Short-Term Momentum and Long-Term Reversal in General Equilibrium

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Work in Progress

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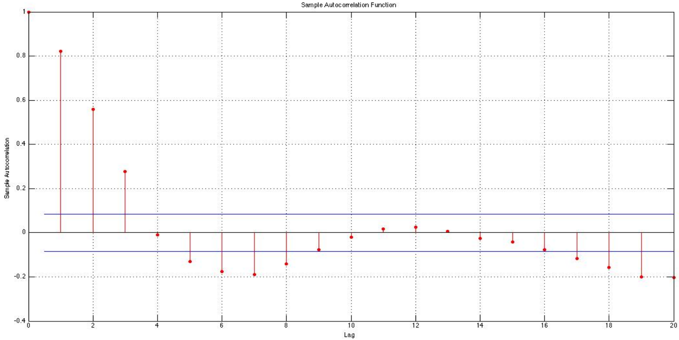

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  2. To show that a GE model with belief heterogeneity AND binding borrowing constraints can yield predictions that are consistent with these facts.

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- IF every agent knows the true dgp is Markov, THEN
  1. PO or CPO (homogeneous beliefs) allocations cannot generate both short-term momentum and long-term reversal.
  2. CPO allocations with heterogeneous beliefs can generate both short-term momentum and long-term reversal.

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$$P_i(A) = P^{\pi_i}(A), \quad \pi_i \in (0, 1) \qquad \text{(DOGMATIC beliefs)}$$

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- Preferences: $u_i(x)$ and discount rate β

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- $\{c_i\}_{i=1,2} \in Y_E^{\infty}(s_0)$ is Constrained Pareto Optimal (CPO) if there is no other $\{\widehat{c}_i\}_{i=1,2} \in Y_E^{\infty}(s_0)$ such that $U_i^{P_i}(\widehat{c}_i) > U_i^{P_i}(c_i^*)$ for all $i = 1, 2$.

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- As in Mehra and Prescott, expected utility is well defined if

$$\sup_{\xi, i} \left\{ \beta \sum_{\xi'} \pi_i(\xi' | \xi) g(\xi')^{1-\sigma} \right\} < 1. \tag{1}$$

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  - T is not a contraction.
  - For any $v_0 > v^*$, $v_n = T^n v_0$ converges from above to v^*.

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- ThePO Law of Motion

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  - The Markov process (s_t, α_t) has a unique invariant measure Ψ_{po}

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- PROPOSITION: *Suppose the true dgp is Markov and both agents know it. In any CESC, there is an invariant measure* Ψ_e *such that,*

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where $P_e \equiv P_e^{F_e}(\Psi_e, \cdot)$ and F_e is the transition function of (s_t, α_t).

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- Let $\overline{R}_{1,e} \equiv R_{1,e} - E^{P_e}\left(R_{1,e}\right).$

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- **Proposition:** *If the τ-period ahead conditional equity premium trends, then the $\tau-$order autocorrelation is positive. If the τ-period ahead conditional equity premium reverts, then the $\tau-$order autocorrelation is negative.*

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- The market belief is:

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- Roughly speaking, the one-period-ahead conditional equity premium trends if the market is more pessimistic (about a positive return) conditional on a positive return than on a negative one.

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- **Proposition:** *Suppose growth is uncorrelated and $corr_a(g) \leq 0$. If $R_{po}(H)(H) \geq R_e(L)(H)$, then the first order autocorrelation of returns is non-negative.*

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- Remark: states 1 and 2 in (b) are those where agent 1 is rich.

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- We calibrated the remaining parameters to match the same 6 moments of the household income data that Alvarez and Jermann (RFS, 2001) used.

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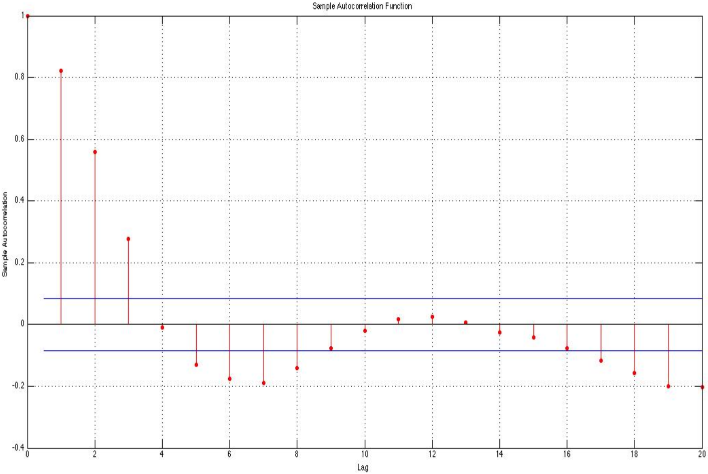

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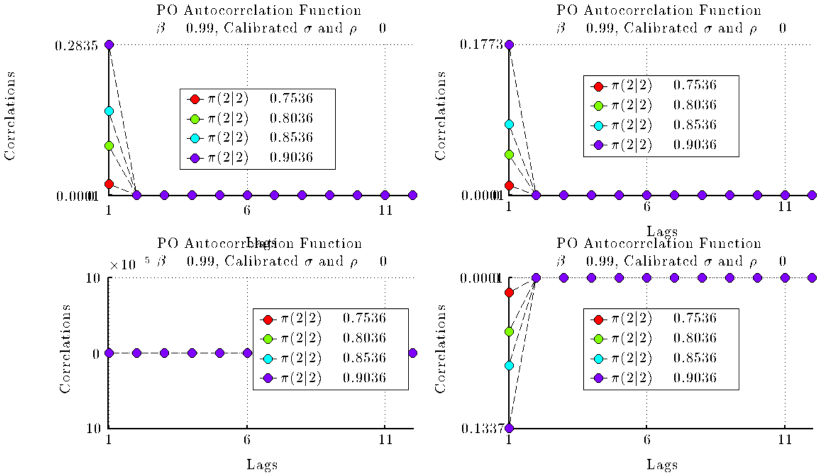

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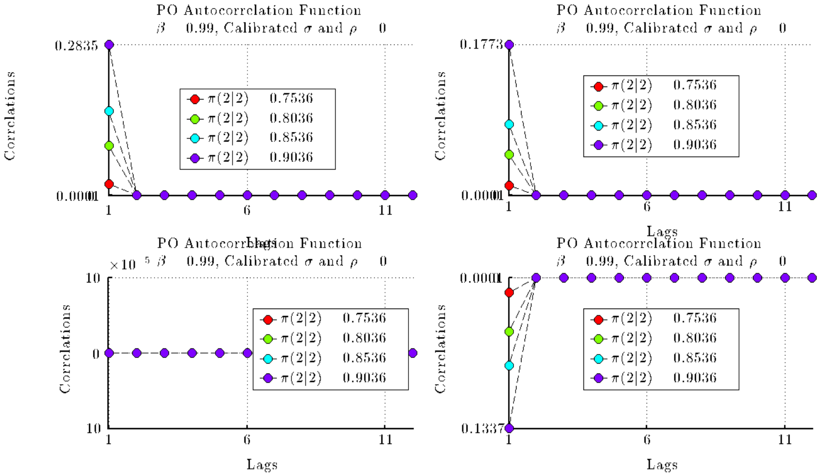

- For each belief about $\pi^*(2|2)$, we choose $\beta = .99$ and σ to match the Equity Premium of 5.91%.

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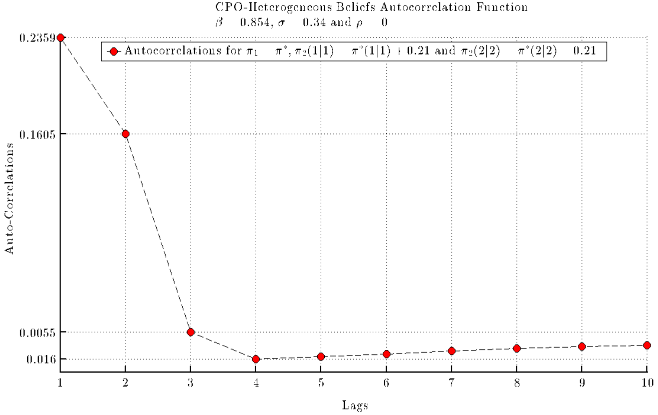
    - Agent 2 has correct beliefs about the correlation of the growth rate if $\varepsilon_1 + \varepsilon_2 = 0$

CPO Allocations

- In this example, agent 2 (correctly) beliefs the growth rate is uncorrelated:



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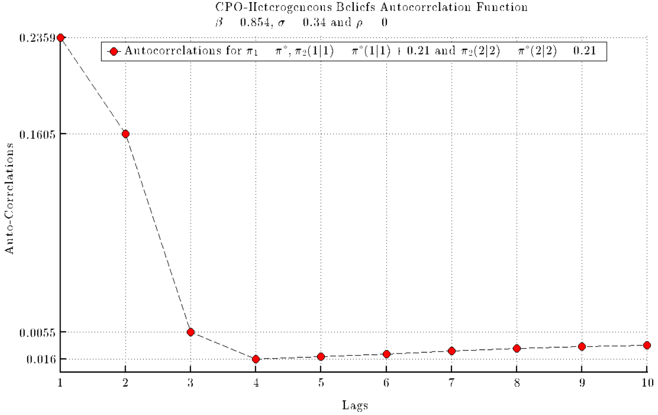

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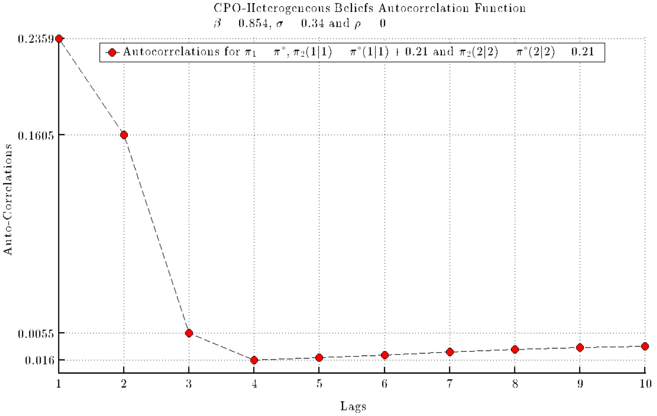

- Equity Premium is 4.19%.
- The Risk-Free rate is way too high.

Conclusions

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  - The (endogenous) dynamics of the wealth distribution induces asset returns "as if" in booms the market becomes (on average) pessimistic about the short-term and optimistic about the long-term.
  - We did not assume agents have psicological biases.