Equilibrium portfolios and equity premium with wealth heterogeneity and uncertainty aversion

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Motivation

- Three salient features of household investments:
  (i) many households do not participate in the stock market
  (ii) poorer are less likely to participate
  (iii) among participants, wealthier households invest a larger share of their wealth in the risky assets

- These features are robust across countries and time periods

- Looking at these features *individually*, there are models explaining each

- *Objectives*:
  (i) provide a *unified* explanation for the above three patterns
  (ii) study the link between endogenous degree of participation and equity premium

Preview of Results

- Tractable model whose key ingredients are:
  (i) wealth heterogeneity
  (ii) wealthier investors have lower absolute risk aversion
  (iii) wealthier investor have lower ambiguity about the mean stock return

- Two possible equilibrium outcomes, with full participation and with limited participation, depending on wealth heterogeneity

- In limited participation equilibrium, the model explains the three salient features of household portfolios

- Proportionally increasing all endowments, equity premium decreases and the degree of participation increases$\Rightarrow$ higher participation is associated with lower risk premium

Related Literature

- Papers explaining non-participation but not increasing wealth share:
  – transaction costs (Cocco (2005), Gomes and Michaelidis (2008))
  – first order risk aversion: uncertainty aversion (Dow and Werlang (1992)), disappointment aversion (Ang, Bekaert, and Li (2005)), loss aversion (Barberis, Huang, and Thaler (2006))
  – other explanations (Berk and Walden (2010))

- Papers explaining increasing wealth share but not non-participation: Peress (2004), Wachter and Yogo (2010), Roussanov (2010)

- Studies looking at the link between participation and equity premium: Saito (1995), Basak and Cuoco (1998), Polkovnichenko (2004), Cao, Wang, and Zhang (2005), Guvenen (2009), Ui (2011)

Basic Setting

- One period setting, two assets: one riskless bond and one risky stock (stock market) whose payoff x is $N(\mu, \sigma^2)$
- Continuum of households $i \in [0, 1]$, heterogenous in endowments of risky stock. Stock endowments are distributed according to (almost) Pareto distribution

$$e_i = \frac{e_H}{1 + i(e_H - e_L)/e_L}, \qquad e_H > e_L$$

Average endowment in the economy is

$$\bar{e} = \int_0^1 e_i di = \int_0^1 \frac{e_H}{1 + i(e_H - e_L)/e_L} di = \frac{e_H e_L}{e_H - e_L} \ln\left(\frac{e_H}{e_L}\right).$$

Ambiguity Aversion

- Investor i has multiple priors over the true expected payoff μ, represented by an interval

$$\mathcal{M}_i = [\mu - u_i, \mu + u_i],$$

  where u_i is household i's degree of uncertainty.

- Household i chooses what share of wealth θ_i to invest in the stock. She solves a max-min problem

$$\max_{\theta_i} \min_{\tilde{\mu} \in \mathcal{M}_i} E\left[-\exp(-\alpha_i w_i)\right],$$

  where α_i is household i's absolute risk aversion.

Modelling Degree of Uncertainty and Risk Aversion

- Wealthier households have lower absolute risk aversion

$$\alpha_i = \gamma/e_i,$$

where $\gamma > 0$ is "relative risk aversion."

- Wealthier households have lower ambiguity about the risky stock payoff

$$u_i = 1/e_i. \tag{1}$$

- Both assumptions are widely supported by the evidence.

Equilibrium

An equilibrium is given by a stock price p^* and investors' stock weights $\theta_i^*, i \in [0,1]$, such that:

(i) given p^*, each θ_i^*, $i \in [0,1]$, solves investor i's optimization problem,

(ii) the total number of stocks demanded by the investors, $\int_0^1 \theta_i^* e_i di$, equals the stock supply $\int_0^1 e_i di$.

Equilibrium with Full Participation

- In the full participation case, we can clean analytic expressions for all endogenous quantities

- Full participation occurs in equilibrium when

$$\frac{1}{e_L} - \frac{1}{\bar{e}} < \gamma\sigma^2.$$

  Hence, the dispersion of wealth should be relatively low.

- Equity premium is

$$\mu - p^* = \gamma\sigma^2 + 1/\bar{e}.$$

  Two terms: the usual risk premium for fundamental uncertainty + the component reflecting the average uncertainty in the economy

- Risky wealth share is $\theta_i^* = \frac{\mu - p^* - 1/e_i}{\gamma\sigma^2}$ – increases in wealth e_i.

Equilibrium with Restricted Participation

- More realistic case, some quantities are characterized implicitly

- Obtains when the earlier participation condition is not satisfied, i.e., when wealth is sufficiently dispersed

- *Participation.* There is a threshold $i^* \in (0, 1)$ such that poor investors $[0, i^*]$ do not invest in the risky asset and rich investors $[i^*, 1]$ do not invest. The threshold is given bye

$$(i^* + e_L/(e_H - e_L)) \ln(1 + i^*(e_H - e_L)/e_L) - i^* = \bar{e}\gamma\sigma^2.$$

- *Risky wealth share.* For participants, θ_i^* is as in the previous slide$\Rightarrow$risky wealth share increases in wealth

- *Equity premium.* $\mu - p^* = \frac{e_L + i^*(e_H - e_L)}{e_H e_L}.$

Restricted Participation and Equity Premium

- Substantial interest in understanding the link between participation and equity premium–restricted participation is a possible explanation of the equity premium puzzle.

- Studies with exogenous restricted participation find a negative link: more participation decreases equity premium.

- Cao, Zhang, and Wang (2005), find the opposite when restricted participation is endogenous due to investors' heterogeneity in degree of ambiguity.

- Cao et al. exogenously assume that otherwise identical investors differ in degree of ambiguity, and their result is obtained by varying their ambiguity levels

- In our model, degree of ambiguity is determined by wealth, so we undertake a similar analysis by varying endowments

Restricted Participation and Equity Premium

- A natural comparative statics: due to economic growth all households become proportionally richer. Introduce parameter k reflecting the level of wealth

- While participation level i^* and equity premium $\mu - p^*$ are described via implicit equation, we obtain analytically the sensitivities of these parameters with respect to k

$$\frac{di^*}{dk} = \frac{\bar{e}\gamma\sigma^2}{\ln\left(1 + i^*(e_H - e_L)/e_L\right)} > 0,$$

$$\frac{d(\mu - p^*)}{dk} = -\frac{i^*\left(1 + i^*(e_H - e_L)/e_L\right)}{k^2 e_H(k\bar{e}\gamma\sigma^2 + i^*)} < 0.$$

Restricted Participation and Equity Premium

- As households get richer the equity premium declines and participation level increases– both observations are consistent with empirical evidence.

- A lower (endogenous) stock market participation is associated with a higher equity premium–unlike Cao, Zhang, and Wang (2005), similar to results of studies with exogenous restricted participation.

Conclusion

- We develop a simple general equilibrium model featuring wealth heterogeneity and ambiguity aversion

- The model is able to explain the three salient features of household portfolios

- Proportional increase in wealth leads to lower equity premium and higher stock market participation

- Negative link between participation and equity premium