint, when it binds, can be used to determine the reinvested profits, T^i_u, in the budget constraint $w^i_{u,L} + w^i_{u,h} = w^i_u + T^i_u$. For a given level of borrowing, w^i_u, the equity invested is $T^i_u = f^{-1}\left(\frac{1-l}{l} w^i_u\right)$.

Lower borrowing due to a requirement on leverage can be balanced by an increase in own funds invested. The effect on total investment and final consumption can be evaluated from equations 25, 26 and 27, which do not change under a binding leverage requirement. Investors can enjoy the same level of consumption in states *uu* and *ud* (equations 29 and 28) and are charged with the same interest rate (equation 30) as the requirement on leverage becomes stricter. Although borrowing goes down, investors reduce their investment in both the safer and riskier assets by less (figure 6 in Appendix II) and enjoy the same levels of final consumption. In doing so, the reinvested funds at

u from previous investment need to increase. Hence, investors increase their portfolio holdings and borrowing at $t = 0$ to account for a binding leverage constraint in state u. This results in a higher loss given default and deadweight loss in state d. Although investors enjoy the same level of final consumption and a lower deadweight loss in state ud, higher risk-taking in the initial period results in higher deadweight loss in the intermediate period should a bad state occur and lower welfare overall (figure 3).

5.3 An alternative requirement

As expectations become more optimistic, investors switch towards the riskier investment. As shown in section 3.2, this creates an externality leading to excessive default relative to the social optimum in state ud, since investors do not factor in the impact that a riskier portfolio has on default. It is also true that higher investment in the safer asset and borrowing are desirable when the prospects of the economy improve (lemma 4). Regulation can provide incentives to investors to behave in a socially optimal way by restricting the level of borrowing that is shifted from safer to riskier investment. Such a requirement can be specified in variety of ways. Herein, we consider a requirement that is equal to the difference between riskier and safer holdings per unit of borrowed funds, i.e.,

$$w^i_{u,H} - w^i_{u,L} \leq \zeta \cdot w^i, \tag{33}$$

where ζ is the regulatory requirement.[12]

Stricter requirements of this type result in a lower rate of default in state ud, since investors reduce

[12] Basel II capital requirements are inadequate for this task, since (internal) model based risk-weights go down during good times (Catarineu-Rabell et al. (2005) and Pederzoli et al. (2010)). Figure 11 in Appendix II presents average risk weights, calculated as the ratio of the aggregate risk weighted assets over aggregate assets, for a panel of 33 international big banks. The panel includes the National Bank of Australia, ANZ, Macquarie, Dexia, China Merchants Bank, BNP Paribas, Credit Agricole, Societe Generale, Natixis, Deutsche Bank, Commerzbank, Unicredit, Monte dei Paschi, ING, Santander, BBVA, Nordea, SEB, Svenska Handelsbanken, UBS, Credit Suisse, Royal Bank of Scotland, Barclays, HSBC, Lloyds, Standard Charted, JP Morgan Chase, Citigroup, Bank of America, Wells Fargo, Bank of NY Mellon, State Street and PNC. Source: Bloomberg. Our proposal results in lower risk-taking accompanied by higher borrowing and higher investment in safer projects. Although relative risk is sometimes not accurately measured, for example when the top tranches of CDOs and MBSs were given too high a rating before the 2007 financial crisis, so that banks which were subject to an RWA, but not a leverage ratio, tended to expand their leverage enormously on the basis of such supposedly risk-free assets, which were not so. To account for such circumstances, our proposal could be augmented by a leverage restriction, in order not only to mitigate risk-taking, but also to reduce investment in projects that are mistakenly perceived to be safer.

their investment in the riskier asset (figure 7 in Appendix II). Moreover, this kind of regulation provides incentives to increase investment in the safer asset, in contrast to leverage regulation whereby investors would reduce investment in both assets. Affecting behaviour in a way that resembles the social planner's solution, the regulatory constraint 33 results in higher borrowing, but lower deadweight loss from default, and higher consumption in state *uu* (figure 8 in Appendix II). In contrast to the prior types of regulation, this one addresses the externality from excessive risk-taking due to optimism more successfully and results in an increase in welfare (figure 3).

6 Summary and Concluding Remarks

The perceived risk profile of investment opportunities changes over time. Agents are Bayesian learners and update their beliefs about future realisations by observing the sequence of past ones. After a prolonged period of good news, expectations are boosted and investors find it profitable to shift their portfolios towards projects that are on average riskier, but promise higher expected returns. Creditors are willing to provide them with funds, since their expectations have improved as well. As a result, the level of borrowing increases, risk premia do not increase and portfolios consist of relatively riskier projects. When bad news does occur, default is higher and the consequences for financial stability are more severe. This creates an externality, since investors do not take into account the level of default when making their portfolio decisions. We examine three types of regulation to correct for the inefficiency in the competitive equilibrium, which are stricter penalties for default, tighter leverage requirements and a novel criterion capturing the relative risk-taking per unit of borrowed funds. Only the latter results in a Pareto improvement.

Our analysis has empirical implications for the identification of points in the leverage cycle where there is a higher risk of future financial instability. In particular, we have constructed a theoretical model to highlight the variables that can be used to develop an index, which could act as a leading indicator for financial distress. In our framework, as expectations become more optimistic due to good realizations, investors start investing in riskier projects and increasing their borrowing. Although the loss given default increases under a riskier portfolio composition, expected default and credit spreads do not adjust. This suggests that not only credit growth, but also portfolio switches to riskier projects should be used to identify the point in the leverage cycle in combination with low (ex-ante) risk premia.

An important element of the identification strategy would be expectations formation. The effectiveness of capturing time-varying transition probabilities between good and bad regimes should be the main objective in model selection for empirical work. One of the conjectures to be tested is that the riskiness of the financial system increases as people become more optimistic.

Measuring the riskiness of financial portfolios over the leverage cycle is not an easy task. As highlighted in this paper, although investors engage in more risky behaviour after a period of good realizations, this results from expectations becoming more optimistic. Commonly used measures to capture risk build-up, such as the volatility of returns on assets or credit spreads, fail to do so due to the fact that they are biased by optimistic expectations. It is evident that market volatility as measured by the VIX index was below its long-term trend before the financial crisis. The same holds for the TED spread, i.e. the difference between the interest rates on interbank loans and short-term US government debt (figure 10 in Appendix II).[13]

The index, which we propose, is the difference between riskier and safer portfolio holdings per unit of leverage. Once expectations become optimistic riskier projects are perceived to be less risky; the same holds for safer ones, which are assessed as being even more safe. Although absolute riskiness goes down for both types, their ranking is preserved. Consider for example risk weighted assets (RWAs) as defined by the Basel Accord II, under which risk weights follow an Internal Rate approach and change over the cycle. As mentioned, the literature on procyclicality has shown that all risk weights go down in good times, as empirical data also suggest (figure 11 in Appendix II). Thus, RWA do not increase as much as they should when banks shift their portfolio towards projects previously regarded as too risky. This procyclicality in measured risk is mitigated once we focus on the difference between projects with a higher and lower risk-weight, assuming that their relative rankings are preserved. Finally, we normalize by leverage, because it is default on debt that causes

[13] Various other indicators, such as credit growth to GDP or housing prices growth, have been proposed in the literature. For a broad overview that differentiates between system-wide and bank-specific variables, see Drehmann et al. (2010).

a financial crisis, a tightening of credit and forced liquidations that lead to fire sales externalities. In figure 9 in Appendix II, we simulate our model for different levels of optimism and show how the proposed index can predict risk-taking and financial instability, whereas the more commonly used volatility measures fails. As a proxy for VIX we calculate the volatility of banking portfolios, which instead moves in the opposite direction. Our analysis suggests that quantity based measures, which capture the risk-taking behaviour of leveraged investors, could be valuable as leading indicators for subsequent financial instability.

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Appendix I

Proof of Lemma 1

Proof. It suffices to show that $\partial a/\partial\pi < 0$. After substituting, equation 13 becomes

$$a = \frac{(X_u^H - R)(1-\lambda) - (X_d^H - \frac{1-\pi R}{1-\pi})\left(1 - \frac{1-\pi}{\pi}\bar{A}\lambda\right)}{(X_d^L - X_d^H)\left(1 - \frac{1-\pi}{\pi}\bar{A}\lambda\right) - (X_u^L - X_u^H)(1-\lambda)}, \tag{34}$$

where $\bar{A} = \dfrac{X_d^L - X_d^H}{X_u^H - X_u^L} > 0$. The gross rate R is fixed by equation 9 and is greater than 1. Also, we choose $\lambda \leq 1$, otherwise agents would never choose to default as they are on-the-verge of defaulting when $\lambda = (1 - 2\gamma c_d)$. The second term in the numerator is increasing with π, thus the numerator is decreasing. Moreover, the first term in the denominator is increasing in π, thus the denominator is increasing. Combining the two we get that $\partial a/\partial\pi < 0$. $\square$

Proof of Proposition 1

Proof. From lemma 1 we know that $\partial a(\pi)/\partial\pi < 0$. Also, $a(\pi)$ is a continuous function of π for $0 < \pi < 1$. We need to show that there exist $\pi^{'}$ and $\pi^{''}$, where $\pi^{'} < \pi^{''}$, such that $a(\pi^{'}) > 1$ and $a(\pi^{''}) < 1$. Then, by the intermediate value theorem there exists π^*, $\pi^{'} < \pi^* < \pi^{''}$, such that $a(\pi^*) = 1$. The limit of $a(\pi)$ as $\pi \to 0$ is $\lim\limits_{\pi\to 0} a(\pi) = \lim\limits_{\pi\to 0} \dfrac{1 - X_d^H}{X_d^L - X_d^H} + \dfrac{\pi^2}{(1-\pi)^2} \dfrac{1-R}{(X_d^L - X_d^H)\bar{A}\lambda} = \dfrac{1 - X_d^H}{X_d^L - X_d^H} > 1$ given that $0 < X_d^H < X_d^L < 1$. Also, $\lim\limits_{\pi\to 1} a(\pi) = -\infty$ given that $R > 1$. The probability $\pi^*(X_d^L, X_u^L, X_d^H, X_u^H, \lambda)$ is given by setting equation 34 equal to 1 and solving for π^*.

In order to prove that $\pi^H < 1$, we just need to show that $a(\pi)$ crosses zero for π^H as π goes from π^* to one, since $a(\pi^*) = 1$ and $a(\pi)$ is continuous and decreasing. The RHS of 34 as $\pi \to 1$ goes to $-\infty$. Thus, there exists a threshold π^H greater than π^* and lower than one, such that the short sales constraint $a \geq 0$ is hit. For $\pi > \pi^H$ the complementary slackness condition implies that $a = 0$ and $\psi > 0$. The probability $\pi^H(X_d^L, X_u^L, X_d^H, X_u^H, \lambda)$ is given by setting equation 34 equal to 0 and solving for π^H.

To complete the proof, we need to show that investors do actually prefer the riskier asset for $\pi \in \left[\pi^H, 1\right]$ over their outside option, which is zero investment and borrowing yielding a utility value of zero. We first show that the equilibrium variables are continuous at π^H as investors choose $a = 0$. For $\pi > \pi^H$ the solution for the amount of borrowing, the borrowing rate and the percentage default are given by

$$\frac{X_u^H - R}{X_d^H - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{R - X_d^H}{X_u^H - R}\lambda}{1-\lambda}, \tag{35}$$

$$\frac{\pi}{1-\pi}\frac{1 - 2\gamma w(X_u^H - R)}{\lambda} = \frac{R - X_d^H}{X_u^H - R}, \tag{36}$$

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{37}$$

We evaluate these conditions as π approaches π^H from the left (denoted by π^{H-}) and compare them with those as π goes to π^H from the right (denoted by π^{H+}). In the former region, investors invest in both assets, but as $\pi \to \pi^{H-}$, $a \to 0$. The equivalent of equations 35 and 36 as $\pi \to \pi^{H-}$ are

$$\frac{X_u^H - R}{X_d^H - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{X_d^L - X_d^H}{X_u^H - X_u^L}\lambda}{1-\lambda} \tag{38}$$

and

$$\frac{\pi}{1-\pi}\frac{1-2\gamma w(X_u^H - R)}{\lambda} = \frac{X_d^H - X_d^L}{X_u^L - X_u^H}. \tag{39}$$

Evaluating the first order conditions 7 and 8 as $\pi \to \pi^{H-}$ we get that $\dfrac{X_d^L - X_d^H}{X_u^H - X_u^L}$ approaches $\dfrac{R(\pi^{H-}) - X_d^H}{X_u^H - R(\pi^{H-})}$. From equations 35 and 38 we get that $R(\pi^{H+}) = R(\pi^{H-})$. Also, $v(\pi^{H+}) = v(\pi^{H-})$ and $w(\pi^{H+}) = w(\pi^{H-})$ from equations 36 and 39. Hence, there is no discontinuity in equilibrium variables when π crosses the threshold π^H and investors choose only the riskier asset.

Investors optimize at π^H. Given the continuity in equilibrium variables it suffices to show that investors' utility is increasing as the probability π goes from π^H to one. The deadweight loss from defaulting in the bad state, $(1-\pi)(1-v)\lambda wR$, can be written as $\lambda w(R-1)$ given that $v = \dfrac{1-\pi R}{R(1-\pi)}$. Investors utility can be written as

$$U = \pi\left(c_u - \gamma c_u^2\right) + (1-\pi)\left(c_d - \gamma c_d^2\right) - \lambda w(R-1). \tag{40}$$

Its derivative with respect to π is

$$\frac{\partial U}{\partial \pi} = c_u - \gamma c_u^2 + \pi(1-2\gamma c_u)\frac{\partial c_u}{\partial \pi} - \left(c_d - \gamma c_d^2\right) - \lambda\frac{\partial(w(R-1))}{\partial \pi}, \tag{41}$$

since c_d is fixed at $\dfrac{1-\lambda}{2\gamma}$. Moreover, $c_u > c_d$, since investors do not default in the good state, and $\pi(1-2\gamma c_u) = \mu_u > 0$. To evaluate $\dfrac{\partial c_u}{\partial \pi}$, we will use equation 7 for $a = 0$ and the derivative of R with respect to π.

For $\pi > \pi^H$, total differentiate equation 35 and recall that $vR = \dfrac{1-\pi R}{1-\pi}$. Then, we get

$$-dR(1-\lambda) = -dvR + \lambda X_d^H \frac{1}{\pi^2}dR\left[\frac{(R-X_d^H)+(X_u^H-R)}{(X_u^H-R)^2}\right] + dvR\frac{1-\pi}{\pi}\frac{R-X_d^H}{X_u^H-R}\lambda - vR\lambda\frac{1}{\pi^2}dR\left[\frac{(R-X_d^H)+(X_u^H-R)}{(X_u^H-R)^2}\right]$$
$$\Rightarrow dR\left[1-\lambda+\frac{\pi}{(1-\pi)^2}+\lambda\frac{1}{\pi^2}(X_d^H - vR)\left[\frac{(R-X_d^H)+(X_u^H-R)}{(X_u^H-R)^2}\right]\right] = -\left[1-\frac{1-\pi}{\pi}\frac{R-X_u^H}{X_u^H-R}\lambda\right].$$

Thus, $dR/d\pi < 0$ for $\pi > \pi^H$. Thus, equation $\mu_u\left(X_u^H - R\right) + \mu_d\left(X_u^L - R\right) = 0$ implies that $\dfrac{\partial \mu_u}{\partial \pi} = \dfrac{\partial(\pi(1-2\gamma c_u))}{\partial \pi} < 0 \Rightarrow \dfrac{\partial c_u}{\partial \pi} > \dfrac{1-2\gamma c_u}{2\gamma\pi} > 0$. Finally, $w = \dfrac{1-\lambda}{2\gamma}\dfrac{1}{X_d^H - \frac{1-\pi R}{1-\pi}}$, thus $\dfrac{\partial(w(R-1))}{\partial \pi} = \dfrac{1-\lambda}{2\gamma}\dfrac{\left(X_d^H - \frac{1-\pi R}{1-\pi}\right) - (R-1)\frac{\pi}{1-\pi}}{\left(X_d^H - \frac{1-\pi R}{1-\pi}\right)^2} = \dfrac{X_d^H - 1}{\left(X_d^H - \frac{1-\pi R}{1-\pi}\right)^2} < 0$. Combining the above we get that investors utility is increasing at π, for $\pi > \pi^H$. $\square$

Proof of Proposition 2

Proof. The optimizing behaviour of investors yields equation 7, which, for $a = 1$, becomes

$$\mu_u \left(X_u^L - R\right) + \mu_d \left(X_d^L - R\right) = 0 \Rightarrow R = \frac{\mu_u X_u^L + \mu_d X_d^L}{\mu_u + \mu_d}$$

The participation constraint of the creditors requires $R > 1$, thus $\frac{\mu_u}{\mu_u + \mu_d} X_u^L + \frac{\mu_d}{\mu_u + \mu_d} X_d^L > 1$ or

$$\frac{\pi u'(c_u)}{\pi u'(c_u) + (1-\pi)u'(c_d)} X_u^L + \frac{(1-\pi)u'(c_d)}{\pi u'(c_u) + (1-\pi)u'(c_d)} X_d^L > 1. \tag{42}$$

Investors optimize when consumption in the good state is higher than in the bad one otherwise they would default in the good state as well. This implies that $\frac{\pi u'(c_u)}{\pi u'(c_u) + (1-\pi)u'(c_d)} < \pi$ and $\frac{(1-\pi)u'(c_d)}{\pi u'(c_u) + (1-\pi)u'(c_d)} > (1-\pi)$. Thus, a necessary, but not sufficient, condition for inequality 42 to hold is

$$\pi X_u^L + (1-\pi) X_d^L > 1 \Rightarrow \pi > \frac{1 - X_d^L}{X_u^L - X_d^L}. \tag{43}$$

For $\pi < \frac{1 - X_d^L}{X_u^L - X_d^L}$ either the participation constraint of creditors is violated or investors' individual rationality is not satisfied. Hence, there is no investment and investors are left with their outside option, which yields a utility value of zero.

Moreover, $\frac{\partial R}{\partial \pi} < 0$ for $\pi \in \left(\frac{1 - X_d^L}{X_u^L - X_d^L}, \pi^*\right)$. This is obtained by total differentiating equation 15. Also, investors' utility is increasing at π. The proof is equivalent to the one outlined in proposition 1. Finally, investors optimize at π^* with positive investment in the safer asset. Hence, there exists a $\pi^L \in \left(\frac{1 - X_d^L}{X_u^L - X_d^L}, \pi^*\right)$ such that investors start investing in the safer asset for $\pi > \pi^L$. The probability threshold π^L is computed by setting investors' indirect utility to the outside option, i.e., zero, and solving for π. The equilibrium solution is obtained by solving equations 15, 16 and 17 for the endogenous variables, and requiring that $R > 1$. □

Proof of Corollary 1

Proof. We showed in propositions 1 and 2 that $dR/d\pi < 0$ for $\pi \in \left(\pi^L, \pi^*\right)$ and $\pi \in \left(\pi^H, 1\right)$. Also, $v = \frac{1 - \pi R}{R - \pi R}$. The numerator increases faster than the denominator as π increases and $dv/d\pi > 0$ for $\pi \in \left(\pi^L, \pi^*\right)$ and $\pi \in \left(\pi^H, 1\right)$. Combining this with equation 10, which holds for $\pi > \pi^*$, and the fact that $v(\pi)$ is continuous at π^* we get the desired result.

□

Proof of Lemma 2

Proof. Assume first that $0 < a^{sp} < 1$, hence $\phi^{sp} = \psi^{sp} = 0$. Combining equations 20 and 21 we get that:

$$\begin{aligned}
&\frac{\left(\pi\left(X_u^L - X_u^H\right) + (1-\pi)\left(X_d^L - X_d^H\right)\right)\cdot\left(1 - a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right)}{a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1} + X_d^L - X_d^H = 0\\
\Rightarrow &\pi\left(X_u^H \cdot X_d^L - X_u^L \cdot X_d^H - X_u^H + X_d^H + X_u^L - X_d^L\right) = 0
\end{aligned}$$

Thus, there is an interior solution for a only if $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$.

We now turn to the two corner solutions $a^{sp} = 1$ and $a^{sp} = 0$. Consider that the social planner chooses to invest in asset L. Then, $a^{sp} = 1$, $\phi^{sp} > 0$ and $\psi^{sp} = 0$. Equations 20 and 21 yields that:

$$\begin{aligned}
&\frac{\left(\pi\left(X_u^L - X_u^H\right) + (1-\pi)\left(X_d^L - X_d^H\right)\right)\cdot\left(1 - X_d^L\right)}{\pi X_u^L + (1-\pi)X_d^L - 1} + X_d^L - X_d^H = \pi\frac{\phi^{sp}}{w^{sp}} > 0\\
&\pi\left(X_u^H \cdot X_d^L - X_u^L \cdot X_d^H - X_u^H + X_d^H + X_u^L - X_d^L\right) > 0 \qquad (44)
\end{aligned}$$

For $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$, equation 44 is satisfied for all π. Similarly, we can show that $\psi^{sp} > 0$ only if $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$. □

Proof of Proposition 3

Proof. The second part of the proposition derives for proposition 1 and lemma 2. To show the first part, we need to compute the probability threshold such that the social planner chooses to invest in the safer asset. Denote this by π^{Lsp}. Then, $\bar{\pi}^L = \max\left\{\pi^L, \pi^{Lsp}\right\}$, where π^L is given by proposition 2. The Lagrange multipliers $\mu_u^{sp} = \pi(1 - 2\gamma c_u^{sp})$ needs to be positive. This is satisfied for $\pi > \pi^{Lsp} = \dfrac{1 - X_d^L}{X_u^L - X_d^L}$.

□

Proof of Lemma 3

Proof. The result for the bad state is obvious, as investors enjoy a fixed consumption in the competitive equilibrium, which is pinned down by the default penalty and their risk-aversion, while in the social planner's solution consumption is zero. Regarding the good state, it suffices to show that $\mu_u > \mu_u^{sp}$ for every $\pi > \pi^*$, where π^* is given by equation 13 for $a = 1$.

We prove that $\mu_u^{sp} < \mu_u$ by construction. The last inequality can be reduced to $\left(\pi\dfrac{X_u^L - X_d^L}{1 - X_d^L} - 1\right)^{-1} < \dfrac{R - X_d^L}{X_u^L - R}$ where R is given by equation 9. The left hand side (LHS) of the inequality is defined for $\pi > \underline{\pi} = \dfrac{1 - X_d^L}{X_u^L - X_d^L}$, since μ_u^{sp} should be positive. The limit of the LHS as $\pi \to \underline{\pi}^+$ is $+\infty$, while the limit as $\pi \to 1$ is $\dfrac{1 - X_d^L}{X_u^L - 1} < \dfrac{R - X_d^L}{X_u^L - R}$, since $R > 1$. Given that $\dfrac{\partial}{\partial\pi}LHS < 0$, there exists a π^{**} such that $\mu_u^{sp} < \mu_u$ for $\pi > \pi^{**}$. It remains to show that $\pi^{**} < \pi^*$. Substitute $\pi^{**} = \dfrac{1 - X_d^L}{R - X_d^L}$ in equation 13 and assume that $a(\pi^{**}) < 1$, which reduces to $(-1+\lambda)(X_u^L - R) > 0$, a contradiction. Thus, $a(\pi^{**}) > 1$. In combination with lemma 1 we get that $\pi^{**} < \pi^*$. For $\pi < \pi^*$ and $\pi > \pi^H$, we do not have an analytical solution for the equilibrium variables and need to resort to numerical approximations to compare equilibrium consumption. □

Proof of Lemma 4

Proof. In the social planner's solution, creditors receive X_d^L in the bad state of the world. In the competitive equilibrium, the repayment is strictly less irrespective of the value for a given that investors enjoy a private benefit, i.e. creditor receive $aX_d^L + (1-a)X_d^H - \frac{1-\lambda}{2\gamma}$ in the bad state. Thus, the percentage repayment for a unit of borrowed funds is lower and percentage default and the borrowing rate are higher in the competitive equilibrium. Combining this result with equations 14, 22 and lemma 3 we get that $w^{sp} > w$. □

Proof of Proposition 4

Proof. Consumption in the good and the bad state are given by $c_u = w\left[a(X_u^L - X_u^H) + X_u^H - R\right]$ and $c_d = \left[a(X_d^L - X_d^H) + X_d^H - \frac{1-\pi R}{1-\pi}\right]$, respectively. Taking the total derivative with respect to v-recall that $\frac{\partial R}{\partial v} = -(1-\pi)R^2$-and setting $\frac{\partial c_u}{\partial v} = \frac{\partial c_d}{\partial v} = 0$ we get that:

$$\frac{\partial a}{\partial v} = \frac{\pi c_u + (1-\pi)c_d}{(X_u^H - X_u^L)\,c_d + \left(X_d^L - X_d^H\right)} \quad \text{and} \quad \frac{\partial w}{\partial v} = -w\frac{\frac{\partial a}{\partial v}\left(X_d^L - X_d^H\right) - \pi R^2}{a\left(X_d^L - X_d^H\right) + X_d^H - \frac{1-\pi R}{1-\pi}}.$$

$\frac{\partial a}{\partial v} > 0$, thus investors have to reduce investment in the riskier asset for a higher delivery. The effect on borrowing depends on the payoffs of assets L and H. If $R^2 > \frac{X_d^L - X_d^H}{X_u^H - X_u^L}$, then $\frac{\partial w}{\partial v} > 0$, otherwise $\frac{\partial w}{\partial v} < 0$. The result is intuitive. If the spread of the two payoffs in the good state is higher than in the bad, i.e. $\frac{X_d^L - X_d^H}{X_u^H - X_u^L} < 1$, then investors have to compensate for the reduction in risky investment with higher borrowing in the safer asset. For $\frac{\partial w}{\partial v} < 0$, it is obvious that an exogenous increase of v results in a lower deadweight loss of default and thus higher welfare given that consumption in both states in preserved. For $\frac{\partial w}{\partial v} > 0$, the deadweight loss of default can be written as $\lambda w(R-1)$ and its derivative with respect to v is $= \lambda\frac{\partial w}{\partial v}(R-1) - w(1-\pi)R^2 = \lambda w\left[\frac{(R-1)\left(\pi R^2 - \frac{\pi c_u + (1-\pi)c_d}{(X_u^H - X_u^L)c_d + \left(X_d^L - X_d^H\right)}\left(X_d^L - X_d^H\right)\right)}{a\left(X_d^L - X_d^H\right) + X_d^H - \frac{1-\pi R}{1-\pi}}(1-\pi)\right]$. The last expression is increasing in π. Denote by $\bar{\bar{\pi}}$, the probability that the derivative becomes zero. It is easy to show that $\bar{\bar{\pi}} < 1$. Then for $\pi \in (\pi^*, \bar{\bar{\pi}})$, an increase in v results in a lower deadweight loss and higher welfare, while for $\pi > \bar{\bar{\pi}}$ this is not the case. □

Appendix II

Table 4: Initial equilibrium variables

Interest rate in $s_t = 0$	r_0=8.27%	Interest rate in $s_t = u$	r_u=8.27%
Interest rate in $s_t = d$	r_d=13.98%	Profits (reinvested) in $s_t = u$	T_u=1.93
Profits (reinvested) in $s_t = d$	T_d=1.09	Profits (distributed) in $s_t = uu$	Π_{uu}=12.60
Profits (distributed) in $s_t = ud$	Π_{ud}=0.86	Profits (distributed) in $s_t = du$	Π_{du}=2.76
Profits (distributed) in $s_t = dd$	Π_{dd}=0.86	Investment in safer asset in $s_t = 0$	$w_{0,L}$=5.76
Investment in riskier asset in $s_t = 0$	$w_{0,H}$=0.01	Loan amount in $s_t = 0$	w_0=5.74
Investment in safer asset in $s_t = u$	$w_{u,L}$=1.17	Investment in riskier asset in $s_t = u$	$w_{u,H}$=13.96
Loan amount in $s_t = u$	w_u=13.74	Investment in safer asset in $s_t = d$	$w_{d,L}$=5.80
Investment in riskier asset in $s_t = d$	$w_{d,H}$=0	Loan amount in $s_t = d$	w_d=4.75
Percentage delivery in state $s_t = u$	v_u=100%	Percentage delivery in state $s_t = d$	v_d=56.67%
Percentage delivery in state $s_t = uu$	v_{uu}=100%	Percentage delivery in state $s_t = ud$	v_{ud}=39.93%
Percentage delivery in state $s_t = du$	v_{du}=100%	Percentage delivery in state $s_t = dd$	v_{dd}=69.79%

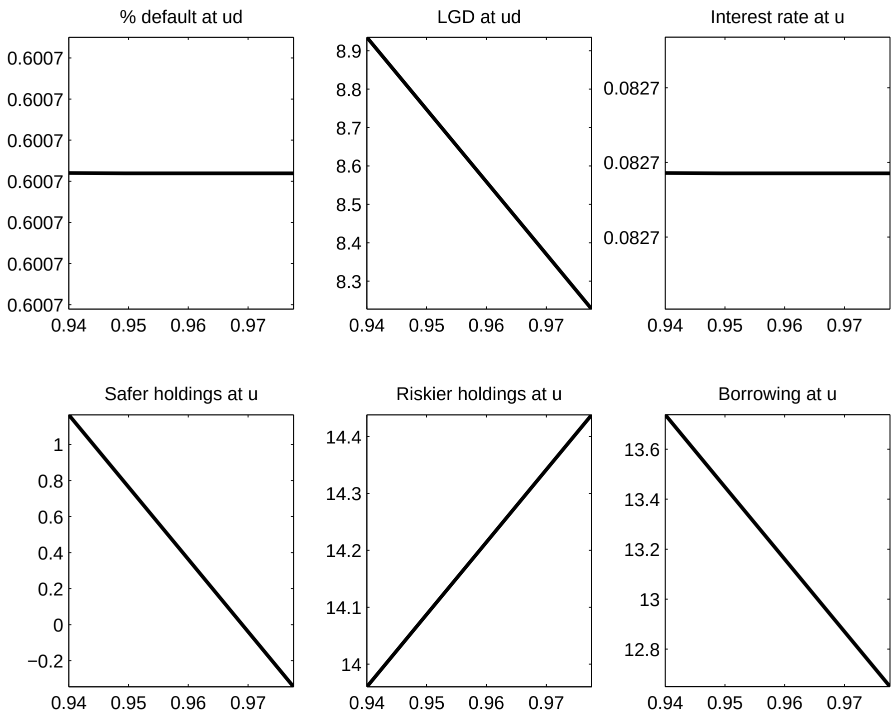

Figure 4: Default, investment and borrowing under various default penalties in state *ud*

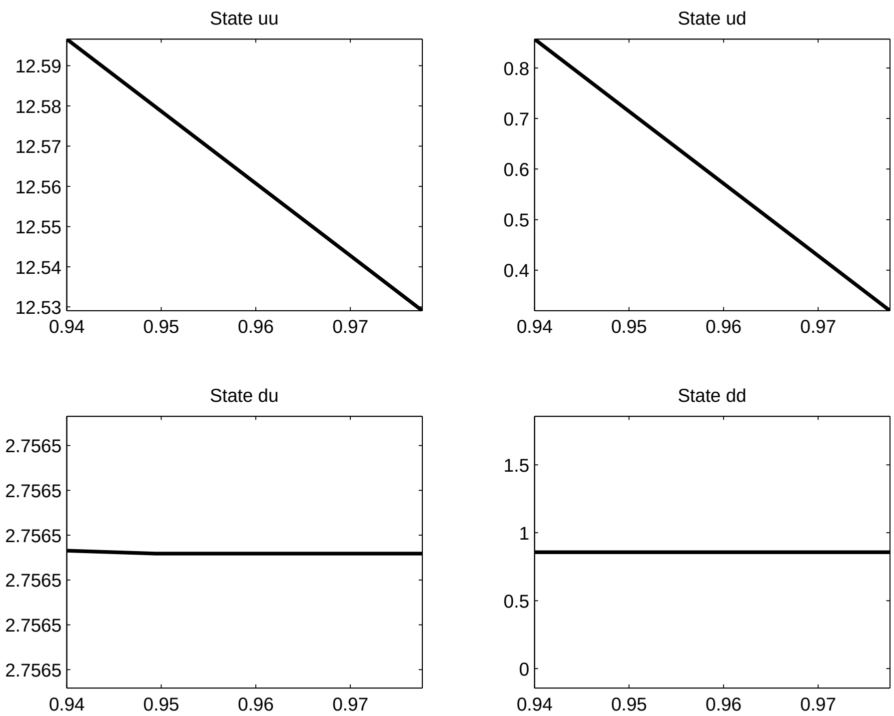

Figure 5: Final consumption under various default penalties in state *ud*

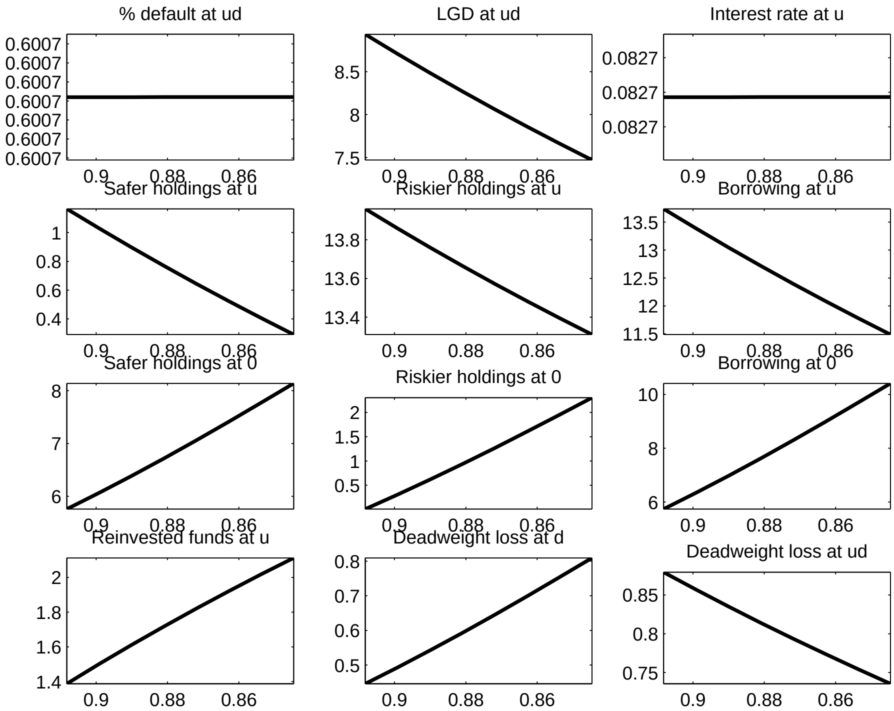

Figure 6: Default, investment and borrowing under various leverage requirements in state *u*

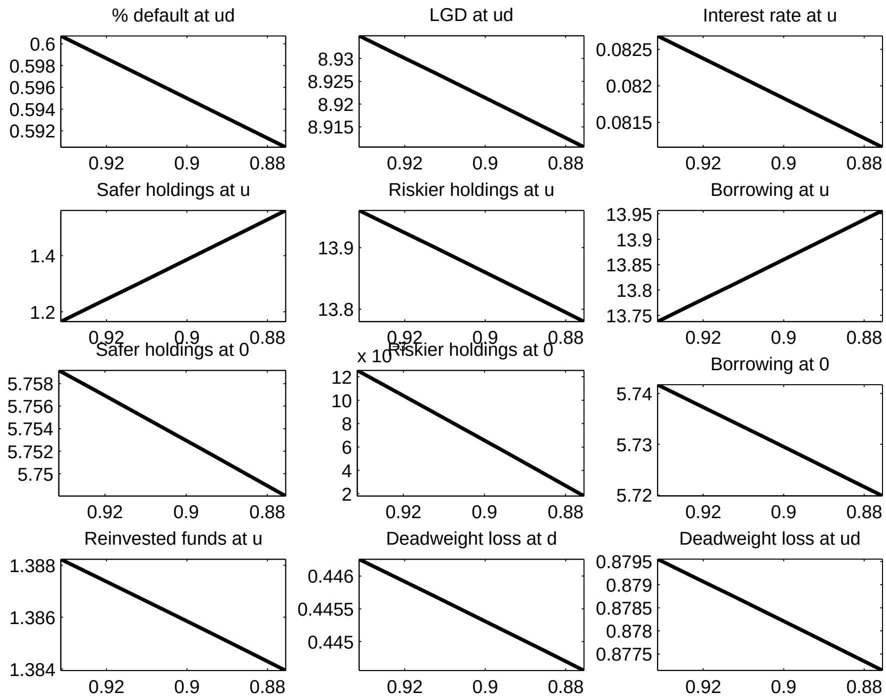

Figure 7: Default, investment and borrowing under various requirements in state *u*

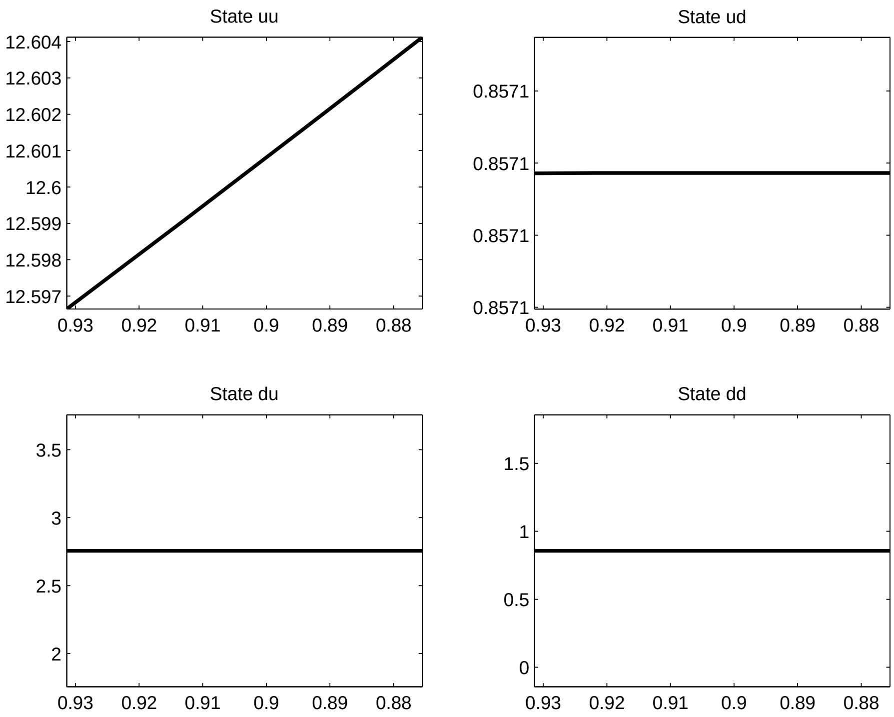

Figure 8: Final consumption under various requirements in state *u*

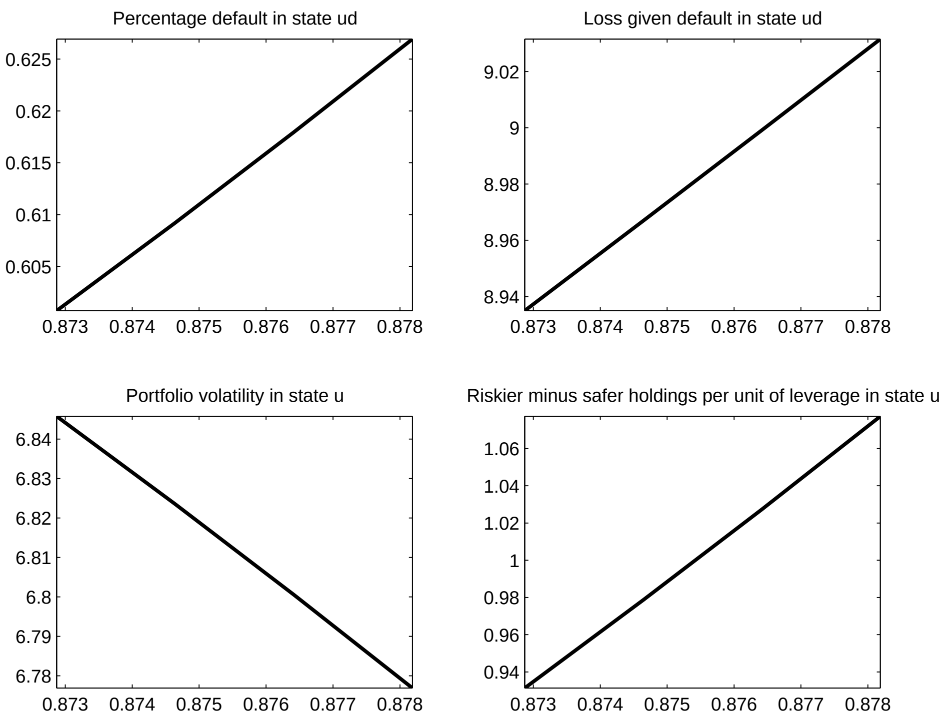

Figure 9: Optimism and riskiness

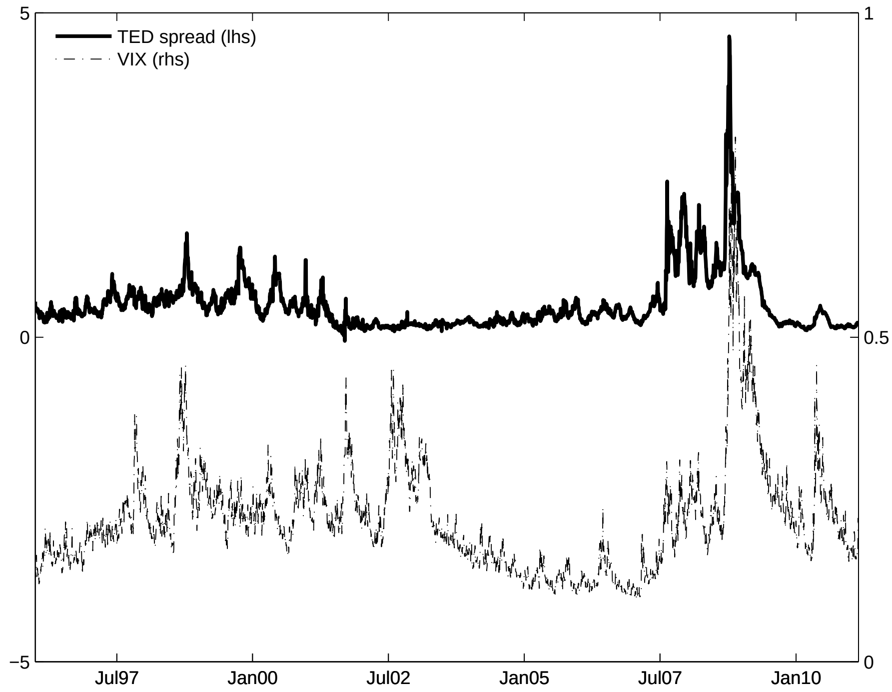

Figure 10: VIX and TED spread evolution over time

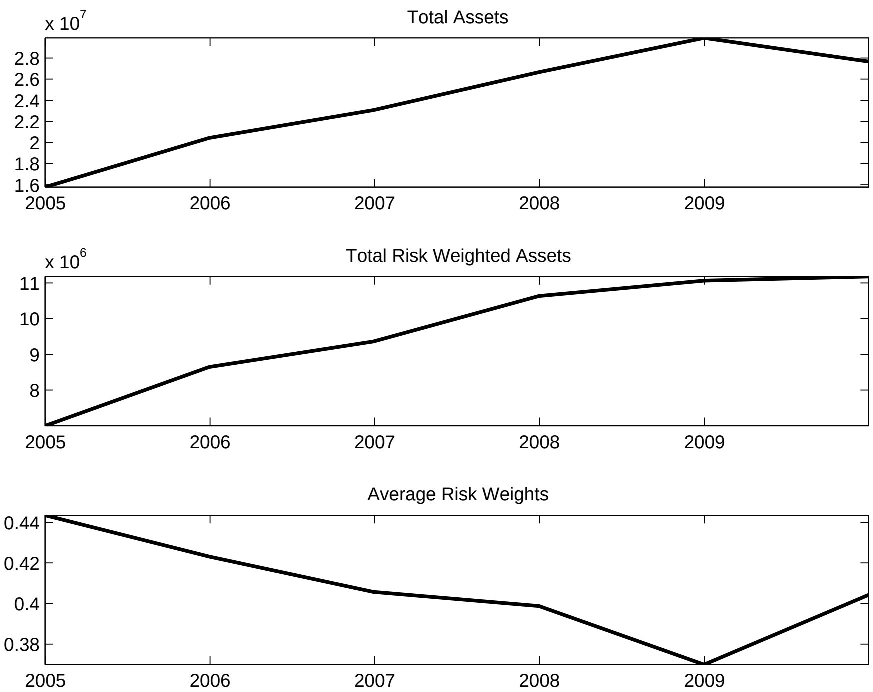

Figure 11: Average risk weights evolution over time