# Information Factor: <br> A One Factor Benchmark Model for Asset Pricing 

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## The Bigger Picture

- Asset prices incorporate information about the SDF and the physical probability distribution (e.g. Ross (2013)) i.e. $\mathbb{Q}$
But: asset prices alone are not sufficient to identify (without additional restrictions) the SDF (e.g. Borovička, Hansen and Scheinkman (2014))
 We bridge the two and show that we can jointly estimate from
the data the in-sample $\mathbb{Q}$ and the sources of priced risk, and
project the SDF and priced risk out-of-sample for pricing and investment purposes.


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## This Paper

(1) extract, in a nonparametric fashion (via entropy), the most likely risk neutral probability measure ( $\mathbb{Q}$ ).
(2) Use the MLE of $\mathbb{Q}$ to estimate (via an EE) the SDF and projects it out-of-sample: the Information-SDF
(3) Construct an out-of-sample mimicking portfolio: Information-Portfolio

## Main Findings:

(1) I-SDF and I-P deliver better cross-sectional pricing than multi-factor models (i.e. better encoding of pricing anomalies).
(2) I-P has high Sharpe ratio, that outperforms standard benchmarks (market, $1 / \mathrm{N}$, Value, Momentum) out-of-sample.
(3) Information factors capture novel information: portable $\alpha$ of $3.5 \%-23.8 \%$ (with high hedged Sharpe Ratio)
(4) results hold for a wide cross section of assets consisting of size, book-to-market-equity, momentum, industry, and long


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## (Other) Closely Related Literature

- Recovery of $\mathbb{Q}$, the SDF, and its characteristics: e.g. Jackwerth and Rubinstein (1996), Ait-Sahalia Lo (1998, 2000), Rosenberg and Engle (2002), Chernov (2003), Ghosh, Julliard and Taylor (2013), Hansen (2014) ....
- Entropy based inference: Stutzer (1996), Kitamura and Stutzer (2002), Julliard and Ghosh (2012), GEL literature
- Cross-sectional asset pricing:

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- Portfolio Investment: Markowitz (1952), MacKinlay and Pastor (2000), Goldfard and lyengar (2003), Jagannathan and Ma (2003), DeMiguel, Uppal, Wang (2007), DeMiguel, Garlappi, Uppal (2007)


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(1) The Big Picture

- Introduction
- (Other) Closely Related Literature
(2) Methodology: An Information Theoretic Approach
- Change of Measure and Relative Entropy
- Why Entropy?
(3) Empirical Analysis
- Cross-Sectional Pricing
- The Information Portfolio
- Portable $\alpha$ : an "Information Anomaly"
(4) Conclusion
- Appendix


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Conclusion

## Change of Measures

- Consider the vector of Euler equations (i.e. no arbitrage restrictions)

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\mathbf{0}=\mathbb{E}\left[M_{t} \mathbf{R}_{t}^{e}\right] \equiv \int M_{t} \mathbf{R}_{t}^{e} d \mathbb{P}
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where $M_{t}$ is the SDF, $\mathbf{R}_{t}^{e}$ is a vector of excess returns and $\mathbb{P}$ is the physical probability measure, and $\mathbf{0}$ is a vector of zeros.

- Under very weak regularity conditions, we have

where $\bar{x}:=\mathbb{E}\left[x_{t}\right], \mathbb{Q}$ is the risk neutral measure and $\frac{M_{t}}{M}=\frac{d \mathbb{Q}}{d \mathbb{P}}$ is the Radon-Nikodym derivative.


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\mathbf{0}=\int \frac{M_{t}}{\bar{M}} \mathbf{R}_{t}^{e} d \mathbb{P}=\int \mathbf{R}_{t}^{e} d \mathbb{Q}=\mathbb{E}^{\mathbb{Q}}\left[\mathbf{R}_{t}^{e}\right]
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- Given a set of excess returns data, we can estimate $\mathbb{Q}$ as

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\hat{\mathbb{Q}}=\underset{\mathbb{Q}}{\arg \min } D(\mathbb{P} \| \mathbb{Q}) \equiv \underset{\mathbb{Q}}{\arg \min } \int \ln \left(\frac{d \mathbb{P}}{d \mathbb{Q}}\right) d \mathbb{P} \text { s.t. } \mathbf{0}=\int \mathbf{R}_{t}^{e} d \mathbb{Q} .
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$\Rightarrow$ The above is a relative entropy (or KLIC) minimization, under the asset pricing restrictions for the cross section of returns.

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The Big Picture

Change of Measure and Relative Entropy Why Entropy?

## Why Minimal Entropy?

I: Maximum Likelihood (i.e. not only an Extremum Estimator):

- Let $\mathbf{z}_{t}$ be a sufficient statistic for the time $t$ state of the economy, then
$M: \mathbf{z} \rightarrow \mathbb{R}_{+}$and $\mathbb{R}^{e}: z \rightarrow \mathbb{R}^{N} \Rightarrow M_{t} \equiv M\left(z_{t}\right)$ and $\mathbb{R}_{t}^{e} \equiv \mathbb{R}^{e}\left(z_{t}\right)$
- Moreover, defining with $p$ and $q$ the pdf's of $\mathbb{P}$ and $\mathbb{Q}$ $D(\mathbb{P} \| Q) \equiv \int \ln \left(\frac{p(z)}{q(z)}\right) p(z) d z \equiv \mathbb{E}[\ln p(z)]-\mathbb{E}[\ln q(z)]$
- Hence, $\hat{\mathbb{Q}}$ solves: arg max $\mathbb{E}[\ln q(\mathbf{z})]$ s.t. $\mathbf{0}=\int \mathbf{R}^{e}(\mathbf{z}) q(\mathbf{z}) d \mathbf{z}$. $\Rightarrow$ non parametric MLE (Owen (2001)) of the R-N measure.
- to recover $M_{t}$ via the Radon-Nikodym derivative simply note that the MLE of $p$ is $p\left(\mathbf{z}_{t}\right)=1 / T \forall t=1, \ldots, T$

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## Why Minimal Entropy? cont'd

II: Naturally imposes non negativity of the pricing kernel
III: Can choose any number of assets and don't need a decomposition of $M$ into short and long run components (cf. Alvarez-Jermann), nor it requires a continuum of option data (cf. Ross)
IV: Occam's razor: $\widehat{\mathbb{Q}}$ adds to the physical measure the minimum amount of extra information needed to price assets.
V: Appropriate for capturing tail risk (Brown and Smith (1986))
VI: Straightforward to add conditional information (i.e. scale the moment function) and/or orthogonality restrictions.
VII: Simple to construct confidence bands (both asymptotic and by "clever" bootstrap)

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\hat{M}_{t}=M\left(\hat{\theta}, \mathbf{R}_{t}^{e}\right)=\frac{1}{T\left(1+\hat{\theta}^{\prime} \mathbf{R}_{t}^{e}\right)}, \quad \forall t \tag{1}
\end{equation*}
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where $\hat{\theta} \in \mathbb{R}^{N}$ is the Lagrange multiplier that solves the dual:

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\begin{equation*}
\hat{\theta}=\arg \min _{\theta}-\sum_{t=1}^{T} \log \left(1+\theta^{\prime} \mathbf{R}_{t}^{e}\right), \tag{2}
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## Out-of-Sample Pricing Factors

## Out-of-Sample -SDF

(1) Divide data $\mathbf{R}_{t}^{e} \in \mathbb{R}^{N}, t=0, \ldots, T-1$, into rolling subsamples of length $\bar{T}$, final date $T_{i}$, and constant $T_{i+1}-T_{i}, i=1,2, \ldots$
(2) Estimate $\theta_{T_{i}}$ over each $i$-th sample with data sampled at $t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(3) O-o-S I-SDF: $M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{+}^{e}\right), t=T_{i}+1, \ldots, T_{i+1}$

## Out-of-Sample I-Portfolio

(a) Hi sub-sample set $\cdot \hat{M_{1}}=M\left(\hat{\theta}_{T_{i}}, R_{t}^{e}\right), t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(2) Project $\hat{M}_{i, t}$ on the space of excess returns to obtain the portfolio weights $\omega_{T_{i}} \in \mathbb{R}^{N}$ (normalised to sum to 1)
(3) O-o-S I-Portfolio: $R_{t}^{I P}=\omega_{T}^{\prime} \mathbf{R}_{+}^{e}, t=T_{i}+1, \ldots, T_{i+1}$


## Out-of-Sample Pricing Factors

## Out-of-Sample I-SDF

(1) Divide data $\mathbf{R}_{t}^{e} \in \mathbb{R}^{N}, t=0, \ldots, T-1$, into rolling subsamples of length $\bar{T}$, final date $T_{i}$, and constant $T_{i+1}-T_{i}, i=1,2, \ldots$
(2) Estimate $\theta_{T_{i}}$ over each $i$-th sample with data sampled at $t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(3) O-o-S I-SDF: $M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{t}^{e}\right), t=T_{i}+1, \ldots, T_{i+1}$

## Out-of-Sample I-Portfolio

(1) $\forall i$ sub-sample set:
(2) Project $\hat{M}_{i, t}$ on the space of excess returns to obtain the portfolio weights $\omega_{T_{i}} \in \mathbb{R}^{N}$ (normalised to sum to 1 )

## Out-of-Sample Pricing Factors

## Out-of-Sample I-SDF

(1) Divide data $\mathbf{R}_{t}^{e} \in \mathbb{R}^{N}, t=0, \ldots, T-1$, into rolling subsamples of length $\bar{T}$, final date $T_{i}$, and constant $T_{i+1}-T_{i}, i=1,2, \ldots$
(2) Estimate $\theta_{T_{i}}$ over each $i$-th sample with data sampled at $t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(3) O-o-S I-SDF: $M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{t}^{e}\right), t=T_{i}+1, \ldots, T_{i+1}$

## Out-of-Sample I-Portfolio

(1) $\forall i$ sub-sample set: $\hat{M}_{i, t}=M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{t}^{e}\right), t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(2) Project $\hat{M}_{i, t}$ on the space of excess returns to obtain the portfolio weights $\omega_{T_{i}} \in \mathbb{R}^{N}$ (normalised to sum to 1)
(3) O-o-S I-Portfolio: $R_{t}^{I P}=\omega_{T_{i}}^{\prime} \mathbf{R}_{t}^{e}, t=T_{i}+1, \ldots, T_{i+1}$


## Out-of-Sample Pricing Factors

## Out-of-Sample I-SDF

(1) Divide data $\mathbf{R}_{t}^{e} \in \mathbb{R}^{N}, t=0, \ldots, T-1$, into rolling subsamples of length $\bar{T}$, final date $T_{i}$, and constant $T_{i+1}-T_{i}, i=1,2, \ldots$
(2) Estimate $\theta_{T_{i}}$ over each $i$-th sample with data sampled at $t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(3) O-o-S I-SDF: $M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{t}^{e}\right), t=T_{i}+1, \ldots, T_{i+1}$

## Out-of-Sample I-Portfolio

(1) $\forall i$ sub-sample set: $\hat{M}_{i, t}=M\left(\hat{\theta}_{T_{i}}, \mathbf{R}_{t}^{e}\right), t=T_{i}+1-\bar{T}, \ldots, T_{i}$
(2) Project $\hat{M}_{i, t}$ on the space of excess returns to obtain the portfolio weights $\omega_{T_{i}} \in \mathbb{R}^{N}$ (normalised to sum to 1)
(3) O-o-S I-Portfolio: $R_{t}^{I P}=\omega_{T_{i}}^{\prime} \mathbf{R}_{t}^{e}, t=T_{i}+1, \ldots, T_{i+1}$

Implementation: $\bar{T}=$ half sample, and $T_{i+1}-T_{i}=1$ year (June)

## Outline

(1) The Big Picture

- Introduction
- (Other) Closely Related Literature
(2) Methodology: An Information Theoretic Approach
- Change of Measure and Relative Entropy
- Why Entropy?
(3) Empirical Analysis
- Cross-Sectional Pricing
- The Information Portfolio
- Portable $\alpha$ : an "Information Anomaly"Conclusion
- Appendix

Cross-Sectional Pricing
The Information Portfolio
Portable $\alpha$

## Cross-Sectional Pricing

Table 1: 25 FF Portfolios, 1929:04-2010:12

| const. | $\lambda_{I P}$ | $\lambda_{\text {sdf }}$ | $\lambda_{R m}$ | $\lambda_{S M B}$ | $\lambda_{H M L}$ | $\bar{R}_{O L S}^{2}$ | $\bar{R}_{G L S}^{2}$ | $T^{2}$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.003 \\ & (5.73) \end{aligned}$ |  | $\begin{gathered} -0.341 \\ (-7.06) \end{gathered}$ |  |  |  | $\begin{gathered} 67.0 \\ {[57.4,100]} \end{gathered}$ | $\begin{gathered} 56.6 \\ {[54.4,100]} \end{gathered}$ | $\begin{gathered} 37.5 \\ (0.182) \end{gathered}$ | $\begin{gathered} 0.077 \\ {[0.0,0.056]} \end{gathered}$ |
| $\underset{(5.70)}{0.003}$ | $\begin{aligned} & 0.023 \\ & (7.32) \end{aligned}$ |  |  |  |  | $\begin{gathered} 68.6 \\ {[52.1,100]} \end{gathered}$ | $\begin{gathered} 59.6 \\ {[75.0,100]} \end{gathered}$ | $\begin{aligned} & 37.10 \\ & (0.096) \end{aligned}$ | $\begin{gathered} 0.072 \\ {[0.0,0.064]} \end{gathered}$ |
| $\underset{(3.40)}{0.011}$ |  |  | $\begin{gathered} -0.004 \\ (-1.41) \end{gathered}$ |  |  | $\begin{gathered} 3.97 \\ {[-3.82,59.8]} \end{gathered}$ | $\begin{gathered} 28.8 \\ {[19.4,40.9]} \end{gathered}$ | $\begin{aligned} & 71.64 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.128 \\ {[0.045,0.145} \end{gathered}$ |
| $\underset{(2.50)}{0.011}$ |  |  | $\begin{gathered} -0.006 \\ (-1.53) \end{gathered}$ | $\underset{(3.86)}{0.002}$ | $\underset{(6.87)}{0.004}$ | $\begin{gathered} 71.3 \\ {[48.9,89.2]} \end{gathered}$ | $\begin{gathered} 40.9 \\ {[31.4,82.8]} \end{gathered}$ | $\begin{aligned} & 51.46 \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.096 \\ {[0.040,0.100} \end{gathered}$ |
| $\begin{aligned} & 0.004 \\ & (1.20) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.025 \\ & (5.26) \\ & \hline \end{aligned}$ |  | $\begin{gathered} 0.0004 \\ (0.132) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.003 \\ & (6.75) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (8.48) \\ & \hline \end{aligned}$ | $\begin{gathered} 86.1 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{gathered} 62.2 \\ {[,]} \end{gathered}$ | $\begin{array}{r} 29.25 \\ (0.083) \\ \hline \end{array}$ | $\begin{gathered} 0.058 \\ {[,]} \end{gathered}$ |
| Panel B: Quarterly |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.028 \\ & (11.33) \end{aligned}$ |  | $\begin{aligned} & -5.46 \\ & (-3.13) \end{aligned}$ |  |  |  | $\begin{gathered} 26.8 \\ {[4.38,100]} \end{gathered}$ | $\begin{gathered} 30.8 \\ {[16.9,69.5]} \end{gathered}$ | $\begin{aligned} & \hline 41.31 \\ & (0.469) \end{aligned}$ | $\begin{gathered} 0.332 \\ {[0.0,0.438]} \end{gathered}$ |
| $\underset{(1.29)}{0.002}$ | $\begin{aligned} & 0.135 \\ & (11.17) \end{aligned}$ |  |  |  |  | $\begin{gathered} 83.7 \\ {[86.9,100]} \end{gathered}$ | $\begin{gathered} 51.6 \\ {[52.2,100]} \end{gathered}$ | $\begin{aligned} & 28.77 \\ & (0.533) \end{aligned}$ | $\begin{gathered} 0.227 \\ {[0.0,0.164]} \end{gathered}$ |
| $\underset{(2.79)}{0.024}$ |  |  | $\begin{aligned} & -0.002 \\ & (-0.308) \end{aligned}$ |  |  | $\begin{gathered} -3.92 \\ {[-4.33,25.7]} \end{gathered}$ | $\begin{gathered} 8.50 \\ {[4.88,11.9]} \end{gathered}$ | $\begin{aligned} & 80.90 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.431 \\ {[0.14,0.42]} \end{gathered}$ |
| $\underset{(2.19)}{0.028}$ |  |  | $\begin{aligned} & -0.015 \\ & (-1.15) \end{aligned}$ | $\underset{(4.95)}{0.007}$ | $\begin{aligned} & 0.013 \\ & (7.20) \end{aligned}$ | $\begin{gathered} 74.7 \\ {[0.52,0.94]} \end{gathered}$ | $\begin{gathered} 17.7 \\ {[0.018,0.56]} \end{gathered}$ | $\begin{aligned} & 59.34 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.351 \\ {[0.066,0.37]} \end{gathered}$ |
| $\begin{aligned} & 0.005 \\ & (0.403) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.108 \\ & (3.71) \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.010 \\ & (0.768) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.008 \\ & (6.60) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.012 \\ & (7.85) \\ & \hline \end{aligned}$ | $\begin{gathered} 83.4 \\ {[,]} \end{gathered}$ | $\begin{gathered} 46.5 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{array}{r} 31.06 \\ (0.054) \\ \hline \end{array}$ | $\underset{[,]}{0.217}$ |

The Big Picture
Methodology
Empirical Analysis

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The Information Portfolio
Portable $\alpha$

## Cross-Sectional Pricing cont'd

Table 2: 10 Momentum Portfolios, 1929:04-2010:12

| const. | $\lambda_{I P}$ | $\lambda_{s d f}$ | $\lambda_{R m}$ | $\lambda_{S M B}$ | $\lambda_{\text {HML }}$ | $\lambda_{M O M}$ | $\bar{R}_{O L S}^{2}$ | $\bar{R}_{G L S}^{2}$ | $T^{2}$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \hline 0.004 \\ & (9.99) \end{aligned}$ |  | $\begin{gathered} -0.27 \\ (-9.16) \end{gathered}$ |  |  |  |  | $\begin{gathered} 90.2 \\ {[78.9,100]} \end{gathered}$ | $\begin{gathered} 68.1 \\ {[56.8,100]} \end{gathered}$ | $\begin{gathered} 12.37 \\ (0.345) \end{gathered}$ | $\begin{gathered} 0.024 \\ {[0.0,0.031]} \end{gathered}$ |
| $\begin{aligned} & 0.002 \\ & (7.80) \end{aligned}$ | $\begin{gathered} 0.034 \\ (13.66) \end{gathered}$ |  |  |  |  |  | $\begin{gathered} 95.4 \\ {[92.0,100]} \end{gathered}$ | $\begin{gathered} 83.6 \\ {[84.3,100]} \end{gathered}$ | $\begin{gathered} 6.37 \\ (0.657) \end{gathered}$ | $\begin{gathered} 0.012 \\ {[0.0,0.015]} \end{gathered}$ |
| $\begin{aligned} & 0.014 \\ & (2.04) \end{aligned}$ |  |  | $\begin{aligned} & -0.009 \\ & (-1.45) \end{aligned}$ |  |  |  | $\begin{gathered} 10.9 \\ {[-12.4,60.1]} \end{gathered}$ | $\begin{gathered} -2.0 \\ {[-11.7,16.9]} \end{gathered}$ | $\begin{aligned} & 40.15 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.074 \\ {[0.026,0.117} \end{gathered}$ |
| $\begin{aligned} & 0.022 \\ & (1.27) \end{aligned}$ |  |  | $\begin{aligned} & -0.013 \\ & (-0.75) \end{aligned}$ | $\begin{aligned} & -0.011 \\ & (-0.61) \end{aligned}$ | $\begin{aligned} & -0.032 \\ & (-1.18) \end{aligned}$ | $\begin{aligned} & 0.007 \\ & (6.02) \end{aligned}$ | $\begin{gathered} 78.9 \\ {[32.9,100]} \end{gathered}$ | $\begin{gathered} 2.59 \\ {[-73.1,98.4]} \end{gathered}$ | $\begin{gathered} 8.81 \\ (0.399) \end{gathered}$ | $\begin{gathered} 0.044 \\ {[0.0,0.823]} \end{gathered}$ |
| $\begin{aligned} & 0.003 \\ & (0.42) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.039 \\ & (4.36) \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.004 \\ & (0.60) \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.005 \\ & (-0.80) \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.015 \\ & (-1.60) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.006 \\ (14.64) \\ \hline \end{gathered}$ | $\begin{gathered} 97.7 \\ {[,]} \end{gathered}$ | $\begin{gathered} 82.1 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{gathered} 2.61 \\ (0.625) \\ \hline \end{gathered}$ | $\begin{gathered} 0.006 \\ {[,]} \\ \hline \end{gathered}$ |
| Panel B: Quarterly |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.008 \\ & (6.39) \end{aligned}$ |  | $\begin{gathered} -1.07 \\ (-7.93) \end{gathered}$ |  |  |  |  | $\begin{gathered} 87.3 \\ {[67.9,100]} \end{gathered}$ | $\begin{gathered} 75.2 \\ {[72.2,100]} \end{gathered}$ | $\begin{gathered} 8.12 \\ (0.529) \end{gathered}$ | $\begin{gathered} 0.056 \\ {[0.0,0.080]} \end{gathered}$ |
| $\begin{aligned} & 0.006 \\ & (5.60) \end{aligned}$ | $\begin{aligned} & 0.107 \\ & (9.90) \end{aligned}$ |  |  |  |  |  | $\begin{gathered} 95.4 \\ {[86.3,100]} \end{gathered}$ | $\begin{gathered} 78.6 \\ {[76.9,100]} \end{gathered}$ | $\begin{gathered} 7.36 \\ (0.581) \end{gathered}$ | $\begin{gathered} 0.047 \\ {[0.0,0.048]} \end{gathered}$ |
| $\begin{aligned} & 0.038 \\ & (2.40) \end{aligned}$ |  |  | $\begin{aligned} & -0.024 \\ & (-1.59) \end{aligned}$ |  |  |  | $\begin{gathered} 14.4 \\ {[-11.4,81.2]} \end{gathered}$ | $\begin{gathered} -6.49 \\ {[-12.5,13.7]} \end{gathered}$ | $\begin{aligned} & 39.55 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.226 \\ {[0.077,0.38]} \end{gathered}$ |
| $\begin{aligned} & 0.061 \\ & (1.08) \end{aligned}$ |  |  | $\begin{aligned} & -0.050 \\ & (-0.83) \end{aligned}$ | $\begin{aligned} & 0.038 \\ & (1.03) \end{aligned}$ | $\begin{aligned} & 0.021 \\ & (0.63) \end{aligned}$ | $\begin{aligned} & 0.022 \\ & (5.46) \end{aligned}$ | $\begin{gathered} 75.4 \\ {[20.9,96.5]} \end{gathered}$ | $\begin{gathered} 1.56 \\ {[-73.6,81.2]} \end{gathered}$ | $\begin{gathered} 9.55 \\ (0.187) \end{gathered}$ | $\begin{gathered} 0.131 \\ {[0.0,1.346]} \end{gathered}$ |
| $\begin{aligned} & 0.036 \\ & (2.43) \end{aligned}$ | $\begin{aligned} & 0.084 \\ & (3.69) \end{aligned}$ |  | $\begin{aligned} & -0.024 \\ & (-1.54) \end{aligned}$ | $\begin{aligned} & 0.032 \\ & (3.28) \end{aligned}$ | $\begin{gathered} 0.0005 \\ (0.05) \end{gathered}$ | $\begin{gathered} 0.021 \\ (20.80) \end{gathered}$ | $\begin{gathered} 98.4 \\ {[,]} \end{gathered}$ | $\begin{gathered} 86.7 \\ {[,]} \end{gathered}$ | $\begin{gathered} 1.35 \\ (0.853) \end{gathered}$ | $\begin{gathered} 0.014 \\ {[,]} \end{gathered}$ |

The Big Picture

Cross-Sectional Pricing
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Portable $\alpha$

## Cross-Sectional Pricing cont'd

Table 3: 25 Portfolios Formed on Long-Term Reversal and Size, 1929:04-2010:12

| const. | $\lambda_{I P}$ | $\lambda_{\text {sdf }}$ | $\lambda_{R m}$ | $\lambda_{S M B}$ | $\lambda_{H M L}$ | $\bar{R}_{O L S}^{2}$ | $\bar{R}_{G L S}^{2}$ | $T^{2}$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.006 \\ & (7.06) \end{aligned}$ |  | $\begin{aligned} & -0.22 \\ & (-2.18) \end{aligned}$ |  |  |  | $\begin{gathered} 13.5 \\ {[-0.83,100]} \end{gathered}$ | $\begin{gathered} 61.5 \\ {[56.4,100]} \end{gathered}$ | $\begin{aligned} & 25.07 \\ & (0.583) \end{aligned}$ | $\begin{gathered} 0.047 \\ {[0.0,0.036]} \end{gathered}$ |
| $\underset{(2.13)}{0.002}$ | $\underset{(7.98)}{0.024}$ |  |  |  |  | $\begin{gathered} 72.3 \\ {[66.4,100]} \end{gathered}$ | $\begin{gathered} 68.0 \\ {[71.0,100]} \end{gathered}$ | $\begin{aligned} & 18.35 \\ & (0.844) \end{aligned}$ | $\begin{gathered} 0.037 \\ {[0.0,0.007]} \end{gathered}$ |
| $\begin{aligned} & 0.005 \\ & (1.38) \end{aligned}$ |  |  | $\underset{(0.78)}{0.002}$ |  |  | $\begin{gathered} -1.6 \\ {[-4.10,44.7]} \end{gathered}$ | $\begin{gathered} 10.1 \\ {[1.83,21.7]} \end{gathered}$ | $\begin{aligned} & 58.14 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.103 \\ {[0.028,0.111]} \end{gathered}$ |
| $\begin{aligned} & 0.002 \\ & (0.68) \end{aligned}$ |  |  | $\begin{aligned} & 0.002 \\ & (0.93) \end{aligned}$ | $\begin{gathered} 0.001 \\ (1.62) \end{gathered}$ | $\underset{(4.96)}{0.007}$ | $\begin{gathered} 74.3 \\ {[56.7,100]} \end{gathered}$ | $\begin{gathered} 26.1 \\ {[12.4,100]} \end{gathered}$ | $\begin{aligned} & 40.37 \\ & (0.018) \end{aligned}$ | $\begin{gathered} 0.077 \\ {[0.01,0.070]} \end{gathered}$ |
| $\begin{aligned} & -0.002 \\ & (-0.86) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.022 \\ & (5.50) \end{aligned}$ |  | $\underset{(2.77)}{0.006}$ | $\begin{gathered} 0.003 \\ (3.99) \end{gathered}$ | $\begin{aligned} & 0.004 \\ & (3.39) \end{aligned}$ | $84.5$ | $\begin{gathered} 66.4 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 16.69 \\ & (0.673) \end{aligned}$ | $\begin{gathered} 0.033 \\ {[,]} \\ \hline \end{gathered}$ |
| Panel B: Quarterly |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.023 \\ & (11.80) \end{aligned}$ |  | $\begin{gathered} -0.33 \\ (-0.300) \end{gathered}$ |  |  |  | $\begin{gathered} -3.94 \\ {[,]} \end{gathered}$ | $\begin{gathered} 27.2 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 54.50 \\ & (0.000) \end{aligned}$ | $\underset{[,]}{0.291}$ |
| $\underset{(3.13)}{0.008}$ | $\begin{aligned} & 0.075 \\ & (5.71) \end{aligned}$ |  |  |  |  | $\begin{gathered} 56.8 \\ {[39.0,100]} \end{gathered}$ | $\begin{gathered} 56.5 \\ {[56.9,100]} \end{gathered}$ | $\begin{aligned} & 23.96 \\ & (0.735) \end{aligned}$ | $\begin{gathered} 0.167 \\ {[0.0,0.064]} \end{gathered}$ |
| $\begin{aligned} & 0.008 \\ & (1.04) \end{aligned}$ |  |  | $\begin{aligned} & 0.013 \\ & (1.81) \end{aligned}$ |  |  | $\begin{gathered} 8.7 \\ {[,]} \end{gathered}$ | $\begin{gathered} 1.33 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 68.46 \\ & (0.000) \end{aligned}$ | $\begin{aligned} & 0.372 \\ & {[,]} \end{aligned}$ |
| $\begin{aligned} & 0.006 \\ & (0.651) \end{aligned}$ |  |  | $\underset{(1.03)}{0.009}$ | $\underset{(2.59)}{0.005}$ | $\underset{(4.76)}{0.020}$ | $\underset{[,]}{77.8}$ | $\begin{gathered} 11.96 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 48.86 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.301 \\ {[,]} \end{gathered}$ |
| $\begin{aligned} & 0.002 \\ & (0.329) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.070 \\ & (5.28) \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.012 \\ & (1.80) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.011 \\ (5.13) \\ \hline \end{array}$ | $\begin{aligned} & 0.007 \\ & (1.47) \end{aligned}$ | $86.7$ | $53.0$ | $\begin{array}{r} 22.61 \\ (0.309) \\ \hline \end{array}$ | $0.153$ |

Cross-Sectional Pricing
The Information Portfolio
Portable $\alpha$

## Cross-Sectional Pricing cont'd

Table 4: Small, Large, Growth, Value, Winners, Losers, 10 Industry, 1929:04-2010:12

| const. | $\lambda_{\text {IP }}$ | $\lambda_{\text {sdf }}$ | $\lambda_{R m}$ | $\lambda_{\text {SMB }}$ | $\lambda_{\text {HML }}$ | $\lambda_{\text {MOM }}$ | $\bar{R}_{\text {OLS }}^{2}$ | $\bar{R}_{G L S}^{2}$ | $T^{2}$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |  |  |  |
| $\underset{(2.49)}{0.001}$ |  | $\begin{aligned} & -1.12 \\ & (-16.7) \end{aligned}$ |  |  |  |  | $\underset{[,]}{94.9}$ | $\underset{[,]}{88.0}$ | $\begin{gathered} 5.39 \\ (0.980) \end{gathered}$ | $\underset{[,]}{0.013}$ |
| $\begin{aligned} & 0.003 \\ & (7.45) \end{aligned}$ | $\begin{aligned} & 0.027 \\ & (8.62) \end{aligned}$ |  |  |  |  |  | $83.0$ | $\begin{gathered} 84.2 \\ {[,]} \end{gathered}$ | $\begin{array}{r} 9.89 \\ (0.770) \end{array}$ | $\underset{[,]}{0.019}$ |
| $\underset{(2.02)}{0.007}$ |  |  | $\begin{gathered} -0.002 \\ (-0.70) \end{gathered}$ |  |  |  | $\begin{gathered} -3.5 \\ {[,]} \end{gathered}$ | $\underset{[,]}{-1.17}$ | $\begin{aligned} & 62.76 \\ & (0.000) \end{aligned}$ | $0.112$ |
| $\underset{(1.25)}{0.003}$ |  |  | $\underset{(1.09)}{0.002}$ | $\underset{(2.42)}{0.002}$ | $\underset{(1.92)}{0.002}$ | $\underset{(8.78)}{0.009}$ | $\underset{[,]}{84.8}$ | $40.3$ | $\begin{aligned} & 27.23 \\ & (0.004) \end{aligned}$ | $\underset{[,]}{0.052}$ |
| $\begin{aligned} & -0.003 \\ & (-1.58) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.035 \\ & (8.10) \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.007 \\ & (4.45) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (5.44) \end{aligned}$ | $\begin{gathered} 0.0003 \\ (0.51) \end{gathered}$ | $\begin{aligned} & 0.008 \\ & (10.65) \\ & \hline \end{aligned}$ | $\begin{gathered} 94.7 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{gathered} 84.8 \\ {[,]} \end{gathered}$ | $\begin{array}{r} 5.73 \\ (0.838) \\ \hline \end{array}$ | $\underset{[,]}{0.012}$ |
| Panel B: Quarterly |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \hline 0.014 \\ & (12.44) \end{aligned}$ |  | $\begin{aligned} & \hline-3.88 \\ & (-6.92) \end{aligned}$ |  |  |  |  | $\underset{[,]}{75.8}$ | $\underset{[,]}{60.9}$ | $\begin{aligned} & \hline 17.15 \\ & (0.248) \end{aligned}$ | $\underset{\substack{0.145 \\[,]}}{ }$ |
| $\underset{(5.81)}{0.009}$ | $\begin{aligned} & 0.100 \\ & (6.61) \end{aligned}$ |  |  |  |  |  | $\begin{gathered} 74.0 \\ {[,]} \end{gathered}$ | $\begin{gathered} 79.9 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 10.85 \\ & (0.698) \end{aligned}$ | $\underset{[,]}{0.077}$ |
| $\underset{(2.10)}{0.021}$ |  |  | $\begin{gathered} -0.005 \\ (-0.52) \end{gathered}$ |  |  |  | $\begin{gathered} -5.1 \\ {[,]} \end{gathered}$ | $\underset{[,]}{0.73}$ | $\begin{aligned} & 67.17 \\ & (0.000) \end{aligned}$ | $0.358$ |
| $\underset{(1.47)}{0.012}$ |  |  | $\xrightarrow[(0.45)]{0.004}$ | $\underset{(2.50)}{0.006}$ | $\underset{(2.01)}{0.005}$ | $\underset{(7.97)}{0.027}$ | ${ }_{[,]}^{82 .} 9$ | $\underset{[,]}{37.3}$ | $\begin{aligned} & 27.86 \\ & (0.003) \end{aligned}$ | $\underset{[,]}{0.178}$ |
| $\begin{aligned} & -0.002 \\ & (-0.26) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.107 \\ & (4.62) \end{aligned}$ |  | $\begin{aligned} & 0.017 \\ & (2.16) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.008 \\ (3.89) \\ \hline \end{array}$ | $\begin{aligned} & 0.002 \\ & (1.10) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.024 \\ & (8.43) \\ & \hline \end{aligned}$ | $\begin{gathered} 89.6 \\ {[,]} \end{gathered}$ | $\begin{array}{r} 75.8 \\ {[,]} \\ \hline \end{array}$ | $\begin{array}{r} 8.27 \\ (0.602) \\ \hline \end{array}$ | $\underset{\substack{0.062 \\[,]}}{ }$ |

The Big Picture

Cross-Sectional Pricing
The Information Portfolio
Portable $\alpha$

## Cross-Sectional Pricing cont'd

Table 5: 30 Industry Portfolios, 1929:04-2010:12

| const. | $\lambda_{I P}$ | $\lambda_{\text {sdf }}$ | $\lambda_{R m}$ | $\lambda_{S M B}$ | $\lambda_{H M L}$ | $\bar{R}_{O L S}^{2}$ | $\bar{R}_{G L S}^{2}$ | $T^{2}$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.002 \\ & (2.58) \end{aligned}$ |  | $\begin{aligned} & -0.15 \\ & (-3.97) \end{aligned}$ |  |  |  | $33.8$ | $\underset{[,]}{50.1}$ | $\begin{aligned} & 12.61 \\ & (0.994) \end{aligned}$ | $\underset{[,]}{0.023}$ |
| $\underset{(1.42)}{0.001}$ | $\underset{(5.74)}{0.022}$ |  |  |  |  | $\begin{gathered} 52.5 \\ {[,]} \end{gathered}$ | $\begin{gathered} 65.6 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 8.56 \\ & (1.00) \end{aligned}$ | $\begin{aligned} & 0.016 \\ & {[,]} \end{aligned}$ |
| $\begin{aligned} & 0.006 \\ & (3.55) \end{aligned}$ |  |  | $\begin{gathered} -0.0001 \\ (-0.12) \end{gathered}$ |  |  | $\begin{gathered} -3.52 \\ {[,]} \end{gathered}$ | $-2.30$ | $\begin{aligned} & 26.69 \\ & (0.535) \end{aligned}$ | $\begin{gathered} 0.047 \\ {[,]} \end{gathered}$ |
| $\begin{gathered} 0.006 \\ (2.46) \end{gathered}$ |  |  | $\begin{aligned} & -0.001 \\ & (-0.36) \end{aligned}$ | $\underset{(0.26)}{0.0004}$ | $\begin{aligned} & -0.001 \\ & (-0.75) \end{aligned}$ | $\begin{gathered} -8.33 \\ {[,]} \end{gathered}$ | $\begin{gathered} -7.32 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 26.00 \\ & (0.463) \end{aligned}$ | $\underset{[,]}{0.046}$ |
| $\begin{aligned} & 0.000 \\ & (0.023) \end{aligned}$ | $\begin{aligned} & 0.025 \\ & (5.79) \end{aligned}$ |  | $\begin{aligned} & 0.005 \\ & (2.93) \end{aligned}$ | $\begin{gathered} -0.0001 \\ (-0.14) \end{gathered}$ | $\begin{aligned} & -0.001 \\ & (-1.07) \end{aligned}$ | $\begin{gathered} 60.4 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{gathered} 64.6 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 7.55 \\ & (1.00) \end{aligned}$ | $\begin{gathered} 0.015 \\ {[,]} \end{gathered}$ |
| Panel B: Quarterly |  |  |  |  |  |  |  |  |  |
| $\underset{(2.93)}{0.011}$ |  | $\begin{aligned} & -1.72 \\ & (-1.64) \end{aligned}$ |  |  |  | $5.55$ | $\underset{[,]}{31.4}$ | $\begin{aligned} & 23.49 \\ & (0.708) \end{aligned}$ | $\begin{gathered} 0.133 \\ {[,]} \end{gathered}$ |
| $\begin{aligned} & 0.010 \\ & (3.48) \end{aligned}$ | $\begin{gathered} 0.043 \\ (2.66) \end{gathered}$ |  |  |  |  | $17.3$ | $39.9$ | $\begin{aligned} & 20.66 \\ & (0.839) \end{aligned}$ | $\underset{[,]}{0.116}$ |
| $\begin{aligned} & 0.017 \\ & (3.55) \end{aligned}$ |  |  | $\begin{aligned} & 0.001 \\ & (0.127) \end{aligned}$ |  |  | $\begin{gathered} -3.51 \\ {[,]} \end{gathered}$ | $\begin{gathered} 2.68 \\ {[,]} \end{gathered}$ | $\begin{aligned} & 35.16 \\ & (0.165) \end{aligned}$ | $\underset{[,]}{0.187}$ |
| $\underset{(2.51)}{0.023}$ |  |  | $\begin{aligned} & -0.006 \\ & (-0.62) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (0.71) \end{aligned}$ | $\frac{-0.002}{(-0.43)}$ | $\begin{gathered} -8.40 \\ {[,]} \end{gathered}$ | $\underset{[,]}{-1.64}$ | $\begin{aligned} & 33.57 \\ & (0.146) \end{aligned}$ | $\underset{[,]}{0.181}$ |
| $\begin{aligned} & 0.020 \\ & (2.58) \end{aligned}$ | $\underset{(2.42)}{0.042}$ |  | $\begin{aligned} & -0.003 \\ & (-0.350) \end{aligned}$ | $\begin{gathered} 0.006 \\ (1.81) \end{gathered}$ | $\begin{aligned} & -0.002 \\ & (-0.562) \end{aligned}$ | $\begin{gathered} 26.6 \\ {[,]} \end{gathered}$ | $\begin{gathered} 42.9 \\ {[,]} \\ \hline \end{gathered}$ | $\begin{aligned} & 16.29 \\ & (0.906) \end{aligned}$ | $\begin{gathered} 0.098 \\ {[,]} \\ \hline \end{gathered}$ |

## The Information Portfolio

Table 8: Summary Statistics of Information Portfolio \& Returns

| Assets | Mean | Vol | SR | Skew | Kurt | CEQ | Ret-gain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |
| Market - Risk Free | 0.004 | 0.045 | 0.091 | -0.567 | 5.028 | 0.003 |  |
| Value - Growth | 0.004 | 0.029 | 0.139 | -0.034 | 5.440 | 0.004 |  |
| Momentum Factor | 0.007 | 0.044 | 0.164 | -1.419 | 13.65 | 0.006 |  |
| $\underset{\substack{R^{I P} \\(F F 25)}}{c_{20}}$ | $\begin{aligned} & 0.021 \\ & (0.007) \end{aligned}$ | $\begin{aligned} & 0.073 \\ & (0.051) \end{aligned}$ | $\underset{\substack{0.288 \\(0.128)}}{ }$ | $\begin{gathered} 0.384 \\ (-0.575) \end{gathered}$ | $\begin{aligned} & 5.541 \\ & (5.589) \end{aligned}$ | $\begin{gathered} 0.018 \\ (0.006) \end{gathered}$ | 0.008 |
| $\underset{(10 \text { Momentum) }}{R^{I P}}$ | $\begin{aligned} & 0.030 \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.127 \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.235 \\ & (0.085) \end{aligned}$ | $\begin{gathered} -0.352 \\ (-0.326) \end{gathered}$ | $\begin{aligned} & 8.022 \\ & (4.793) \end{aligned}$ | $\begin{aligned} & 0.022 \\ & (0.003) \end{aligned}$ | 0.007 |
| $\begin{gathered} R_{(25} \mathrm{L}-\mathrm{T} \text { Reversal \& Size) } \end{gathered}$ | $\begin{aligned} & 0.013 \\ & (0.007) \end{aligned}$ | $\begin{aligned} & 0.064 \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.206 \\ & (0.137) \end{aligned}$ | $\begin{gathered} -0.212 \\ (-0.444) \end{gathered}$ | $\begin{aligned} & 5.111 \\ & (5.865) \end{aligned}$ | $\begin{aligned} & 0.011 \\ & (0.006) \end{aligned}$ | 0.003 |
| $(\mathrm{S}, \mathrm{~B}, \mathrm{G}, \mathrm{~V}, \mathrm{w}, \mathrm{~L}, 10 \mathrm{Ind} .)$ | $\begin{aligned} & 0.027 \\ & (0.005) \end{aligned}$ | $\underset{(0.046)}{0.088}$ | $\begin{aligned} & 0.306 \\ & (0.106) \end{aligned}$ | $\underset{(-0.490)}{\substack{0.679}}$ | $\underset{(4.953)}{6.180}$ | $\begin{aligned} & 0.023 \\ & (0.004) \end{aligned}$ | 0.009 |
| $\begin{gathered} R^{\prime P} \\ (30 \text { Industry }) \end{gathered}$ | $\begin{aligned} & 0.002 \\ & (0.005) \end{aligned}$ | $\begin{aligned} & 0.083 \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.018 \\ & (0.112) \end{aligned}$ | $\begin{gathered} 0.040 \\ (-0.522) \end{gathered}$ | $\begin{aligned} & 6.318 \\ & (5.708) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.001 \\ (0.004) \end{gathered}$ | -0.004 |

Note: $1 / N$ portfolio in parenthesis.

## The Information Portfolio

Table 8: Summary Statistics of Information Portfolio \& Returns

| Assets | Mean | Vol | SR | Skew | Kurt | CEQ | Ret-gain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel B: Quarterly |  |  |  |  |  |  |  |
| Market - Risk Free | 0.013 | 0.087 | 0.150 | -0.435 | 3.635 | 0.009 |  |
| Value - Growth | 0.012 | 0.060 | 0.204 | 0.109 | 4.754 | 0.010 |  |
| Momentum Factor | 0.020 | 0.081 | 0.254 | -1.411 | 10.13 | 0.017 |  |
| $\begin{gathered} R^{I P} \\ (\text { FFF25) } \end{gathered}$ | $\begin{aligned} & 0.080 \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.194 \\ & (0.103) \end{aligned}$ | $\begin{gathered} 0.413 \\ (0.207) \end{gathered}$ | $\begin{gathered} 0.410 \\ (-0.183) \end{gathered}$ | $\begin{aligned} & 3.955 \\ & (3.576) \end{aligned}$ | $\begin{gathered} 0.061 \\ (0.016) \end{gathered}$ | 0.022 |
| $\underset{(10 \text { Momentum) }}{R^{I P}}$ | $\begin{aligned} & 0.085 \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.239 \\ & (0.093) \end{aligned}$ | $\begin{aligned} & 0.354 \\ & (0.143) \end{aligned}$ | $\begin{gathered} -0.090 \\ (-0.231) \end{gathered}$ | $\begin{aligned} & 5.295 \\ & (3.805) \end{aligned}$ | $\begin{aligned} & 0.056 \\ & (0.009) \end{aligned}$ | 0.020 |
| $\begin{gathered} R^{(P)} \\ \text { (25 L-T Reversal \& Size) } \end{gathered}$ | $\begin{gathered} 0.042 \\ (0.023) \end{gathered}$ | $\begin{aligned} & 0.134 \\ & (0.104) \end{aligned}$ | $\begin{aligned} & 0.313 \\ & (0.220) \end{aligned}$ | $\begin{gathered} 0.168 \\ (-0.057) \end{gathered}$ | $\begin{aligned} & 3.833 \\ & (3.865) \end{aligned}$ | $\begin{aligned} & 0.033 \\ & (0.018) \end{aligned}$ | 0.010 |
|  | $\begin{aligned} & 0.083 \\ & (0.016) \end{aligned}$ | $\begin{aligned} & 0.173 \\ & (0.090) \end{aligned}$ | $\begin{aligned} & 0.480 \\ & (0.175) \end{aligned}$ | $\begin{gathered} 0.181 \\ (-0.315) \end{gathered}$ | $\begin{aligned} & 3.463 \\ & (3.794) \end{aligned}$ | $\begin{aligned} & 0.068 \\ & (0.012) \end{aligned}$ | 0.027 |
| $\begin{gathered} R^{1 P} \\ \text { (30 Industry) } \\ \hline \end{gathered}$ | $\begin{aligned} & 0.007 \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.180 \\ & (0.093) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.041 \\ & (0.186) \end{aligned}$ | $\begin{gathered} 0.029 \\ (-0.298) \end{gathered}$ | $\begin{aligned} & 2.934 \\ & (3.942) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.000 \\ (0.013) \end{gathered}$ | -0.013 |

Note: $1 / N$ portfolio in parenthesis.

## $R^{I P}$ Cumulated Returns

## Long-Short Strategy



## $R^{I P}$ portfolio weights

Time Series of Portfolio Weights


The Big Picture
Methodology
Empirical Analysis

Cross-Sectional Pricing
The Information Portfolio
$R^{I P}: S, M, V$ and $G$ aggregated weights
Monthly Information Portfolio Weights Extracted from FF25


Cross-Sectional Pricing
The Information Portfolio Portable $\alpha$

## Portable $\alpha$ : an "Information Anomaly"

Table 7: $R^{I P} \alpha$ With Respect to FF and Momentum Factors, 1929:04-2010:12

| Assets | $\alpha(\%)$ | $\beta_{R m}$ | $\beta_{S M B}$ | $\beta_{H M L}$ | $\beta_{M O M}$ | $\bar{R}_{\text {OLS }}^{2}$ | Info-Ratio |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Monthly |  |  |  |  |  |  |  |
| FF25 | 1.31 | 0.47 | 0.21 | 1.30 |  | 26.9 | 0.211 |
| Momentum | $(4.89)$ | $(7.44)$ | $(2.41)$ | $(13.82)$ |  |  |  |
|  | 1.37 | 0.94 | -0.30 | 0.26 | 1.67 | 36.1 | 0.135 |
| Reversal \& Size | 0.69 | 0.40 | 0.35 | 0.92 |  | 20.7 | 0.121 |
|  | $(2.79)$ | $(6.92)$ | $(4.39)$ | $(10.56)$ |  |  |  |
| V, W, L, 10 Ind. | 0.93 | 0.59 | 0.35 | 1.28 | 1.25 | 46.6 | 0.145 |
| Industry | $(3.28)$ | $(9.00)$ | $(3.90)$ | $(12.87)$ | $(19.70)$ |  |  |
|  | 0.29 | 0.81 | -0.87 | -0.59 |  | 28.3 | 0.041 |
|  | $(0.96)$ | $(11.43)$ | $(-8.79)$ | $(-5.57)$ |  |  |  |

Annualised $\alpha$ : 3.5\%-17.7\%

## Portable $\alpha$ : an "Information Anomaly" cont'd

Table 7: $R^{I P} \alpha$ With Respect to FF and Momentum Factors, 1929:04-2010:12

| Assets | $\alpha$ (\%) | $\beta_{R m}$ | $\beta_{\text {SMB }}$ | $\beta_{\text {HML }}$ | $\beta_{\text {MOM }}$ | $\bar{R}_{\text {OLS }}$ | Info-Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel B: Quarterly |  |  |  |  |  |  |  |
| FF25 | $\begin{aligned} & 5.49 \\ & (4.01) \end{aligned}$ | $\begin{aligned} & 0.59 \\ & (3.29) \end{aligned}$ | $\underset{(1.72)}{0.43}$ | $\begin{aligned} & 1.12 \\ & (4.85) \end{aligned}$ |  | 15.4 | 0.310 |
| 10 Momentum | $\begin{aligned} & 2.63 \\ & (1.77) \end{aligned}$ | $\begin{aligned} & 1.20 \\ & (6.52) \end{aligned}$ | $\begin{gathered} -0.21 \\ (-0.82) \end{gathered}$ | $\begin{aligned} & 0.43 \\ & (1.76) \end{aligned}$ | $\underset{(10.76)}{1.92}$ | 42.6 | 0.147 |
| 25 L-T Reversal \& Size | $\begin{aligned} & 2.39 \\ & (2.57) \end{aligned}$ | $\begin{aligned} & 0.50 \\ & (4.16) \end{aligned}$ | $\underset{(0.06)}{0.01}$ | $\begin{aligned} & 0.93 \\ & (5.94) \end{aligned}$ |  | 18.0 | 0.199 |
| S, B, G, V, W, L, 10 Ind. | $\begin{aligned} & 3.21 \\ & (2.77) \end{aligned}$ | $\begin{aligned} & 0.67 \\ & (4.70) \end{aligned}$ | $\begin{aligned} & 0.29 \\ & (1.42) \end{aligned}$ | $\begin{aligned} & 1.16 \\ & (6.15) \end{aligned}$ | $\begin{aligned} & 1.23 \\ & (8.89) \end{aligned}$ | 33.6 | 0.230 |
| 30 Industry | $\begin{aligned} & 1.16 \\ & (1.00) \\ & \hline \end{aligned}$ | $\begin{aligned} & 1.02 \\ & (6.73) \\ & \hline \end{aligned}$ | $\begin{array}{r} -1.01 \\ (-4.72) \\ \hline \end{array}$ | $\begin{array}{r} -0.66 \\ (-3.37) \\ \hline \end{array}$ |  | 29.1 | 0.077 |

Annualised $\alpha: 4.7 \%-23.8 \%$

## Why does it work?

- The maximum Sharpe Ratio portfolio must price the cross-section of assets (since it pins down the Capital Allocation Line).
- Standard methods for forecasting this portfolio use the asset side i.e. tangency of the Efficient Frontier and the Capital Allocation Line
$\Rightarrow$ They don't work empirically due to measurement error on $N+N(N+1) / 2$ parameters (mean plus covariance matrix) e.g. DeMiguel, Garlappi, Uppal (2007) with $N=25$ this method requires 350 parameters!
- Our method uses the SDF side i.e. tangency of the Indifference Curve to the Capital Allocation Line
$\Rightarrow$ need to estimate only N parameters of the dual solution.


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E.g. : with $\mathrm{N}=25$ this method requires 350 parameters!
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## Outline

(1) The Big Picture

- Introduction
- (Other) Closely Related Literature
(2) Methodology: An Information Theoretic Approach
- Change of Measure and Relative Entropy
- Why Entropy?
(3) Empirical Analysis
- Cross-Sectional Pricing
- The Information Portfolio
- Portable $\alpha$ : an "Information Anomaly"

4) Conclusion

- Appendix


## Conclusion

The paper provides a novel (ML) method for the extraction of the SDF, and its mimicking portfolio, that:

- Prices assets out-of-sample as well or better than canonical factor models.
- Identifies a large amount of risk not spanned by traditional factors.
- Provides a tradeable portfolio that:
(1) is statistically indistinguishable from the maximum Sharpe Ratio portfolio;
(2) has very high Sharpe Ratio even when hedged w.r.t. traditional factors;
(3) has low turnover, hence low trading costs.

