Skewing the Odds

Taking risks for rank-based rewards

Dawei Fang[1] Thomas Noe[2]

[1]Department of Economics
U. of Gothenburg

[2]Said Business School/Balliol College
Oxford

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RISK TAKING IN RANK-BASED COMPETITION: IT'S IMPORTANT

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  - Optimal Bayesian persuasion (Meyer, 1991), (Kamenica and Gentzkow, 2011)
- policy level, in a world where agents are motivated by rank-based incentives, contest risk taking has significant social externalities:
  - stability of the financial system
  - speed of technical innovation
  - corporate investment and financial policies
  - the cost of military and diplomatic conflicts
- The social externalities generated by contestant risk taking are context specific
- But contestant risk-taking itself simply depends on rank-based rewards
- So a general theory of rank-based risk taking can be applied in specific contexts to mitigate externalities

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- Develops a theory of risk taking in rank-based competitions with
  - An arbitrary (finite) number of contestants,
  - An arbitrary number of distinct contest rewards,
  - in which agent strategy sets are restricted only by a capacity constraint.
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GENERALITY IS IMPORTANT

- Contests with multiple contestants, varying reward levels, and endogenous choice of risk distributions induce fundamentally different equilibrium behavior than "simpler" contests
- Most rules for allocating rank-based rewards proposed in economics literature (e.g., Ziph's law, PAM models) entail non-binary reward structures.
- Differences between the solution of random contest games and other games (e.g., all pay auctions) are obscured if attention is restricted to two-player models.

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▸ Why?

Endogenous performance distributions matter

- For a very large class of rank-based prize allocation systems, which includes the power law, Gilbrat's law, winner-take all, convex, and concave allocations,
- equilibrium performance distributions *never* satisfy the symmetry and unimodality restrictions imposed by parametric models of risk taking. (e.g., (Klette and Meza, 1986), (Hvide, 2002), (Gaba, Tsetlin and Winkler, 2004), (Goel and Thakor, 2008), and (Kräkel, 2008))

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MULTIPLE PRIZE LEVELS (> 2) MATTER

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  - Interior mode performance distributions
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- The mechanical link between the dispersion and skewness breaks

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EXECUTIVE SUMMARY OF RESULTS

- Optimal contest strategies are always bounded.
- The upper bound is increasing in the ratio between the reward for the highest performance level and the *fair share* division of rewards.
- The modality of the performance distribution depends on the sign pattern of the second differences of the rank-ordered rewards.
- Increasing *real inequality* between rewards increases risk taking.
- Increasing the convexity of the relation between reward and rank increases the skewness of contestant performance.

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Risk taking with fixed contest payoff functions

THE CONTEST

- An allocation of rewards based on performance level, x. Rewards determined by
- *Contest payoff function*, P, which is
  - non-negative,
  - nondecreasing, and
  - continuous

  function defined over the compact interval $[0, \bar{x}]$, $\bar{x} > 0$.
- $\min_{x \in [0,\bar{x}]} P(x) := \underline{v} \geq 0$
- $\max_{x \in [0,\bar{x}]} P(x) := \bar{v} > \underline{v}$

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THE CONTESTANT'S CHALLENGE

- Pick a non negative random performance variable, $\tilde{X}$, with associated *performance distribution*, F, to maximize $\mathbb{E}[P(\tilde{X})]$
- subject only to the capacity constraint:

$$\mathbb{E}[\tilde{X}] = \int_0^{\bar{x}} x\, dF(x) \leq \mu.$$

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SOLUTION TO CONTESTANT'S PROBLEM

LEMMA 1. OPTIMALITY CONDITIONS FOR F

A probability distribution function solving the contestants problem exists. For any such solution, there exist multipliers $\alpha \geq 0$ and $\beta > 0$ such that the solution, F, satisfies the condition

$$P(x) \leq \alpha + \beta x \quad \forall x \geq 0;$$
$$dF\{x \geq 0 : P(x) < \alpha + \beta x\} = 0.$$

VALUE MINIMIZING CONTEST DESIGNS

- Now think of the contest designer's problem
- Contest designer aims minimize the contestant's payoffs over all contest payoff functions
  - with performance range $[0, \bar{x}]$ and
  - minimum and maximum prizes, $\underline{v}$ and $\bar{v}$.

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OBSERVATIONS

- The contestant's optimal strategy will exploit any convexity in the payoff schedule through randomization
- $\Rightarrow$ contestant payoff actually obtained equals the payoff against the concave majorant of the payoff function, $\hat{P}$.
- *Concave majorant*: the least concave function that majorizes P. The majorant is also the
  - pointwise infimum of all upper support lines of P
  - At any performance level x, $\hat{P}(x)$ equals the maximum payoff the contestant can attain over using any random performance strategy with mean x.
  - An optimal design (designer's perspective) always produces a uniform concave upper envelope.

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Risk taking in contest games

RANDOM PERFORMANCE CONTESTS

- In random performance contests, the contest payoff function is not fixed by a designer.
- Rather, it is the product of the strategies played by the contestants.
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THE CONTEST GAME

- $n \geq 2$ homogeneous contestants who simultaneously choose performance distributions
- All the contestants pick a distribution, F, subject to only to the constraint that the distribution has non-negative support and its expectation is less than μ
- There are n prizes, v_i is the value of the ith prize, $v_1 \leq \cdots \leq v_n$.
- Each contestant's realized performance is a draw from the performance distribution she choses.
- Prize allocation is *rank-based*: highest realized performance wins the nth prize,...
- Ties are broken by a proration rule.

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LEMMA 8

Equilibrium random performance is the same for all contestants and is

a. invariant under increasing affine transformations of the prize schedule,
b. proportional to capacity, μ,
c. and distributed continuously over its support $[0, \mu\, n((v_n - v_1)/V)]$, where V represents total prize payments in excess of the lowest prize, v_1.

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fraction of normalized real gains received by highest performer : fair-share fraction

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$I_p(i, n-i)$	*Component Beta distributions* with parameters $a = i$ and $b = n - i$.

The shape of performance distributions

CARROTS, STICKS, AND RISKS

- Consider a contest prize schedule with three distinct prize levels and four contestants.
  - A "stick" worth 0.0.
  - A "hay bundle" worth 0.5
  - A "carrot" worth 1.0
- Low performers receive the sticks, middling performers the hay, and top performers the carrots.
- The number of carrots and stick combined with the number of contestants determines the prize vector.
- Consider three possible carrot-and-stick prize schedules:
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- How will increasing the number of sticks affect risk-taking?

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STICKS

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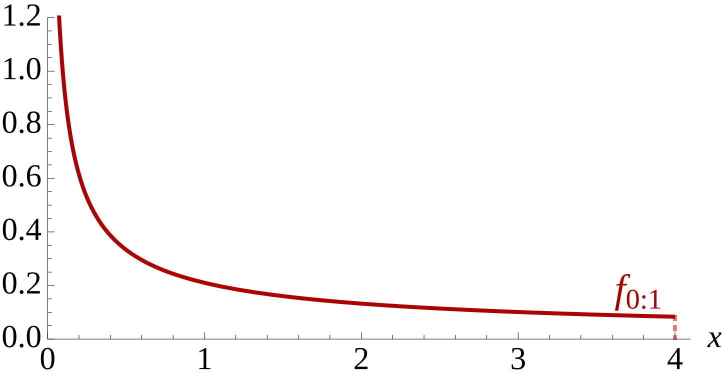
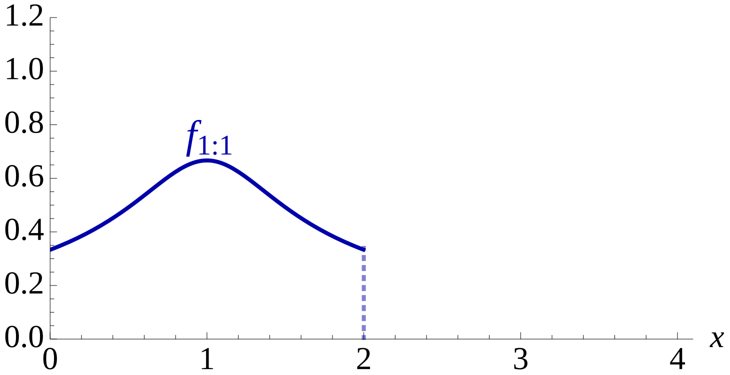


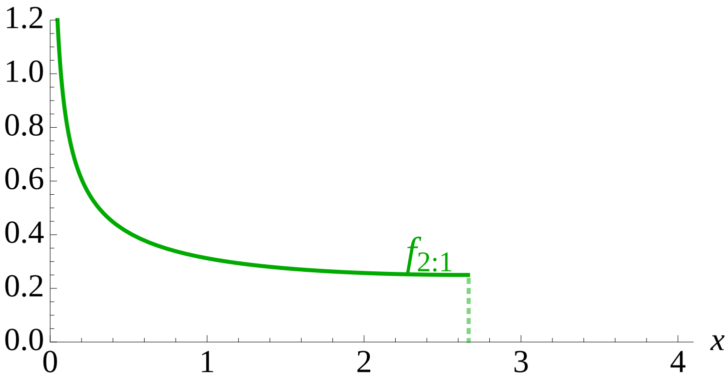
STICKS

One stick/One carrot



STICKS

Two sticks/One carrot



Sticks

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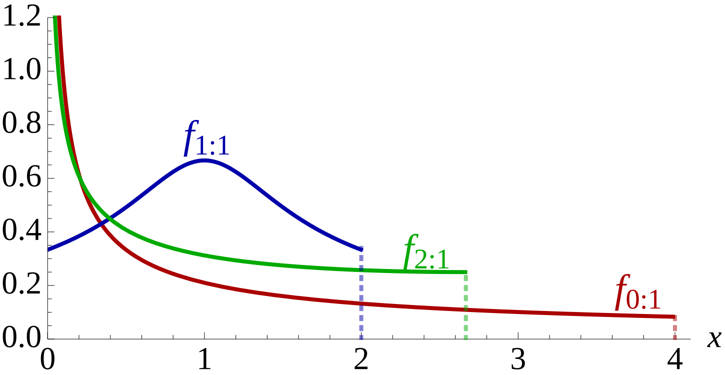

TAIL-BEHAVIOR OF PERFORMANCE DISTRIBUTIONS [PROPOSITION 3]

- Determined by the second differences of the prize schedule: roughly,
  - if the second difference for the lowest prize level with a non vanishing second difference is positive (negative) then PDF is initially decreasing (increasing);
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- *Example*: Tail behavior of balanced carrot/stick contests.

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	two-carrots/two-sticks	one-carrot/one stick

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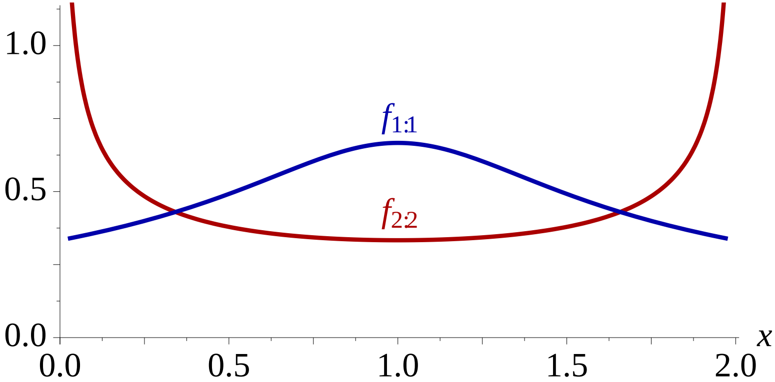
CARROTS, STICKS, AND MODALITY OF CONTESTANT PERFORMANCE DISTRIBUTIONS



GLOBAL MODALITY

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If the sequence of second differences of the prize schedule has at most one sign change, then equilibrium performance PDF that is either quasiconvex or quasiconcave, and thus the global behavior of the PDF is determined by its tail behavior.

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Performance dispersion and the prize schedule

DISPERSION AND INEQUALITY

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Let F and G be two distributions. F is more dispersed than G in the sense of convex order if, for all convex functions, $w : \Re^+ \to \Re$

$$\int w(x)\, dF(x) \geq \int w(x)\, dG(x).$$

INEQUALITY

Let x and y be ordered nonnegative vectors $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$, $x_1 \leq \ldots \leq x_n$, $y_1 \leq \ldots \leq y_n$, where n is a natural number. Then x *majorizes* y if

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REAL PRIZE INEQUALITY AND DISPERSION

- For a prize vector $v = (v_1, \dots, v_n)$, define its *normalized real gain* vector as
$$\bar{v}^r = (\bar{v}_1^r, \dots, \bar{v}_n^r) = \left(\frac{v_1 - v_1}{\sum_{i=1}^n (v_i - v_1)}, \dots, \frac{v_n - v_1}{\sum_{i=1}^n (v_i - v_1)} \right).$$
- Increasing the inequality of the *normalized real gains* offered by the prize schedule, always increases the dispersion of equilibrium performance distribution.

PROPOSITION 5. PRIZE IN EQUALITY $\Rightarrow$ PERFORMANCE DISPERSION

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- Reducing the inequality of the (nominal) prize schedule can *increase* the dispersion of the performance distribution:
- *Example:* Ultra-robin hood transfer from the highest prize to the lowest prize:

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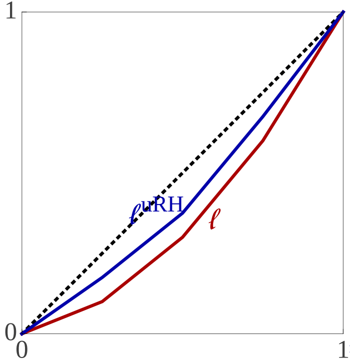
- The ultra-RobinHood transfer reduces nominal prize inequality, measured by Lorentz curve of the prize vector
- The ultra-RobinHood transfer increases real prize inequality, measured by the Lorentz curve for normalized real gains.
- Increase in real prize inequality leads to *more* performance dispersion, i.e., more risk taking.



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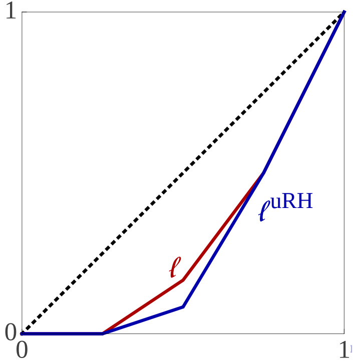
- Increase in real prize inequality leads to *more* performance dispersion, i.e., more risk taking.



REAL NOT NOMINAL INEQUALITY MATTERS

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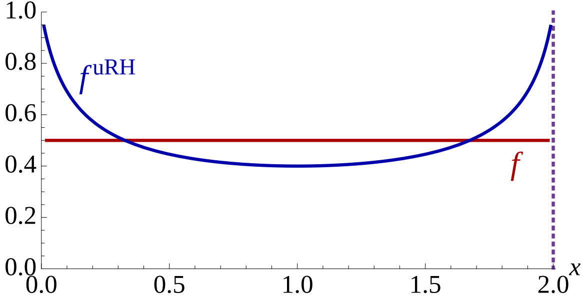

Effect of contest size on risk taking

Effect of new entrants

- Adding a matching number of new entrants and minimum prizes to a contest increases real prize inequality.
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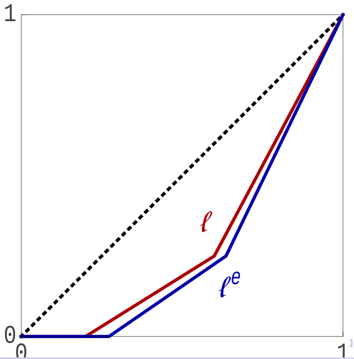
Parameters?



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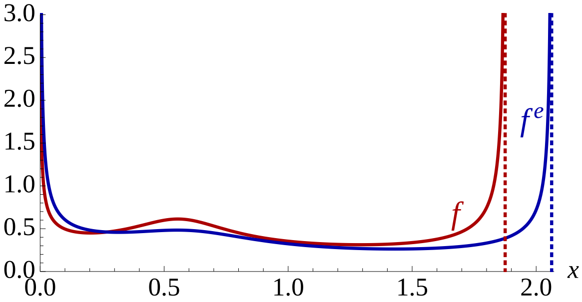

EFFECT OF s-FOLD SCALING OF THE PRIZE SCHEDULE

Up scaling contest size does not affect real prize inequality.

Parameters?

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Up scaling contest size does not affect real prize inequality.
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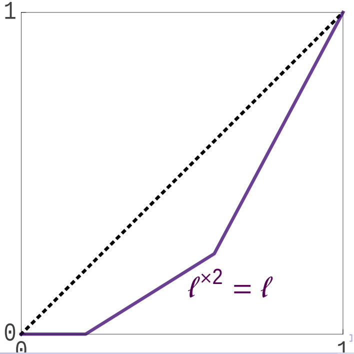

Scaling in the limit

- As the scaling factor increases without limit, the performance distribution converges to a discrete distribution whose support is determined by the number of non-vanishing prize differences of the original prize schedule.
- Even when the scaling factor is modest, the clustering of performance around the limit points is quite apparent.

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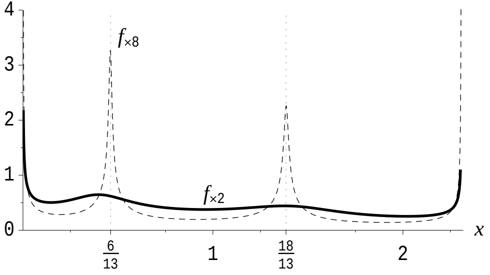

▸ Parameters?

The skewness of performance distributions

SKEWNESS: DEFINITIONS

VANZWET SKEWNESS ORDER

Let F and G be two CDFs, both of which are strictly increasing and twice continuously differentiable on the corresponding support. The distribution G is more skewed to the right (left) than F in the sense of (Zwet, 1964) if and only if $G^{-1} \circ F$ is convex (concave) on the support of F.

PRIZE VALUE TRANSFORMATION FUNCTION

A function $h : \Re \to \Re$ is a *prize value transformation function* if it is nondecreasing. The *prize value transformation* of prize vector v generated by h, which we represent by v^h, is defined by $v_i^h = h(v_i)$ for all $i \in \{1, \ldots, n\}$.

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CONVEX TRANSFORMATION INCREASE SKEWNESS

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- Consider the power law (coefficient .50) prize schedule for a contest with eight prizes:

$$v_p(i) = (n+1-i)^{-1/2}, \quad i = 1, 2 \ldots 8.$$

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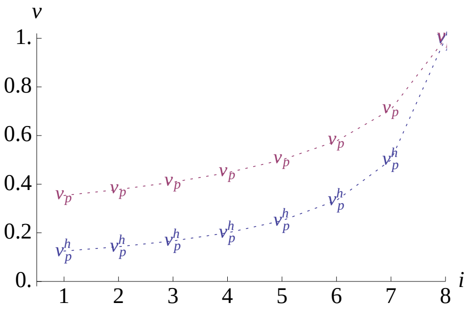

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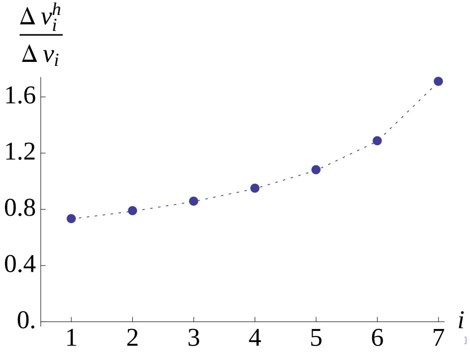
- h is convex, so the difference ratio $i \to \Delta v_p^h(i)/\Delta v_p(i)$ is increasing



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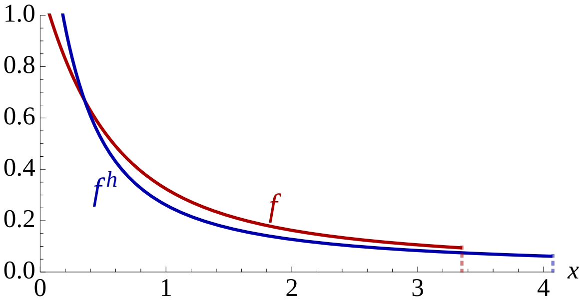

PREDICTIONS

- Given Chevalier and Ellison (1997) and Chevalier and Ellison (1999)
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- Limitations on the upper range of CEO compensation should be industry specific:
  - Reducing CEO pay slice in industries where risk taking is socially harmful (e.g., banking)
  - Not restricting CEO pay slice in industries where equilibrium risk-taking is below socially optimal levels (e. g., biotechnology)
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Thank you for your time!

MULTIPLE CONTESTANTS > 2 ALSO MATTER

- In an all-pay auction, but not a rank-based contest game, a bidder has a walk-away option: the bidder can walk away from the auction and keep his wealth intact.
- When there only two bidders, it is never optimal for a bidder to exercise the walk-away option
- In the two bidder case, an isomorphism can be established between all pay auction equilibria and random performance contest equilibria.
- when the number of participants (bidders in the all-pay auction contestants in the random performance contest) exceeds two, the walk away option can rationally exercised in equilibrium. The isomorphism breaks
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UNIFORM EQUILIBRIUM CONTEST PAYOFF FUNCTION

- The uniform bound puts a floor on $\bar{x}$
- The equilibrium condition that $\bar{x}$ is a best response puts a ceiling on $\bar{x}$
- The floor and the ceiling are the same.
- The ceiling can only be reached if the concave majorant of the contest payout function is uniform.
- So, the concave majorant of the contest payoff function is uniform with upper bound, $\bar{x}$.
- If the contest payoff function breaks contact with its concave majorant at any performance level, that performance level is not a best response and so will not be played.
  - Which ensures that if the contest payoff function "goes flat" it never returns to contact
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EQUILIBRIUM CONDITIONS ON DISTRIBUTIONS

EQUILIBRIUM DISTRIBUTION EQUATION

There exists a unique equilibrium. In this equilibrium, every contestant chooses the same performance distribution. If F_v represents the equilibrium performance distribution associated with prize schedule v, then $\text{Supp}\{F_v\} = [0, \mu n(v_n - v_1)/V]$, where

$$V = \sum_{i=1}^{n} (v_i - v_1), \tag{1}$$

and, over $\text{Supp}\{F_v\}$, F_v is uniquely determined by

$$\sum_{i=0}^{n-1} (v_{i+1} - v_1) \binom{n-1}{i} F_v(x)^i (1 - F_v(x))^{n-1-i} = \frac{V}{\mu n} x.$$

◂ Return

QUANTILE REPRESENTATIONS

QUANTILE FUNCTION

$$Q_v(p) = \frac{\mu n}{V} \sum_{i=0}^{n-1} (v_{i+1} - v_1) \binom{n-1}{i} p^i (1-p)^{n-1-i}.$$

$$Q_v(p) = \frac{\mu n}{V} \sum_{i=1}^{n-1} \Delta v_i I_p(i, n-i)$$

Return

QUANTILE REPRESENTATIONS (CONT.)

QUANTILE DENSITY

$$q_v(p) = \frac{\mu\, n\,(n-1)}{V} \left(\sum_{i=0}^{n-2} \Delta v_{i+1} \binom{n-2}{i} p^i \,(1-p)^{n-2-i} \right).$$

DERIVATIVE OF QUANTILE DENSITY

$$q'_v(p) = \frac{\mu\, n\,(n-1)}{V} \left(\sum_{i=0}^{n-3} (\Delta v_{i+2} - \Delta v_{i+1}) \binom{n-2}{i} (n-2-i) p^i (1-p)^{n-3-i} \right).$$

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QUANTILE REPRESENTATIONS (CONT.)

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DERIVATIVE OF QUANTILE DENSITY

$$q_v'(p) = \frac{\mu\, n\,(n-1)}{V} \left(\sum_{i=0}^{n-3} (\Delta v_{i+2} - \Delta v_{i+1}) \binom{n-2}{i} (n-2-i) p^i (1-p)^{n-3-i} \right).$$

Return

PARAMETERS

$$\textit{Entrants}: \quad v = (0, 0, \overbrace{1, \dots, 1}^{4}, \overbrace{3, \dots, 3}^{4})$$

$$v^e = (0, 0, 0, \overbrace{1, \dots, 1}^{4}, \overbrace{3, \dots, 3}^{4})$$

$$\textit{Scaling}: \quad v^{\times 2} = (\overbrace{1, \dots, 1}^{4}, \overbrace{1, \dots, 1}^{8}, \overbrace{3, \dots, 3}^{8})$$

$$\textit{Clustering}: \quad v^{\times 2} = (0, 0, \overbrace{1, \dots 1}^{4}, \overbrace{3, \dots 3}^{4}, 5, 5)$$

$$v^{\times 8} = (\overbrace{0, \dots 0}^{8}, \overbrace{1, \dots 1}^{16}, \overbrace{3, \dots 3}^{16}, \overbrace{5, \dots 5}^{8})$$

Return to Entrants Return to Scaling Return to Clustering