Optimal Capital Allocation: VaR, C-VaR, Spectral Measures and Beyond in Russian Markets

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- Introduction

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- Optimal Capital Allocations: Mean-Variance and Mean C-VaR

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The purpose of a risk measure is twofold:

- *Determination of risk capital*, that is determine the amount of capital a financial institution needs to cover unexpected losses.

- *Management tool*, that is risk measures are used by management to limit the amount of risk a unit within the firm may take.

The problem of defining an opportune risk measure has always been particularly important, both in the academia and in financial institutions.

A proposal that, nowadays, has been accepted by academics, but not yet entirely by financial professionals, is the one proposed by Artzner, Delbaen, Eber and Heath (1999), that we will simply refer as ***ADEH*** from now on.

If we define $\Psi(\Delta P_t)$ the risk measure of ΔP_t, ADEH affirm that $\Psi(\Delta P_t)$ should have the following properties:

- **Translation Invariance**: given a random variable ΔP_t, the risk-free title ΔG and a generic constant $\theta_G \in \mathbb{R}$, then

$$\Psi(\Delta P_t + \theta_G \Delta G) = \Psi(\Delta P_t) - \theta_G \tag{1}$$

- **Sub-additivity**: given two price variations (or returns) $\Delta P_{t,1}$ and $\Delta P_{t,2}$, it holds that

$$\Psi(\Delta P_{t,1} + \Delta P_{t,2}) \leq \Psi(\Delta P_{t,1}) + \Psi(\Delta P_{t,2}) \tag{2}$$

- **Positive Homogeneity**: given ΔP_t and a not negative constant λ:

$$\Psi(\lambda \Delta P_t) = \lambda \Psi(\Delta P_t) \tag{3}$$

- **Monotonicity**: given two price variations (or returns) $\Delta P_{t,1}$ and $\Delta P_{t,2}$, such that $\Delta P_{t,1} \leq \Delta P_{t,2}$ then

$$\Psi(\Delta P_{t,2}) \leq \Psi(\Delta P_{t,1}) \tag{4}$$

Let us suppose that the density function $f(\Delta\hat{P}_t)$ of the random variable $\Delta\hat{P}_t = \Delta P_t/\Delta G$ is continuous and defined on $\mathbb{R}$.

For sake of simplicity and without losing generality, we consider here the case with $\Delta G = 1$, so that $\Delta\hat{P}_t = \Delta P_t$.

The first step is to derive the cumulative distribution function of ΔP_t, defined as

$$F(\Delta P_t) = \int_{-\infty}^{\Delta P_t} f(x)dx$$

Definition 1.1. *The* ***Expected Shortfall*** *(ES_α) is the simple arithmetic mean of all the losses that we have with probability equal or smaller than α:*

$$ES_\alpha = -\frac{1}{\alpha}\int_0^\alpha F^{-1}(\Delta P_t)d\Delta P_t \tag{5}$$

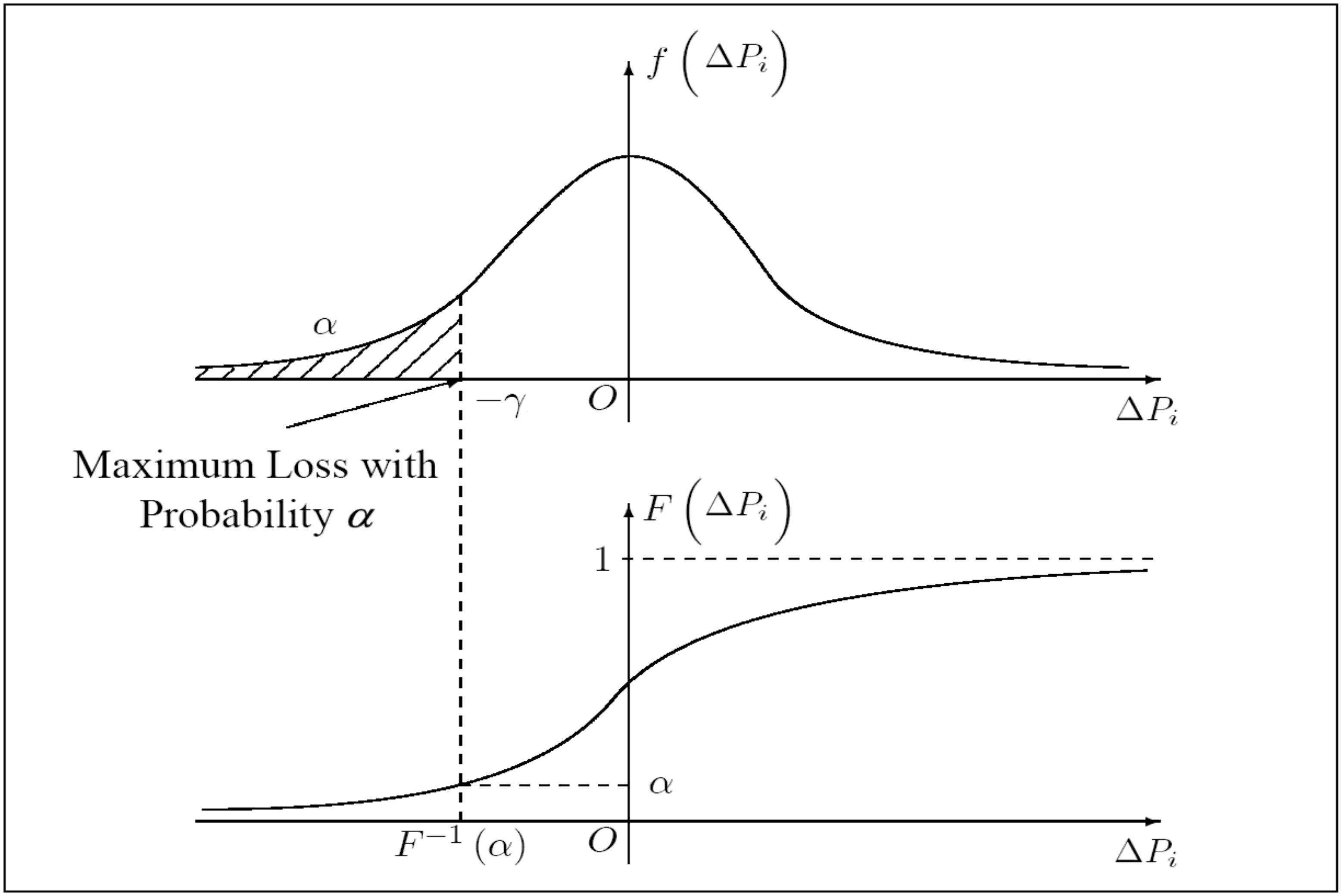

Figure 1: *Density Function, Cumulative distribution function and Inverse function.*

The Expected Shortfall has been defined as the simple arithmetic average of the α worst losses. However, instead of computing the simple average, we can consider a weighted average, thus generalizing the Expected Shortfall.

If the denote the weight function by $\phi(\Delta P_t)$, the resulting risk measure is the following one:

Definition 1.2. *A* ***risk measure*** $M_\phi(\Delta P_t)$ *is defined as* ***spectral*** *if*

$$M_\phi(\Delta P_t) = -\int_0^1 \phi(\Delta P_t) F^{-1}(\Delta P_t) d\Delta P_t \tag{6}$$

where $\phi(\Delta P_t)$ *is known as the risk spectrum or risk-aversion function.*

It is clear from definition (6) that the Expected Shortfall implies a discontinuous $\phi(\Delta P_t)$ that takes the value 0 for profits or small losses and takes a constant value for high losses.

More formally, we have an indicator function:

$$\phi(\Delta P_t) = \frac{1}{\alpha}\mathbf{1}_{\Delta P_t < \alpha}, \quad \text{where} \tag{7}$$

$$\mathbf{1}_{\Delta P_t < \alpha} = \begin{cases} 1, & \Delta P_t < \alpha \\ 0 & \text{otherwise} \end{cases}$$

Then, Acerbi (2002) proved the following result:

Theorem 1.1. *A spectral risk measure is coherent if and only if*

1. $\phi(\Delta P_t)$ *is not negative;*
2. $\phi(\Delta P_t)$ *is not increasing;*
3. $\int_0^1 \phi(\Delta P_t) d\Delta P_t = 1$

Definition 1.3. *The VaR at level α is the maximus loss one could expect to lose with probability α over a specific period of time.*

Similarly to the expected shortfall, the VaR can be obtained as a special case of the spectral risk measure (6).

The VaR implies a $\phi(\Delta P_t)$ function that takes the degenerate form of a *Dirac delta function*: by construction, the Dirac delta function is zero over all the real line except 0 where it is ∞, while its integral is equal to 1. If we set,

$$\phi(\Delta P_t) = \text{Dirac}(\Delta P_t - \alpha), \quad \text{we get}$$

$$Var_\alpha = -\int_0^1 \text{Dirac}(\Delta P_t - \alpha) F^{-1}(\Delta P_t) d\Delta P_t = -F^{-1}(\alpha) \tag{8}$$

that is exactly the VaR definition. In eq. (8), we used the Dirac function that accumulates the whole density on the value α (the function's argument is zero only when $\Delta P_t = \alpha$).

The VaR spectrum is reported in Figure 2.

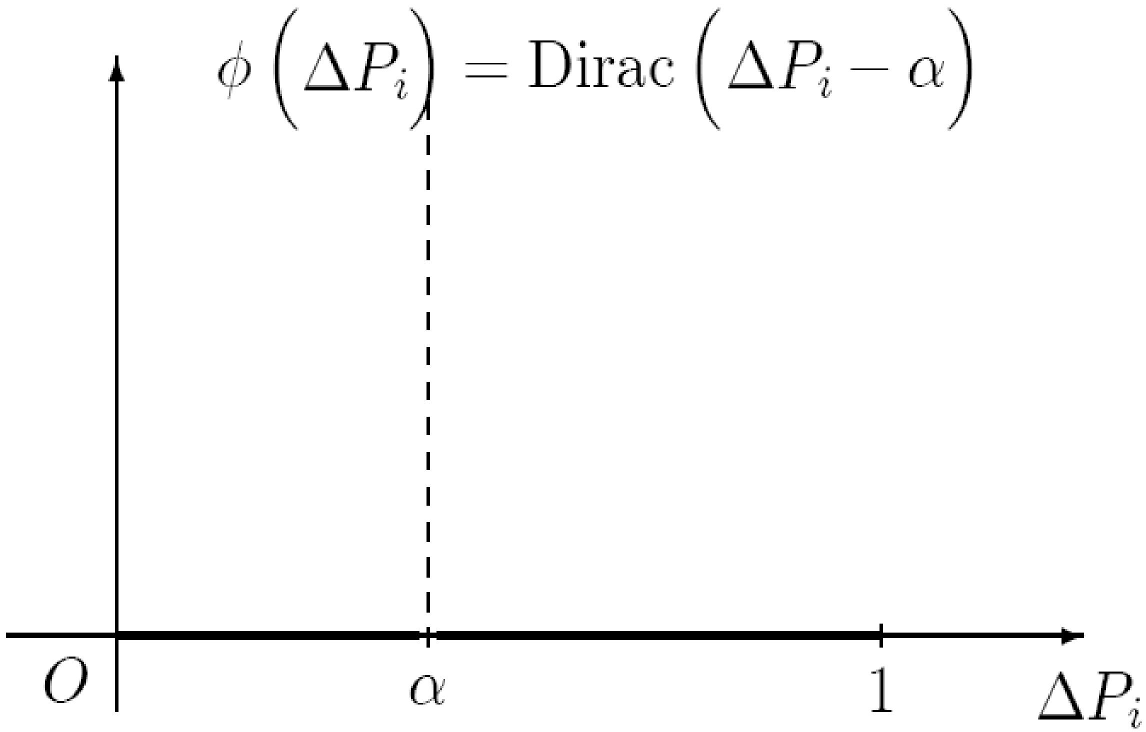

Figure 2: *Graphical representation of VaR spectrum*

Even though the VaR spectrum is never negative and sums to one (due to the Dirac function properties) it is not monotonous decreasing:

$\Rightarrow$ it is 0 for values before and after α, (formally, it first increases and then decreases). The VaR spectrum does not satisfy the second point of Theorem 1.1 and, therefore, *it is not a coherent risk measure.*

One may ask which property among the four needed to be a coherent risk measure is not satisfied: *the VaR is not sub-additive.* That is, it's possible to construct two portfolios in such a way that

$$VaR(\Delta P_{t,1} + \Delta P_{t,2}) > VaR(\Delta P_{t,1}) + VaR(\Delta P_{t,2})$$

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Let W be a wealth to invest in a portfolio of d financial risky assets $W = \sum_{i=1}^{d} w_i$, and where w_i are the capital amount invested in each asset i. We ca express the previous balance constraint (in monetary terms) in percentage terms, $1 = \sum_{i=1}^{d} x_i$ where $x_i = w_i/W$.

Let $\boldsymbol{x} = (x_1, \ldots, x_d)'$ be the decision vector whose elements represent the capital percentages held in each asset, while $\boldsymbol{y} = (y_1, \ldots, y_d)$ represents the future log-returns of each asset i over the given holding period.

Suppose the multivariate distribution of $\boldsymbol{y}$ has a density function $p(\boldsymbol{y})$, then the portfolio's loss is defined as the negative log-return of the portfolio, given by:

$$f(\boldsymbol{x}, \boldsymbol{y}) = -[x_1 y_1 + \ldots + x_d y_d] = -\boldsymbol{x}'\boldsymbol{y}.$$

Then, the portfolio's VaR for a given confidence level α is defined as

$$VaR_\alpha(\boldsymbol{x}) = \min\{\gamma | \Psi(\boldsymbol{x}, \gamma) \geq \alpha\},$$

where $\Psi(\boldsymbol{x}, \gamma)$ is the cumulative distribution function of the loss associated to the portfolio with weights $\boldsymbol{x}$. The portfolio's CVaR is defined as

$$CVaR_\alpha(\boldsymbol{x}) = \frac{1}{1-\alpha} \int_{f(\boldsymbol{x},\boldsymbol{y}) \leq VaR_\alpha(\boldsymbol{x})} f(\boldsymbol{x}, \boldsymbol{y}) p(\boldsymbol{y}) d\boldsymbol{y},$$

The ***mean-variance*** and ***mean-CVaR portfolio*** selections are defined:

$$\begin{aligned} \min_{\boldsymbol{x}} \boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x} \\ s.t. \\ \boldsymbol{x}'\boldsymbol{\mu} = \overline{y} \\ \boldsymbol{x}'\mathbf{1} = 1 \end{aligned} \qquad \begin{aligned} \min_{\boldsymbol{x}} CVaR_\alpha(\boldsymbol{x}) \\ s.t. \\ \boldsymbol{x}'\boldsymbol{\mu} = \overline{\boldsymbol{y}} \\ \boldsymbol{x}'\mathbf{1} = 1 \end{aligned}$$

where $\boldsymbol{\Sigma}$ is the $d \times d$ variance-covariance matrix of the d asset log-returns.

Given that the $CVaR_\alpha$ includes the VaR_α directly, it is very difficult to optimize this function. Instead, following Rockafellar and Uryasev (1999, 2002) we use this simpler auxiliary function

$$F_\alpha(\boldsymbol{x},\gamma) = \gamma + \frac{1}{1-\alpha} \int_{f(\boldsymbol{x},\boldsymbol{y})\geq\gamma} (f(\boldsymbol{x},\boldsymbol{y}) - \gamma)p(\boldsymbol{y})d\boldsymbol{y},$$

which can also be written as

$$F_\alpha(\boldsymbol{x},\gamma) = \gamma + \frac{1}{1-\alpha} \int [f(\boldsymbol{x},\boldsymbol{y}) - \gamma]^+ p(\boldsymbol{y})d\boldsymbol{y}, \quad \text{where} \quad z^+ = \max(z,0)$$

Rockafellar and Uryasev (1999, 2002) showed that the previous function has the following properties:

- $F_\alpha(\boldsymbol{x},\gamma)$ is a convex function of γ;
- VaR_α is a minimizer of $F_\alpha(\boldsymbol{x},\gamma)$;
- the minimum value of the function $F_\alpha(\boldsymbol{x},\gamma)$ is $CVaR_\alpha(\boldsymbol{x})$

$\Rightarrow$ Therefore $CVaR_\alpha$ can be found by optimization of the function $F_\alpha(\boldsymbol{x},\gamma)$ w.r.t. $\boldsymbol{x}$ and the VaR γ.

If the loss function $f(\boldsymbol{x}, \boldsymbol{y})$ is a convex function of the portfolio variables $\boldsymbol{x}$, then $F_\alpha(\boldsymbol{x}, \gamma)$ is also a convex function of $\boldsymbol{x}$, which can be solved using standard optimization techniques.

The last equation can be further simplified by generating q historical or Monte Carlo scenarios, $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_q$, of the random vector $\boldsymbol{y}$, sampled from the probability distribution $p(\boldsymbol{y})$:

$$\widetilde{F}_\alpha(\boldsymbol{x}, \gamma) = \gamma + \frac{1}{q(1-\alpha)} \sum_{i=1}^{q} [f(\boldsymbol{x}, \boldsymbol{y}) - \gamma]^+ \tag{9}$$

Eq. (9)is minimized s.t. $\boldsymbol{x}'\boldsymbol{\mu} = \overline{y}$ and $\boldsymbol{x}'\mathbf{1} = 1$. By repeating the whole procedure for different values of $\overline{y}$, we obtain the CVaR efficient frontier.

How to generate the the scenarios for the d asset log-returns?

We considered three alternatives approaches (so far...):

- The log-returns $\boldsymbol{y}$ of the d portfolio assets during the time step $(t, t+1)$ are sampled from a multinormal distribution $N(\boldsymbol{\mu}_{t+1}, \boldsymbol{\Sigma}_{t+1})$, where the mean return vector is assumed to be equal to $\mathbf{0}$, while the elements of the $d \times d$ variance-covariance matrix $\boldsymbol{\Sigma}_{t+1}$ are the forecasted variances and covariances using ***Exponentially Weighted Moving Averages (EWMA)***, see RiskMetrics Group (1994& ff.).

- The log-returns $\boldsymbol{y}$ are generated using ***Filtered Historical Simulation*** (see Barone-Adesi et al. (1999)) , which requires only univariate volatility models (here we used a simple GARCH(1,1) model), without the need to specify the multivariate dependence structure.

- Simple ***Historical Simulation***.

Brief Review of FHS.

The data is filtered using estimated and forecasted variances from GARCH type models:

$$z_{i,t-k+1} = \frac{y_{i,t-k+1}}{\sigma_{i,t-k+1}}, \quad i = 1, \ldots, d \quad k = 1, \ldots, T-50$$

These standardized log-returns are multiplied by the forecasted volatility $\sigma_{i,t+1}$ obtaining the filtered log-returns:

$$y'_{i,t-k+1} = z_{i,t-k+1}\sigma_{i,t+1}, \quad i = 1, \ldots, d \quad k = 1, \ldots, T-50$$

The previous filtered log-returns are closer to be i.i.d., since they have the same variance $\sigma^2_{i,t+1}$, $i = 1, \ldots, d$.

The scenarios are then sampled by drawing a row of historical filtered log-returns with a bootstrap procedure in $\{\boldsymbol{y}'_{t-k+1}\}$, where $\boldsymbol{y}'_{t-k+1} = (y'_{1,t-k+1}, \ldots, y'_{1,t-k+1})$, $k = 1, \ldots, T-50$.

Some Clarification about Mean-Variance, Mean-VaR and Mean-CVaR

Let's consider the case of continuous distributions so that CVaR coincides with Expected Shortfall (see DeGiorgi (2002) for a proof a the following statements):

- Under the assumption of multivariate Gaussian distributed returns, the set of efficient portfolios under value-at-risk and expected shortfall is a subset of the set of efficient portfolios under the standard deviation.

- The equivalence of these optimization problems under multivariate normal distribution has been first stated by Rockafellar and Uryasev (1999, Proposition 4.1), whereas Hürlimann (2002) proved the equivalence of mean-ES and mean-variance analysis for elliptic distributions.

- Although mean-variance, mean-VaR and mean-ES(CVaR) analysis are equivalent under multivariate Gaussian distributed returns, the mean-variance efficient portfolios can be inefficient under mean-ES(CVaR) or mean-VaR portfolio selection, see figures below.

- (μ, VaR_α) and $(\mu, CVaR_\alpha)$ efficient frontiers could be empty for value of α greater than a given level.

- In the presence of a risk-free asset, the set of efficient portfolios under the various risk measures are identical, unless one of these is empty.

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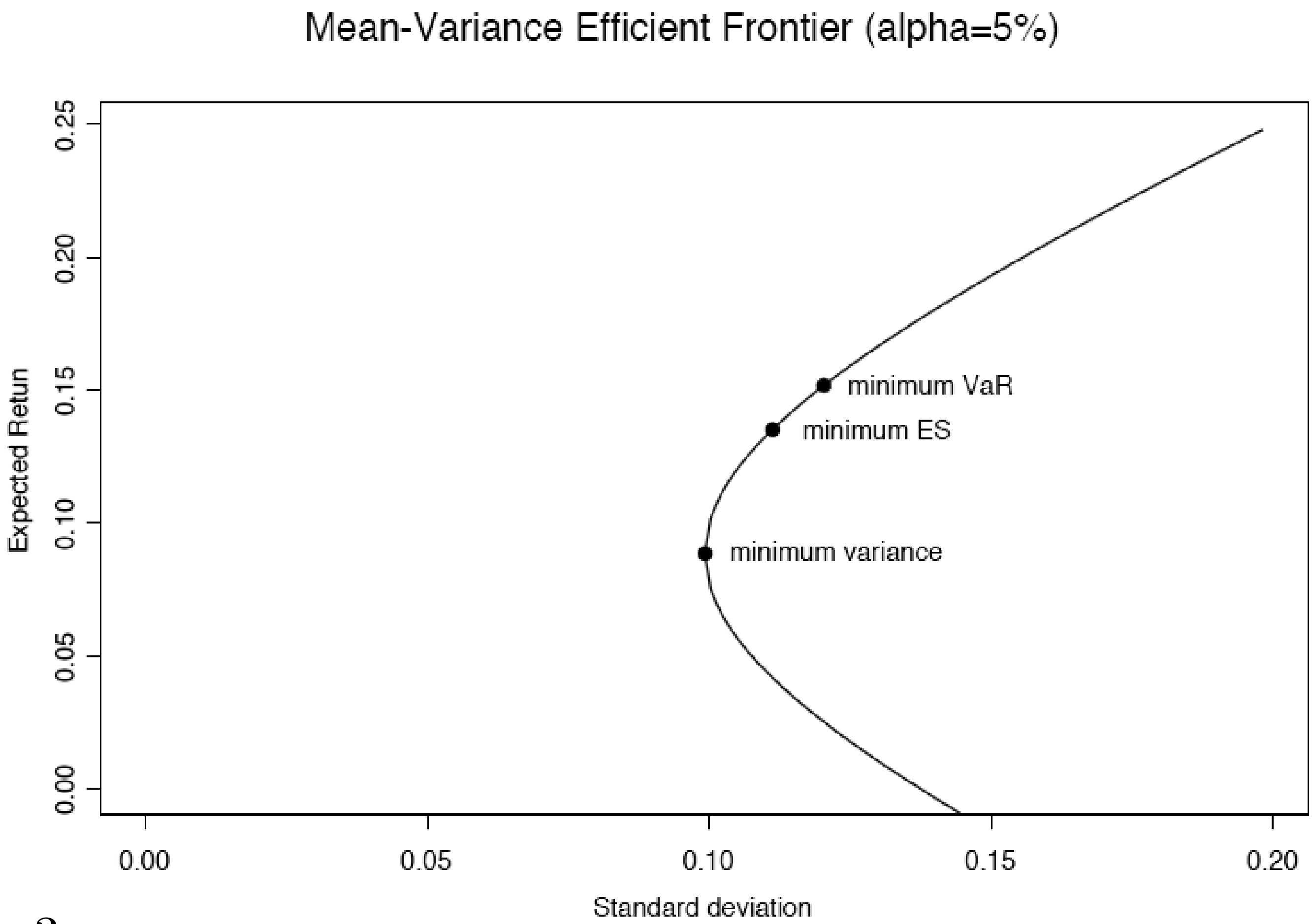

Figure 3: (μ, σ)-boundary with the global minimum variance portfolio, the global minimum $VaR_{5\%}$ portfolio and the global minimum ES5% portfolio. The efficient frontiers under the various measures, are the subset of boundary above the corresponding minimum global risk portfolios. We see that under VaR5% and ES5% the set of efficient portfolios is reduced with respect to the variance. Source: Degiorgi (2002,p.12)

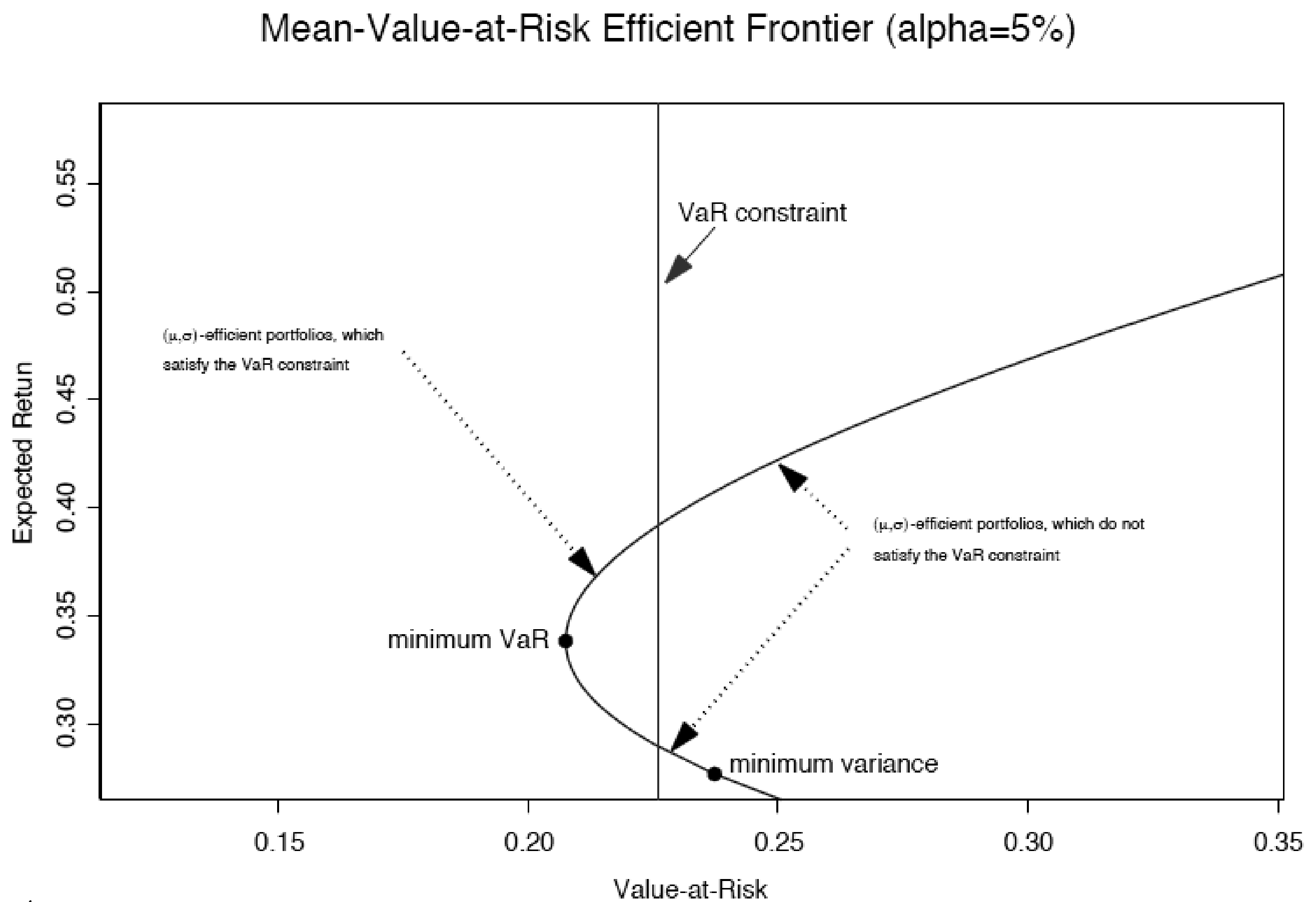

Figure 4: $(\mu, VaR_{5\%})$-boundary with the global minimum variance portfolio. Portfolios on the $(\mu, VaR_{5\%})$-boundary between the global minimum $VaR_{5\%}$ portfolio and the global minimum variance portfolio, are (μ, σ)-efficient. The VaR constraint (vertical line) could force (μ, σ) investors with high variance to reduce the variance and (μ, σ) investors with low variance to increase the variance, to be on the left side of the VaR constraint.Source: Degiorgi (2002,p.12)

Empirical Analysis: Portfolio Allocations

Initial dataset: 25 most liquid Russian stocks on MICEX by the daily trading volume.

Period: 2006 - 2011.

Final dataset: stocks number 13, 16, 24, 25 were then omitted because of insufficient number of observations

Optimization methods considered: Mean-Variance, Mean-VaR and Mean-CVaR (the MIN Variance, MIN VaR, MIN CVaR portfolios are chosen, respectively)

Performance Statistics: 1-day ahead (out-of-sample), without transaction costs, and short selling constraints imposed.

Figure 5: RTS Index 2006-2011

Empirical Analysis: Portfolio Allocations

Historical simulation	Mean-Variance	Mean - VaR	Mean - CVaR
Cumulated return (%)	-75.42	-7.39	-38.21
VaR 99%	0.3073	0.4765	0.5135
CVaR 99%	0.3179	0.5158	0.5350
Sharpe	-1.6264	-0.0444	-0.2513
Cond. Sharpe	-0.4328	-0.0269	-0.1308
Omega	0.0218	0.8989	0.5748
MDD	0.2860	0.6876	0.8248

EWMA	Mean-Variance	Mean - VaR	Mean - CVaR
Cumulated return (%)	-89.12	126.51	144.38
VaR 99%	0.5109	2.0852	0.9309
CVaR 99%	0.5158	3.0043	1.0086
Sharpe	-1.0377	0.4696	0.4771
Cond. Sharpe	-0.3150	0.1596	0.2095
Omega	0.0362	3.4100	3.4129
MDD	0.6331	2.4690	1.9117

FHS	Mean-Variance	Mean - VaR	Mean - CVaR
Cumulated return (%)	-80.1891	-69.0769	-84.3471
VaR 99%	0.3857	0.6823	0.4711
CVaR 99%	0.5234	1.0734	0.6118
Sharpe	-0.6742	-0.4779	-0.6271
Cond. Sharpe	-0.2795	-0.1175	-0.2515
Omega	0.2533	0.3459	0.2670
MDD	0.9047	2.6261	1.1073

Work in progress:

Mean-Variance vs Mean-CVaR vs Mean-Spectral Risk Measure (SRM)

We used the Mean-SRM approach discussed in Adam, Houkari, and Laurent (2008, Journal of Banking and Finance pp. 1876-1877), with the spectrum given by the exponential risk-aversion function employed in Cotter and Dowd (2006, Journal of Banking and Finance) and Acerbi (2004, *Risk Measures for the 21st Century*, Wiley, pp. 147-207).:

$$\phi(p) = \frac{ke^{-kp}}{1 - e^{-k}}$$

where $k \in (0, \infty)$ is the user's coefficient of absolute risk-aversion.

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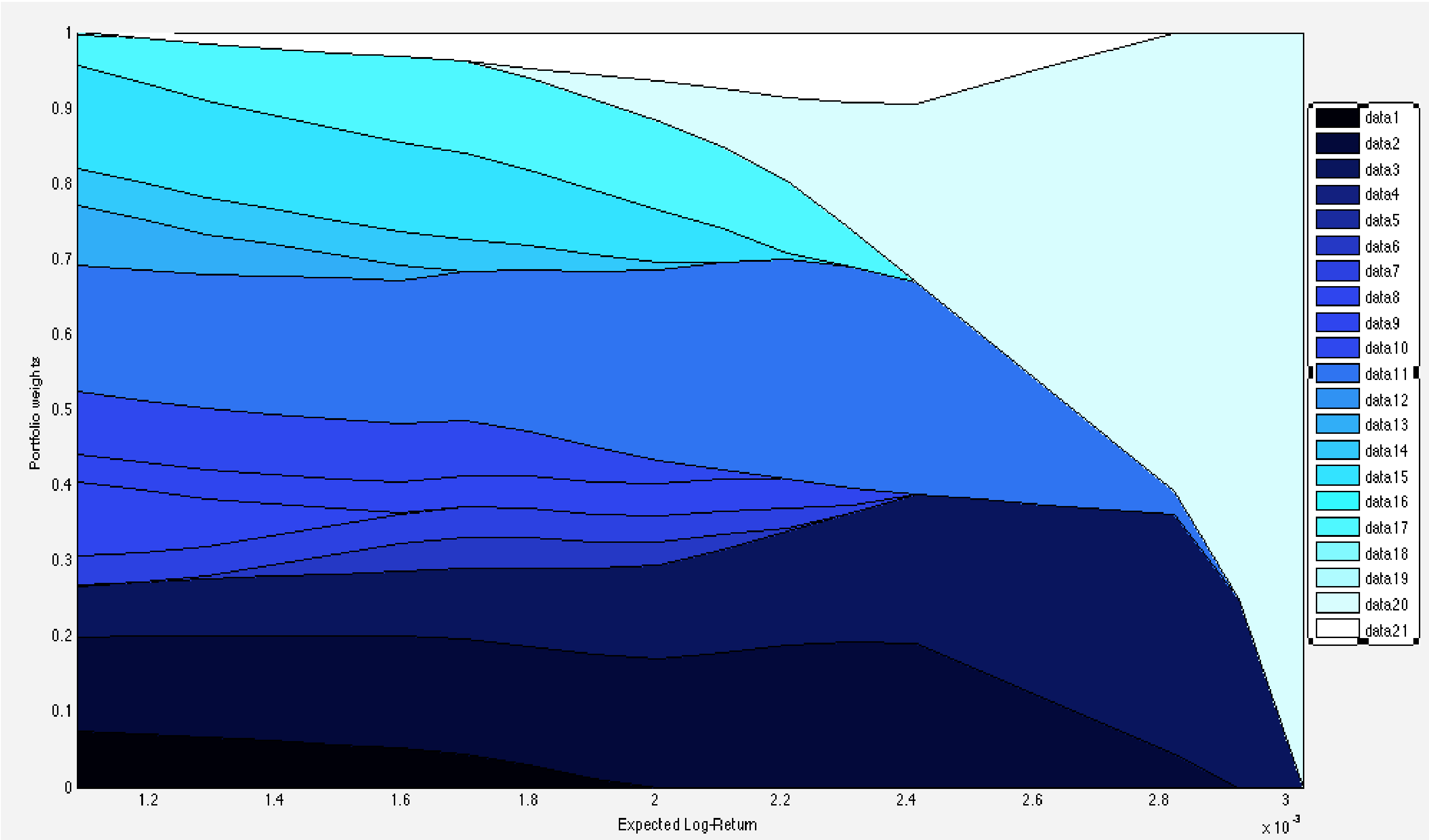

Figure 6: Portfolio Weights from Mean-Variance optimization

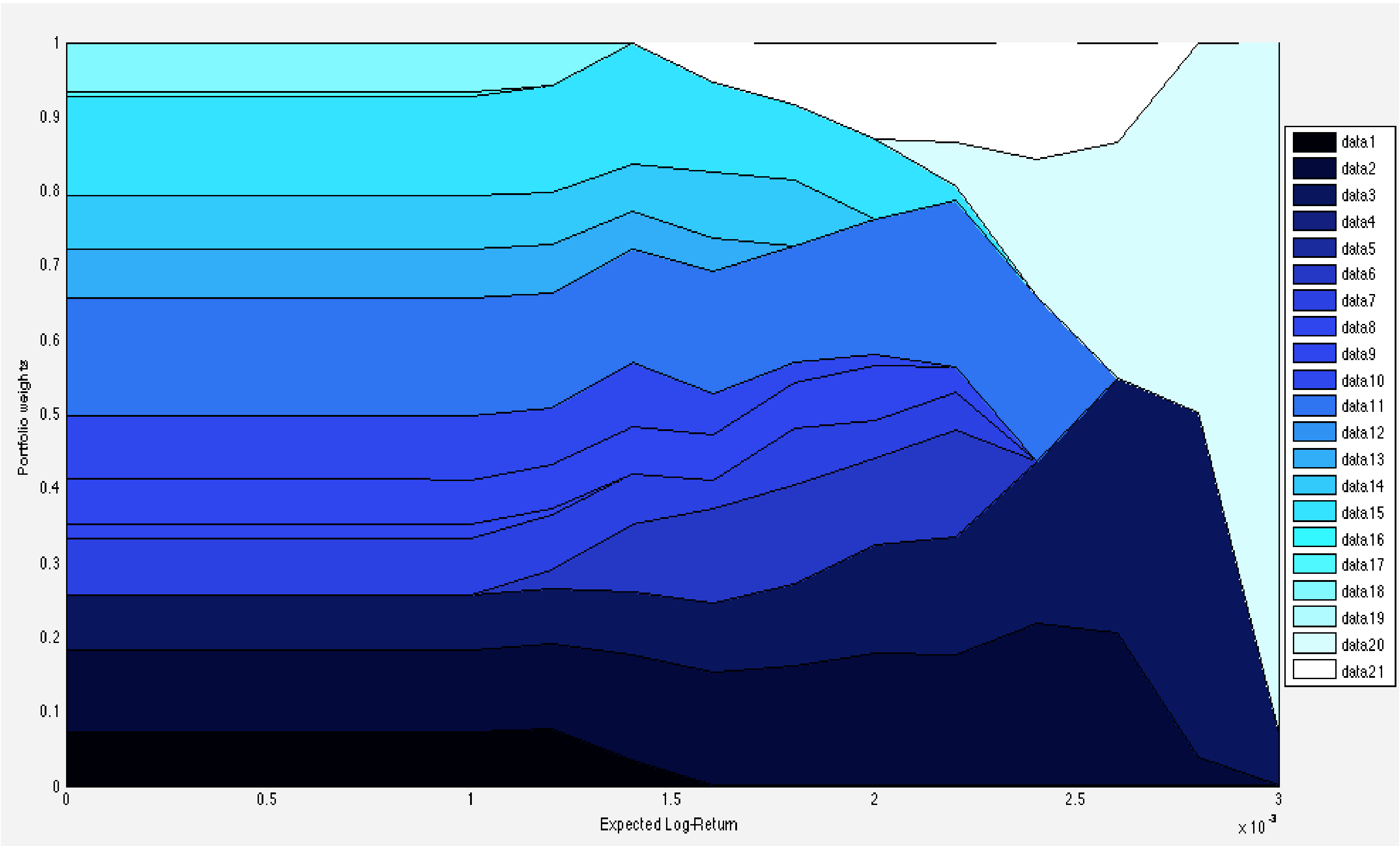

Figure 7: Portfolio Weights from Mean-CVaR optimization

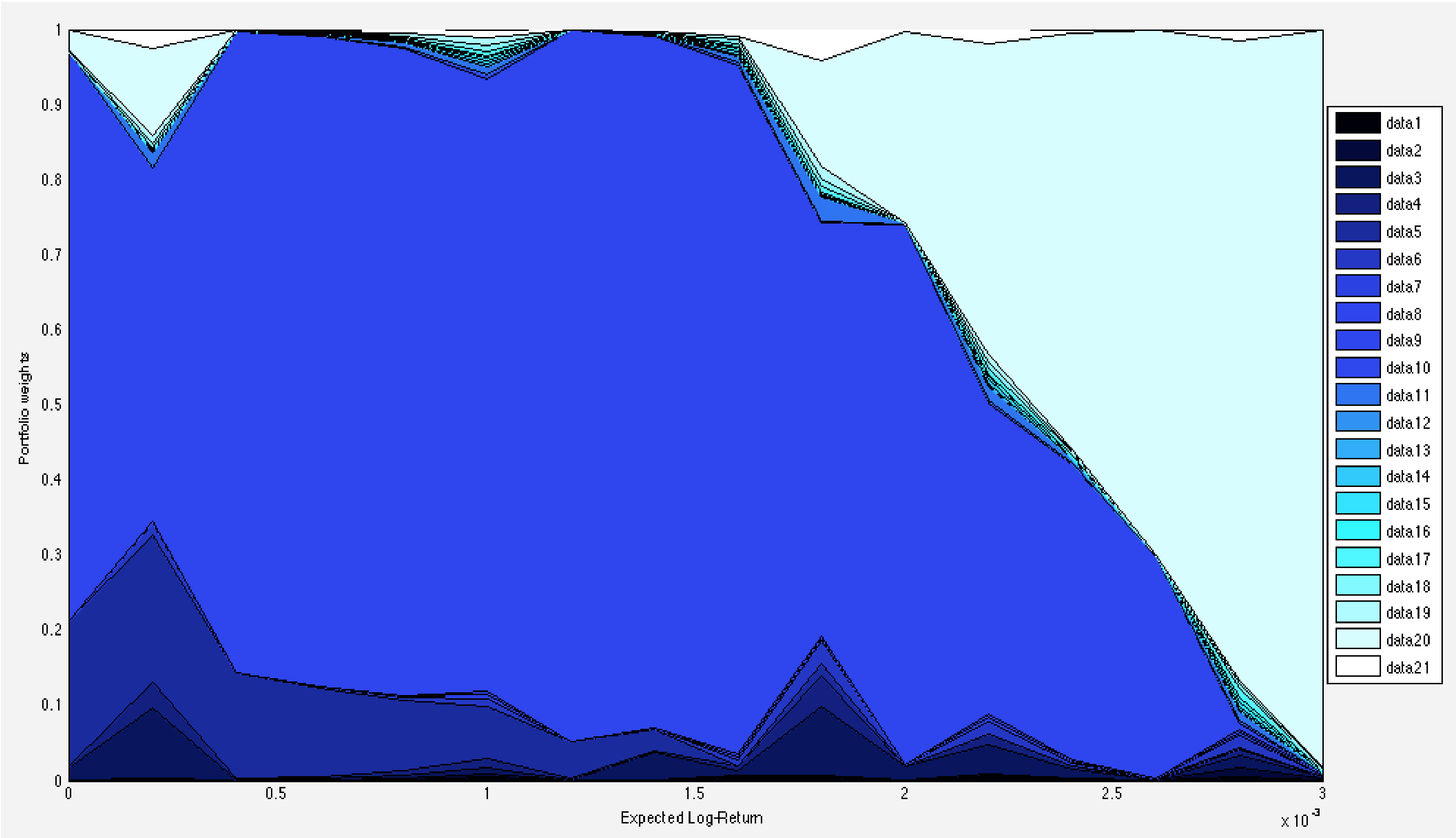

Figure 8: Portfolio Weights from Mean-Spectral Risk Measure ($k = 10$)

$\Longrightarrow$ ***The Mean-SRM approach is very sensitive to the change of the user's coefficient of absolute risk-aversion*** k***:***

$\Longrightarrow$ ***Moreover, the Mean-SRM approach tends to corner solutions in many cases, as recently proved by Brandtner (2011)***

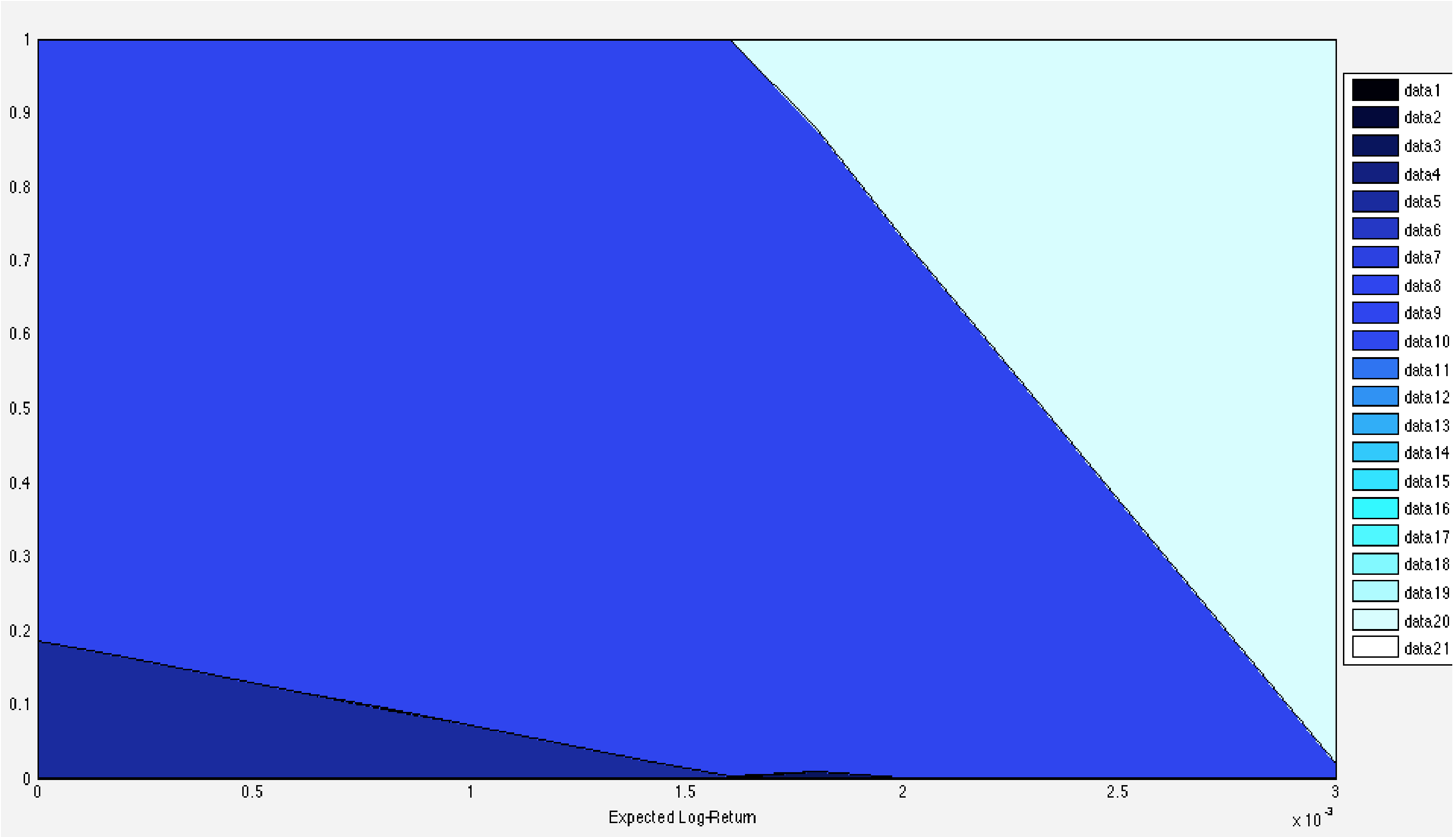

Figure 9: SRM: $k = 5$

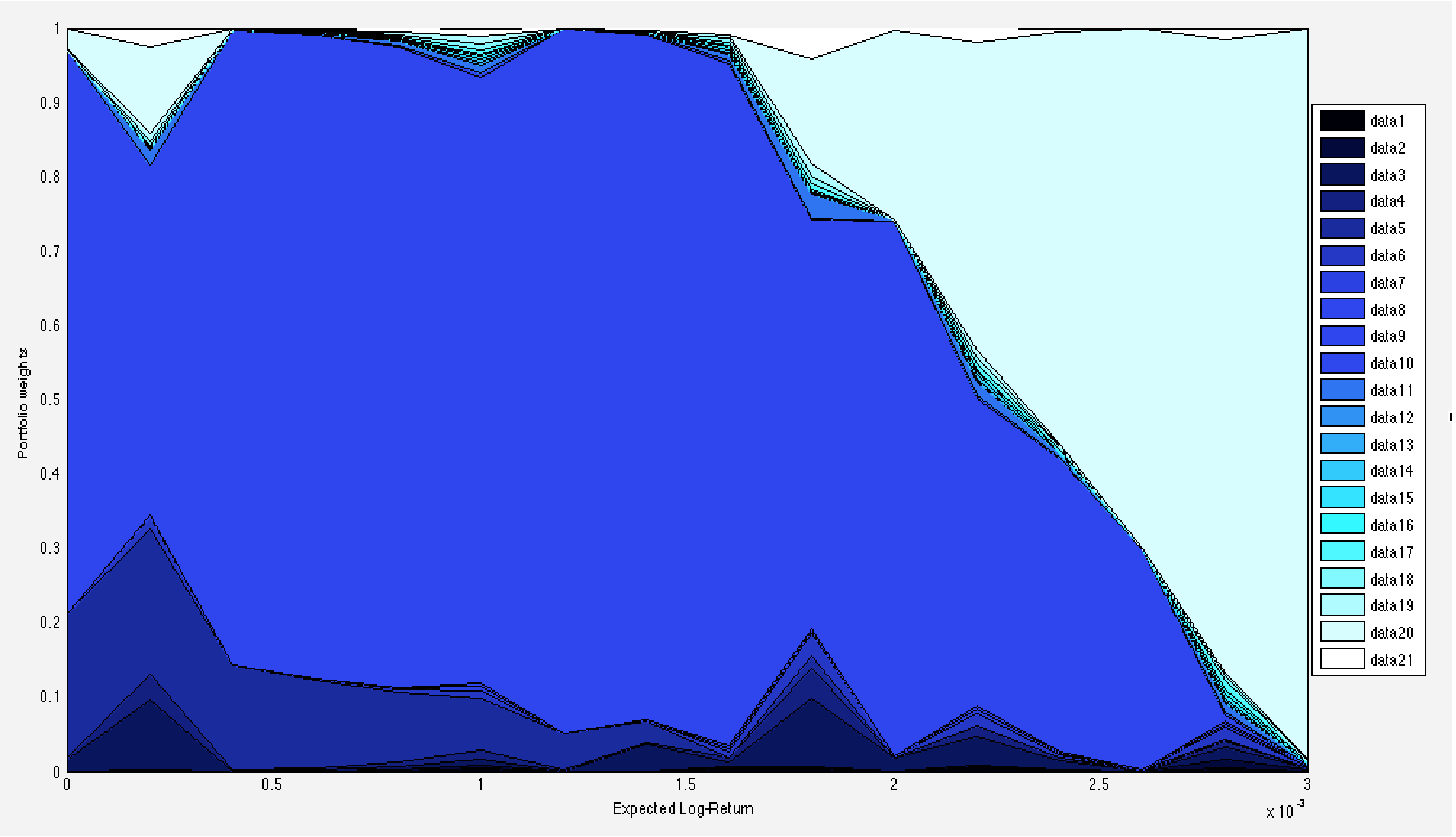

Figure 10: SRM: $k = 10$

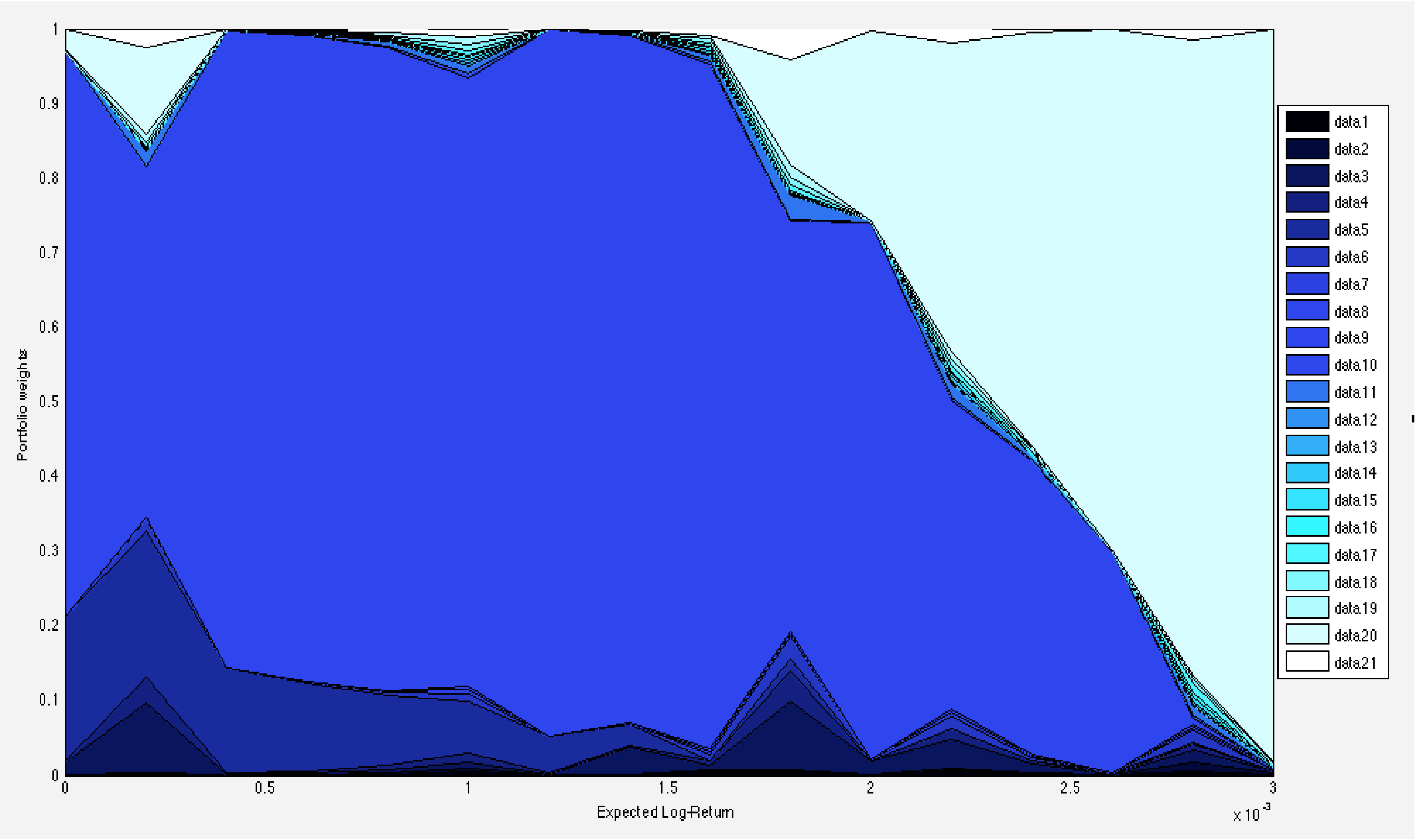

Figure 11: SRM: $k = 15$

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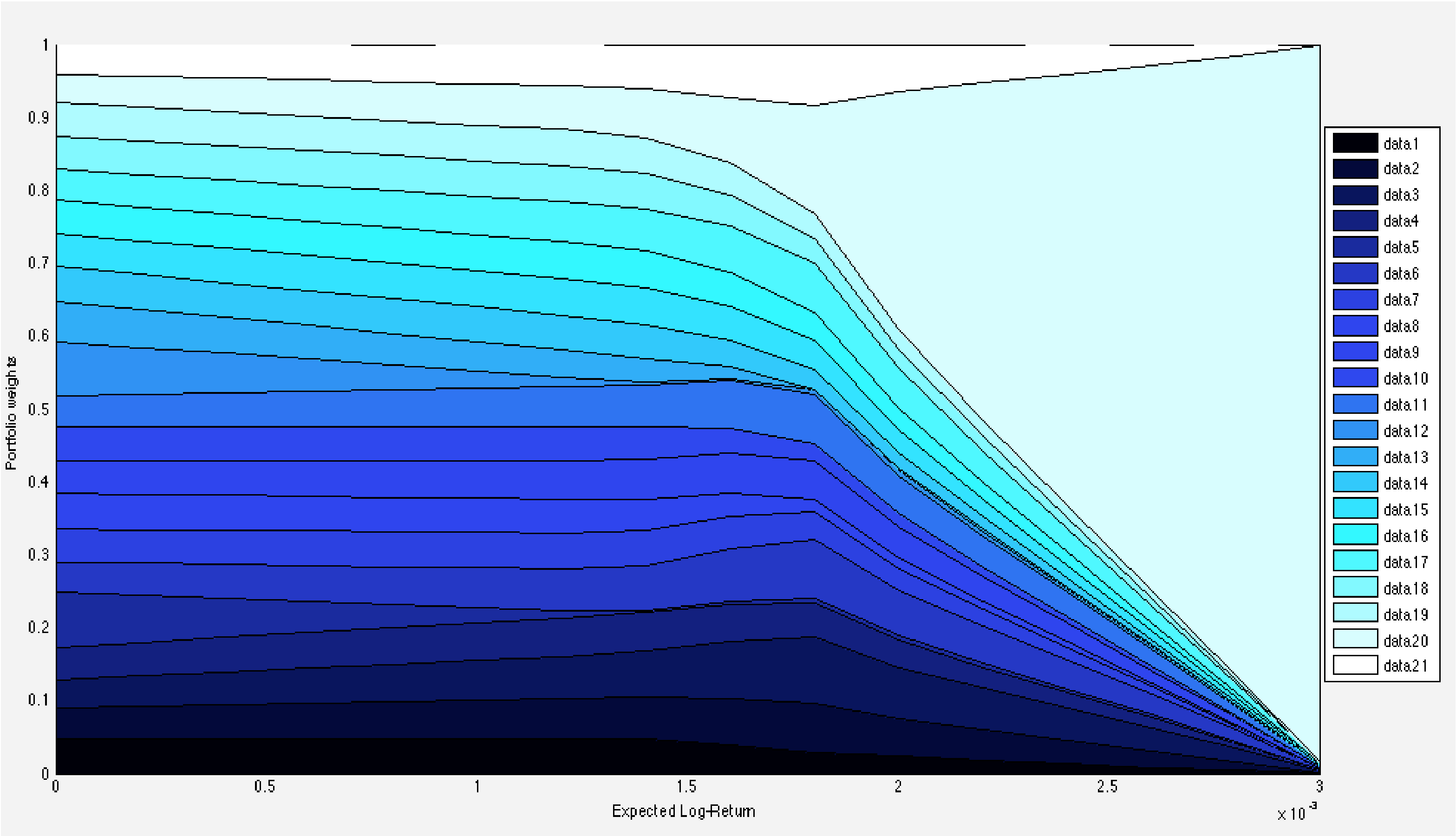

Figure 12: SRM: $k = 20$

The following models were used for volatility forecasting with our dataset of the most liquid Russian stocks[a]:

- ARCH(q):

$$y_t = h_t^{1/2} \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + u_t.$$

- GARCH(p, q):

$$y_t = h_t^{1/2} \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j} + u_t.$$

- Exponential GARCH (EGARCH(p, q)):

$$\log h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \{\theta_1 \varepsilon_{t-i} + \theta_2 (|\varepsilon_{t-i}| - E|\varepsilon_t|)\} + \sum_{i=1}^{p} \beta_i \log h_{t-i} + u_t.$$

[a]Hereinafter NaN value in table means the algorithm did not converge or converged to inappropriate values of parameters, e.g. sum of GARCH coefficients greater then 1

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- GJR-GARCH(p, q):

$$y_t = h_t^{1/2} \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i h_{t-i} + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \gamma_j S_{t-j}^{-} \varepsilon_{t-j} + u_t,$$

where S_{t-j}^{-} is a dummy variable that take the value 1 when γ_j is negative and 0 when it is positive.

- HAR with the Realized Volatility (RV):

$$\log RV_t = \alpha_0 + \alpha_d \log RV_{t-1} + \alpha_w \log RV_{t-5:t-1} + \alpha_m \log RV_{t-22:t-1} + u_t.$$

- ARFIMA$(p, d, q))$ with RV:

$$\Phi(L)(1-L)^d (\log(RV_t) - \mu) = \Theta(L) \varepsilon_t + u_t,$$

where L is a lag operator, $\Phi(\cdot)$ and $\Theta(\cdot)$ are polinomials of order p and q respectively.

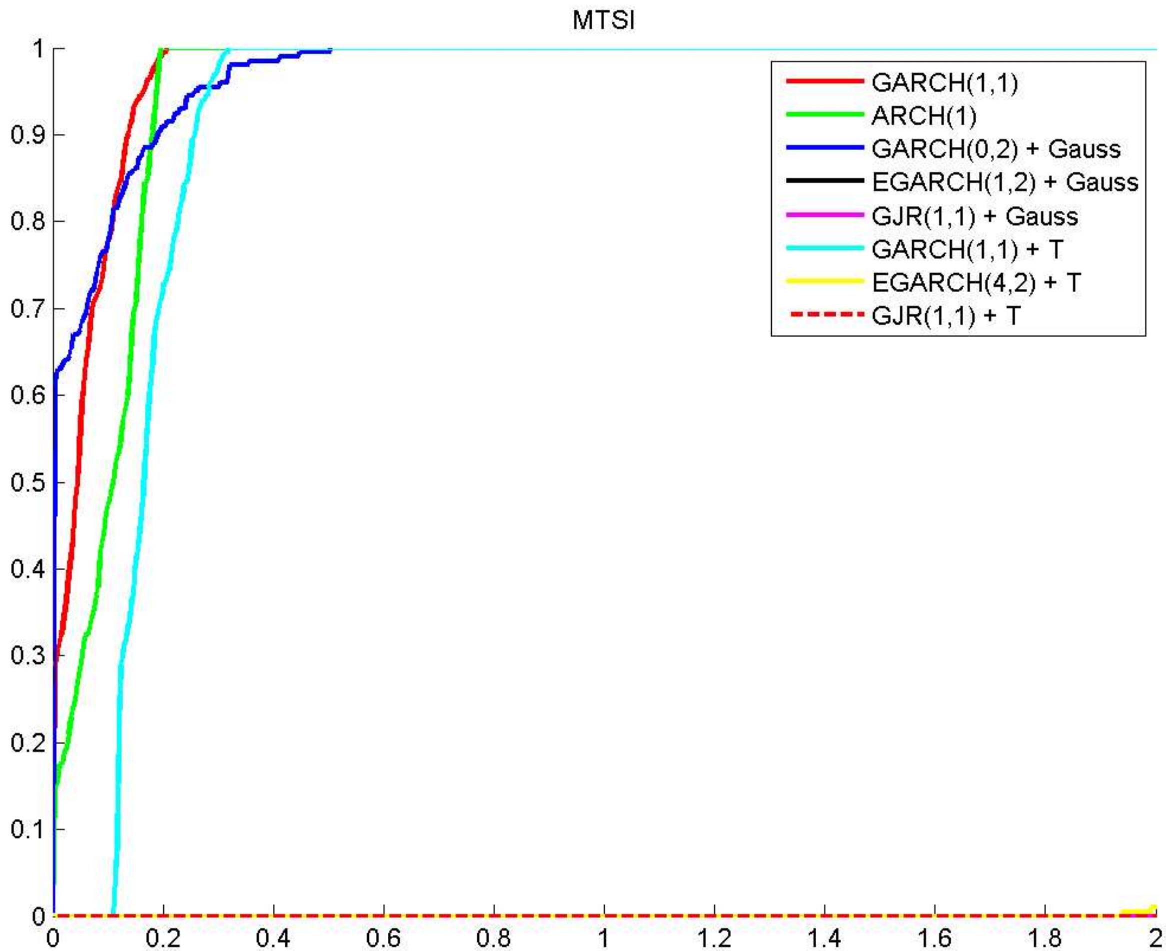

Figure 13: Dolan-More' performance profiles: AFLT (GARCH type models only)

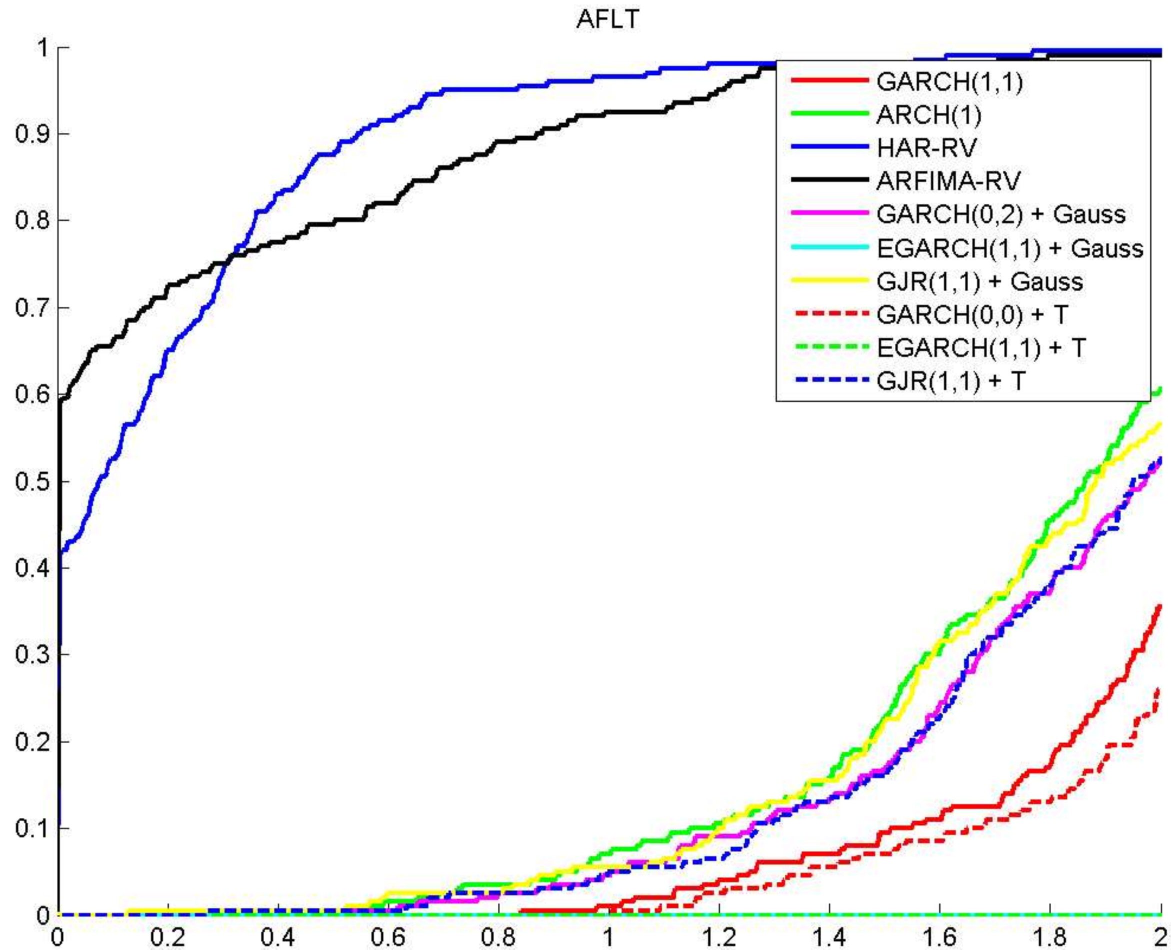

Figure 14: Dolan-More' performance profiles: AFLT (All Volatility models)

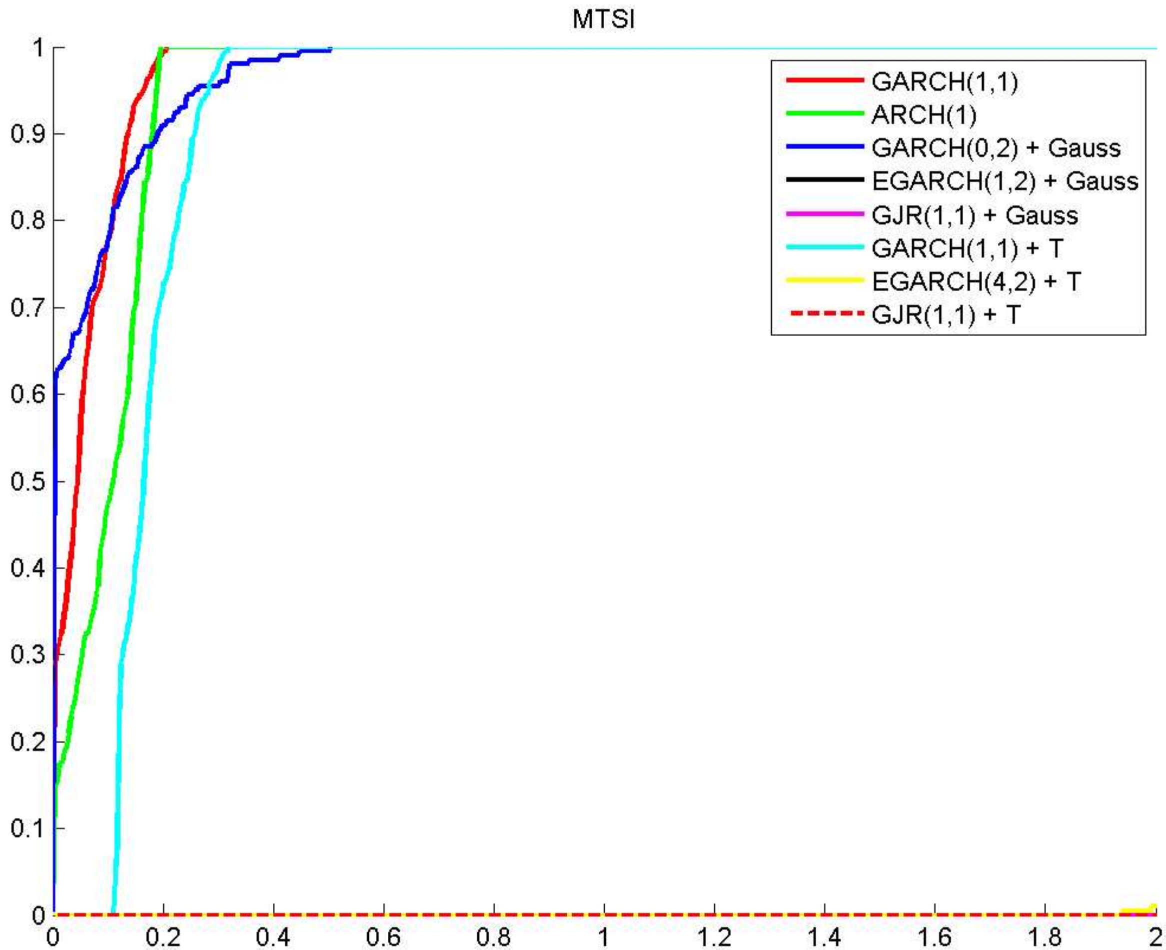

Figure 15: Dolan-More' performance profiles: MTSI (GARCH type models only)

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,0014	-0,00036	0,067558	-0,07013	0,021252	0,045521	5,505101	0,001706
CHMF	-0,00384	-0,00223	0,056886	-0,09149	0,025786	-0,34062	4,327895	0,018679
GAZP	-0,00217	-0,00254	0,102531	-0,05516	0,020478	1,090875	9,087711	0,001
GMKN	-0,00237	-0,00185	0,049209	-0,07877	0,020395	-0,25397	4,373653	0,020067
HYDR	-0,00249	-0,00187	0,09963	-0,09055	0,025662	0,280432	5,784658	0,001
LKOH	-0,00094	-0,00019	0,040431	-0,05295	0,01604	-0,07615	3,972233	0,084893
MAGN	-0,00256	7,76E-05	0,061785	-0,09539	0,024632	-0,8742	5,766759	0,001
MTSI	-0,00087	0,000109	0,037648	-0,05611	0,015176	-1,10823	6,866685	0,001
NLMK	-0,00319	-0,00188	0,052626	-0,10033	0,025716	-0,44892	4,457522	0,010529
NOTK	-0,00031	0,00152	0,067828	-0,06786	0,020375	0,076772	6,1011	0,001
OGKC	-0,00266	-0,00215	0,094134	-0,13244	0,031266	-0,5378	6,546597	0,001
OGKE	-0,00134	-0,00027	0,060058	-0,05749	0,021412	0,186259	4,07205	0,051029
PMTL	0,000171	-0,00013	0,060923	-0,04483	0,020502	0,679714	4,015478	0,010927
RASP	-0,002	-0,00163	0,159507	-0,11714	0,031461	1,108109	11,6082	0,001
ROSN	-0,00289	-0,00263	0,100025	-0,06502	0,023263	0,508054	6,646015	0,001
RTKM	-0,00329	-0,00082	0,087923	-0,15746	0,026822	-1,65458	14,03631	0,001
SBER	-0,00316	-0,00264	0,088377	-0,0839	0,024011	0,194248	5,880377	0,001
SIBN	-0,00392	-0,00159	0,098509	-0,09456	0,025543	-0,16727	6,396572	0,001
SNGS	-0,00211	0,000506	0,082403	-0,08323	0,020856	0,034593	7,079792	0,001
TATN	-0,00097	-0,00032	0,05606	-0,06421	0,017477	-0,07068	5,33424	0,002439
TRNF	-0,0035	0,000251	0,102974	-0,12147	0,032644	-0,19141	5,117879	0,003693

Table 1: Original Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,03514	0,051889	2,356126	-3,72642	1,004356	-0,985	5,75503	0,002352
CHMF	0,003479	-0,03563	2,306261	-1,86844	1,010179	0,489364	3,034681	0,221897
GAZP	-0,0135	-0,03753	1,839387	-2,53438	1,005415	-0,02838	2,589464	0,5
GMKN	-0,01336	0,056523	1,87439	-2,334	1,010339	-0,25641	2,456736	0,441913
HYDR	-0,05978	0,027597	2,318542	-2,57451	1,039544	0,076683	2,975126	0,5
LKOH	-0,01407	0,036695	2,806719	-2,69407	1,010151	0,045342	3,964027	0,230243
MAGN	-4,5E-06	0,034911	3,037043	-2,33933	1,010037	0,337413	4,140191	0,080961
MTSI	0,058292	0,174096	2,811406	-2,8718	1,003377	-0,35275	4,822294	0,022846
NLMK	-0,00047	0,027334	2,424606	-2,13088	1,010134	0,456848	3,195971	0,259329
NOTK	0,013757	0,146078	3,292559	-2,23181	1,115967	0,516414	3,733734	0,092493
OGKC	0,048124	0,104139	2,185196	-3,50159	1,050343	-0,58296	4,469512	0,026475
OGKE	-0,00028	0,069061	3,494444	-1,87463	0,991135	1,22172	6,232535	0,001
PMTL	-1,7E-05	0,040712	2,777977	-2,36491	1,009904	0,096494	3,863779	0,304183
RASP	0,058887	0,113188	2,166736	-1,92725	1,011591	-0,12632	2,681609	0,5
ROSN	-0,02242	-0,02879	3,194616	-2,88774	1,130523	0,059818	3,630189	0,5
RTKM	0,010152	0,020299	3,252481	-3,00837	1,002737	-0,01908	6,198757	0,003095
SBER	0,015037	-0,01013	2,628956	-3,27104	1,105826	-0,18111	3,539177	0,5
SIBN	0,015232	0,128321	2,951456	-3,36874	1,132936	-0,17353	3,941336	0,205022
SNGS	-0,00022	0,117125	3,124401	-3,11089	1,010191	0,023963	4,565129	0,047918
TATN	-0,02162	0,110632	2,237407	-3,40951	1,011628	-0,87293	5,181118	0,00569
TRNF	-0,02527	-0,01923	3,889538	-2,21454	1,005761	0,861812	6,400932	0,001316

Table 2: GARCH(1,1) Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	0,005263	0,064529	2,055938	-3,52681	1,01016	-0,99023	5,379313	0,003595
CHMF	0,003473	-0,03564	2,306186	-1,86842	1,010162	0,489359	3,034675	0,221906
GAZP	-0,00378	-0,03823	2,117843	-2,57418	1,01018	0,073435	2,808907	0,5
GMKN	-0,01146	0,046046	1,934381	-2,24585	1,010087	-0,17686	2,476073	0,5
HYDR	-0,04488	0,0347	2,026734	-2,33336	1,009229	-0,13707	2,994517	0,5
LKOH	-0,01234	0,041589	2,775836	-2,71929	1,01007	0,006378	3,934318	0,259395
MAGN	1,29E-05	0,034933	3,037475	-2,33963	1,010175	0,337413	4,140191	0,080961
MTSI	0,092705	0,226733	2,069125	-4,2381	1,005844	-1,84217	9,830373	0,001
NLMK	-0,00046	0,027352	2,424665	-2,1309	1,010148	0,456852	3,195982	0,259322
NOTK	0,000109	0,115203	3,062123	-1,80596	1,010223	0,664562	4,00855	0,039114
OGKC	0,051203	0,062294	3,410485	-3,33398	1,008882	0,04702	6,308365	0,002626
OGKE	9,51E-06	0,070381	3,53547	-1,88907	1,010869	1,222136	6,233213	0,001
PMTL	3,72E-05	0,04073	2,775555	-2,36275	1,009003	0,096494	3,863784	0,304179
RASP	0,084066	0,142731	2,338467	-2,24208	1,006561	-0,10859	2,982073	0,5
ROSN	-1,3E-05	0,012572	2,750227	-2,82126	1,010017	-0,12398	3,884265	0,27398
RTKM	0,000979	0,008721	3,07033	-3,07739	1,010196	-0,20487	6,07699	0,003551
SBER	-2,4E-06	-0,03113	2,43599	-3,12306	1,009724	-0,21635	3,782868	0,295635
SIBN	2,73E-05	0,085014	2,676291	-3,1562	1,010198	-0,25417	4,362168	0,060363
SNGS	-0,00028	0,117064	3,12424	-3,11081	1,010147	0,023973	4,565164	0,047915
TATN	0,002803	0,112524	2,429271	-3,11035	1,010172	-0,53979	4,862912	0,016265
TRNF	-0,00707	-0,00125	3,831631	-2,29164	1,01013	0,78716	6,080572	0,002125

Table 3: ARCH(1) Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,03048	0,023365	2,398703	-3,47945	1,009695	-0,6534	5,133428	0,008985
CHMF	3,11E-17	-0,0396	2,386091	-1,85722	1,010153	0,506513	3,059344	0,19732
GAZP	-3,1E-17	-0,04223	2,406676	-2,50164	1,010153	0,161231	2,894001	0,5
GMKN	-0,01146	0,046046	1,934381	-2,24585	1,010087	-0,17686	2,476073	0,5
HYDR	-0,05978	0,027597	2,318542	-2,57451	1,039544	0,076683	2,975126	0,5
LKOH	8,88E-18	0,050925	2,612201	-2,84808	1,010153	-0,06971	3,864469	0,312768
MAGN	-3,6E-17	0,03492	3,037394	-2,33959	1,010153	0,337413	4,140191	0,080961
MTSI	0,121394	0,202384	2,730542	-2,56898	1,002759	-0,15587	4,110317	0,126366
NLMK	-1,8E-17	0,027207	2,416882	-2,12415	1,010153	0,452612	3,183009	0,26929
NOTK	1,33E-17	0,115085	3,061799	-1,80594	1,010153	0,664562	4,00855	0,039114
OGKC	0,048124	0,104139	2,185196	-3,50159	1,050343	-0,58296	4,469512	0,026475
OGKE	-0,09454	0,048883	3,224508	-2,3178	1,017219	0,423483	4,199265	0,058388
PMTL	3,55E-17	0,040739	2,778679	-2,36548	1,010153	0,096494	3,863784	0,304179
RASP	0,058887	0,113188	2,166736	-1,92725	1,011591	-0,12632	2,681609	0,5
ROSN	-5,3E-17	0,012587	2,75061	-2,82163	1,010153	-0,12398	3,884265	0,27398
RTKM	-3,6E-17	0,007196	3,065377	-3,07766	1,010153	-0,20308	6,065714	0,003612
SBER	-6,2E-17	-0,03114	2,437027	-3,12439	1,010153	-0,21635	3,782868	0,295635
SIBN	0	0,084983	2,676143	-3,15609	1,010153	-0,25417	4,362168	0,060363
SNGS	4E-17	0,103812	2,962664	-3,06868	1,010153	0,066723	4,316489	0,081267
TATN	-0,20517	-0,06553	2,108703	-2,87947	0,988662	-0,51185	3,806652	0,084988
TRNF	1,33E-17	-0,01378	3,690233	-2,2068	1,010153	0,882687	5,738031	0,002837

Table 4: GARCH + Gaussian Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,14906	-0,06099	2,477638	-2,78056	1,100889	-0,01142	2,713366	0,5
CHMF	0,105525	0,156418	2,747681	-2,41428	1,112907	0,262569	2,776272	0,5
GAZP	0,017876	-0,01168	2,165854	-2,50364	1,125561	0,087909	2,262983	0,429258
GMKN	-0,09935	-0,10777	1,732437	-1,85142	1,047034	-0,03728	1,731265	0,091611
HYDR	-0,06559	-0,01765	2,21495	-2,32064	1,039784	-0,02816	2,763753	0,5
LKOH	0,021028	0,149569	2,102697	-2,54955	1,125982	-0,47998	2,617184	0,185184
MAGN	-0,32052	-0,33597	1,649076	-2,3036	0,962183	0,051342	2,259279	0,441022
MTSI	0,156146	0,307536	2,04699	-2,41112	1,015482	-0,40629	2,594186	0,283135
NLMK	-0,05446	-0,07625	1,946442	-2,48854	1,023361	-0,13509	2,693622	0,5
NOTK	0,001866	0,186203	2,399978	-2,53683	1,11834	-0,23891	2,476729	0,49919
OGKC	0,21786	0,340373	2,593912	-2,60368	1,044467	-0,4545	3,26896	0,247044
OGKE	0,154106	0,181828	2,520445	-1,98507	0,995423	0,075212	2,655814	0,5
PMTL	-0,00903	-0,0452	2,669109	-2,37182	1,03051	0,31489	3,043741	0,5
RASP	-0,01425	0,041049	1,878161	-2,86784	1,030847	-0,36044	2,808997	0,443535
ROSN	-0,02856	-0,04722	2,132756	-2,46747	1,13852	-0,03603	2,230305	0,41209
RTKM	-0,18123	-0,21561	2,094862	-2,4107	0,956692	-0,0506	3,218734	0,5
SBER	0,069693	0,135091	1,676417	-2,01197	1,020637	-0,27583	2,010688	0,134454
SIBN	-0,03975	0,076291	2,979542	-2,37695	1,129028	0,179743	2,702651	0,5
SNGS	-0,20242	-0,16712	1,960543	-2,38044	1,046385	0,120744	2,333783	0,499151
TATN	0,074079	0,267117	2,724944	-2,44207	1,139238	-0,15056	2,711411	0,5
TRNF	0,157705	0,156671	3,254843	-2,11639	0,969905	0,354015	3,996181	0,103872

Table 5: EGARCH + Gaussian Standartized Returns Statistics

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Table 6: Add caption

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,02128	0,100667	3,187058	-3,03502	1,008119	-0,23295	5,172738	0,014414
CHMF	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
GAZP	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
GMKN	0,002138	0,062308	1,989427	-2,13467	1,011972	-0,17214	2,300651	0,405206
HYDR	-0,0092	0,10497	2,550468	-2,5922	1,059642	-0,01309	3,108352	0,5
LKOH	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
MAGN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
MTSI	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NLMK	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NOTK	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
OGKC	0,14052	0,229024	2,2234	-2,41824	1,059595	-0,32234	2,869005	0,5
OGKE	-0,1342	0,012321	2,921439	-2,33071	1,111427	0,198165	3,19072	0,5
PMTL	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
RASP	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
ROSN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
RTKM	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SBER	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SIBN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SNGS	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
TATN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
TRNF	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

Table 7: GJR + Gaussian Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,06988	-0,02438	1,634242	-3,01094	0,852267	-0,96578	5,273329	0,004241
CHMF	0,021682	-0,01787	2,40446	-1,83296	1,00875	0,506513	3,059344	0,19732
GAZP	0,000859	-0,04136	2,406741	-2,49996	1,009819	0,161231	2,894001	0,5
GMKN	-0,01214	0,045156	1,930871	-2,24609	1,008743	-0,17703	2,47693	0,5
HYDR	-0,05906	0,028481	2,319281	-2,57597	1,039822	0,076091	2,975683	0,5
LKOH	-0,00999	0,037762	2,439332	-2,68048	0,947165	-0,06971	3,864469	0,312768
MAGN	-0,00842	0,023571	2,773889	-2,15152	0,925317	0,337413	4,140191	0,080961
MTSI	-0,01846	0,116883	2,227672	-2,90261	0,893877	-0,42521	5,298306	0,009779
NLMK	0,029841	0,056945	2,437548	-2,08624	1,006318	0,452612	3,183009	0,26929
NOTK	0,047456	0,157946	2,987008	-1,68638	0,969821	0,664562	4,00855	0,039114
OGKC	0,005022	0,07225	2,17806	-3,73967	1,043102	-0,66449	5,106138	0,009156
OGKE	0,051508	0,11711	3,34734	-1,70953	0,942354	1,222136	6,233213	0,001
PMTL	-0,01721	0,019013	2,453788	-2,12077	0,898301	0,096494	3,863784	0,304179
RASP	0,058643	0,112795	2,16537	-1,92585	1,010801	-0,1255	2,681674	0,5
ROSN	-0,00759	0,004825	2,704938	-2,79015	0,996166	-0,12398	3,884265	0,27398
RTKM	-0,00015	1,78E-05	0,06956	-0,07013	0,022971	-0,20308	6,065714	0,003612
SBER	-0,00564	-0,03673	2,428125	-3,12584	1,008799	-0,21635	3,782868	0,295635
SIBN	-0,01535	0,066885	2,57438	-3,06954	0,977536	-0,25417	4,362168	0,060363
SNGS	0,010345	0,114216	2,974701	-3,06009	1,010729	0,066723	4,316489	0,081267
TATN	-0,02623	0,032977	2,064008	-1,9226	0,664655	-0,04617	5,270556	0,013167
TRNF	0,036381	0,025747	2,883549	-1,66626	0,779375	0,882687	5,738031	0,002837

Table 8: GARCH + T Standartized Returns Statistics

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Table 9: Add caption

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	0,048584	0,213897	2,810131	-2,84208	1,081509	-0,09093	3,205072	0,5
CHMF	0,133518	0,175274	2,882666	-2,48409	1,151174	0,267151	2,816521	0,5
GAZP	0,033842	0,036532	1,791975	-2,44666	1,048184	-0,1012	2,202067	0,3615
GMKN	-0,11907	-0,10073	1,56653	-1,71273	0,969361	-0,03944	1,732266	0,091773
HYDR	-0,06972	-0,05072	2,487977	-2,19491	1,029421	0,171996	2,948384	0,5
LKOH	0,156518	0,086407	7,940398	-2,64177	1,479033	3,03553	17,20488	0,001
MAGN	-0,31046	-0,32604	1,640958	-2,43605	0,983745	0,011839	2,26411	0,455716
MTSI	-0,00805	-0,05875	4,940743	-1,68115	1,057953	2,114414	10,88863	0,001
NLMK	-0,07086	-0,09734	1,955722	-2,50974	1,020902	-0,14314	2,728781	0,5
NOTK	0,02873	0,017874	2,205128	-1,48353	0,855091	0,507406	2,704987	0,171133
OGKC	0,025396	0,229477	2,125916	-3,30919	1,273562	-0,75058	3,033812	0,054445
OGKE	0,130089	0,162306	2,430395	-1,77552	0,948777	0,223197	2,690098	0,5
PMTL	-0,04968	0,034404	2,383174	-3,33373	1,016226	-0,14229	4,506937	0,051093
RASP	0,025802	0,079586	1,968244	-2,74656	1,018101	-0,28568	2,752818	0,5
ROSN	-0,15443	-0,16191	1,919392	-2,50784	1,109472	-0,07038	2,211358	0,383051
RTKM	-0,32551	-0,01511	1,187678	-14,6452	2,120614	-6,33252	43,31629	0,001
SBER	0,082257	0,185385	1,897761	-2,02925	0,995028	-0,19155	2,101522	0,223818
SIBN	-0,17491	-0,14561	2,531178	-2,54843	1,129754	0,297789	2,7761	0,5
SNGS	-0,14374	-0,16265	2,428703	-2,385	1,110526	0,314269	2,620783	0,458667
TATN	0,063111	0,303984	2,318685	-2,35075	1,091534	-0,35126	2,588187	0,370044
TRNF	-0,27395	0,122224	2,249676	-10,1801	1,687274	-4,26491	25,23871	0,001

Table 10: EGARCH + T Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,0213	0,106725	3,159468	-3,00188	0,998463	-0,26122	5,210422	0,013194
CHMF	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
GAZP	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
GMKN	0,001446	0,061533	1,98448	-2,13356	1,01015	-0,17279	2,301192	0,40511
HYDR	-0,00827	0,105962	2,552785	-2,58208	1,058856	-0,01067	3,110268	0,5
LKOH	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
MAGN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
MTSI	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NLMK	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NOTK	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
OGKC	0,089361	0,176133	2,167911	-3,4077	1,080353	-0,55651	4,050223	0,051426
OGKE	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
PMTL	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
RASP	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
ROSN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
RTKM	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SBER	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SIBN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
SNGS	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
TATN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
TRNF	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

Table 11: GJR + T Standartized Returns Statistics

Ticker	mean	median	max	min	std	skewness	kurtosis	JB test p-value
AFLT	-0,03555	-0,02192	1,363043	-1,85641	0,668046	-0,31283	3,439079	0,421674
CHMF	-0,07554	-0,11955	2,563161	-2,18602	1,078385	0,410237	3,139489	0,352906
GAZP	-0,13886	-0,13235	2,88551	-2,39308	1,107022	0,339954	2,927121	0,5
GMKN	-0,08491	-0,08369	2,332407	-2,05095	0,917613	0,191163	2,866126	0,5
HYDR	-0,18412	-0,10415	1,685577	-2,0297	0,86295	-0,06577	2,582177	0,5
LKOH	-0,10492	-0,0826	1,945995	-2,05535	0,854914	0,107763	2,983927	0,5
MAGN	-0,01063	0,004057	2,379577	-1,72014	0,775269	0,340769	3,87362	0,14517
MTSI	0,013044	0,129224	1,464008	-2,3883	0,727277	-0,66287	4,630886	0,017767
NLMK	-0,08339	-0,02533	2,019856	-1,63892	0,863774	0,409292	2,921977	0,362369
NOTK	-0,06617	0,013424	2,115311	-1,73853	0,685437	0,433852	3,951314	0,088099
OGKC	-0,13668	-0,06906	1,329499	-2,3763	0,731162	-0,3651	3,709154	0,195255
OGKE	0,058789	0,120249	1,563944	-1,49228	0,567238	0,203932	4,025367	0,147235
PMTL	-0,09013	-0,07254	1,539088	-1,45473	0,586198	0,149674	3,587684	0,5
RASP	-0,0289	0,046182	1,032337	-1,37766	0,62587	-0,26286	2,446658	0,42291
ROSN	-0,22023	-0,21886	2,212273	-2,22168	1,038859	0,231488	2,720335	0,5
RTKM	-0,09764	-0,0188	1,573503	-2,05168	0,712877	-0,52203	3,486075	0,126963
SBER	-0,12799	-0,16559	2,681542	-2,59546	1,063085	0,104207	3,303131	0,5
SIBN	-0,11233	-0,05205	2,067053	-2,03345	0,837512	-0,01614	3,120197	0,5
SNGS	-0,11237	-0,07024	2,192992	-2,2036	0,83436	0,268397	3,348859	0,5
TATN	-0,05757	-0,01007	1,955932	-2,10914	0,757789	-0,02486	4,279622	0,089546
TRNF	-0,02003	0,026329	2,524162	-2,53973	0,995739	-0,00586	3,626583	0,5

Table 12: RV Standardized Returns Statistics

How to measure the forecasting performance of the alternative volatility models?

Table 13: Specification for *loss functions* (see Hansen and Lunde (2005) for details)

$$\mathrm{MSE}_1 \equiv \frac{1}{n}\sum_{t=1}^{n}(\sigma_t - h_t)^2\,; \qquad \mathrm{MSE}_2 \equiv \frac{1}{n}\sum_{t=1}^{n}\left(\sigma_t^2 - h_t^2\right)^2;$$

$$\mathrm{QLIKE} \equiv \frac{1}{n}\sum_{t=1}^{n}\left(\log(h_t^2) + \frac{\sigma_t^2}{h_t^2}\right); \qquad \mathrm{R^2LOG} \equiv \frac{1}{n}\sum_{t=1}^{n}\left(\log\frac{\sigma_t^2}{h_t^2}\right)^2;$$

$$\mathrm{MAE}_1 \equiv \frac{1}{n}\sum_{t=1}^{n}|\sigma_t - h_t|; \qquad \mathrm{MAE}_2 \equiv \frac{1}{n}\sum_{t=1}^{n}|\sigma_t^2 - h_t^2|;$$

	GARCH(1,1)	ARCH(1)	HAR	ARFIMA	GARCH + Gaussian	EGARCH + Gaussian
AFLT	0,000313932	6,72E-05	2,45E-07	1,58E-07	0,000107035	1,15778421
CHMF	NaN	0,000165269	3,18E-08	NaN	0,000117723	1,25967585
GAZP	0,000714161	7,62E-05	2,99E-07	9,15E-09	4,58E-05	1,758965297
GMKN	0,000493352	0,000110852	2,15E-08	1,97E-08	0,000110852	2,090208736
HYDR	4,59E-05	9,90E-05	2,67E-08	9,45E-09	4,59E-05	222,1406535
LKOH	NaN	6,69E-05	2,01E-08	1,13E-08	3,01E-05	18483,21974
MAGN	NaN	0,000139897	5,24E-08	5,22E-08	6,03E-05	3,419279922
MTSI	3,13E-05	3,40E-05	1,17E-08	4,42E-09	3,46E-05	15529,40997
NLMK	NaN	0,000158774	9,21E-08	7,74E-08	8,94E-05	2,406363955
NOTK	2,82E-05	7,80E-05	3,99E-07	1,85E-07	2,77E-05	19,53022852
OGKC	4,74E-05	9,80E-05	1,56E-07	1,35E-07	4,74E-05	72,13881874
OGKE	0,00054583	6,25E-05	6,12E-05	3,76E-07	0,000172518	5,599373413
PMTL	0,000139597	0,000140539	1,43E-06	1,31E-06	4,31E-05	8,390780541
RASP	0,000163566	0,000153006	8,17E-08	3,03E-07	0,000163566	1,314893652
ROSN	5,82E-05	0,00011499	1,26E-08	1,19E-08	5,49E-05	1,525893197
RTKM	NaN	7,29E-05	1,00E-06	9,23E-07	3,48E-05	1,276971269
SBER	5,16E-05	0,000145241	1,25E-08	6,11E-08	5,92E-05	4,21012829
SIBN	4,06E-05	9,99E-05	1,68E-07	4,31E-08	5,09E-05	1,218791491
SNGS	NaN	6,60E-05	2,36E-08	1,42E-08	3,15E-05	0,796068083
TATN	0,000216713	4,91E-05	1,97E-07	4,69E-08	0,00014116	1,42261428
TRNF	NaN	0,000141606	1,86E-07	5,27E-08	0,000100617	1,007468291

Table 14: MSE Performance Measure: 1-day ahead volatility forecasts

	GARCH(1,1)	ARCH(1)	HAR	ARFIMA	GARCH + Gaussian	EGARCH + Gaussian
AFLT	0,017636	0,007861	0,000313	0,00026	0,009797	1,076002
CHMF	Inf	0,012758	0,000124	4,32E-05	0,010848	1,12235
GAZP	0,026629	0,008514	0,000459	6,47E-05	0,006765	1,326258
GMKN	0,022119	0,010128	8,57E-05	8,54E-05	0,010128	1,445752
HYDR	0,006595	0,009669	0,00014	7,72E-05	0,006595	14,90438
LKOH	Inf	0,007743	0,000112	6,93E-05	0,005486	135,953
MAGN	Inf	0,011444	0,000171	0,000175	0,00776	1,849125
MTSI	0,005159	0,005563	9,13E-05	5,08E-05	0,005274	124,6171
NLMK	Inf	0,012284	0,000223	0,000214	0,00945	1,55124
NOTK	0,005182	0,008336	0,000415	0,000244	0,005245	4,419301
OGKC	0,006214	0,009112	0,00023	0,00024	0,006214	8,493457
OGKE	0,023354	0,007447	0,004454	0,000397	0,01244	2,366296
PMTL	0,010524	0,010615	0,000519	0,000442	0,00654	2,896681
RASP	0,011523	0,011182	0,000221	0,000476	0,011523	1,146685
ROSN	0,007548	0,010494	7,61E-05	7,48E-05	0,007406	1,235269
RTKM	Inf	0,007884	0,000533	0,000314	0,00588	1,129986
SBER	0,007109	0,011404	7,99E-05	0,00022	0,007691	2,051859
SIBN	0,006284	0,009377	0,000335	0,000148	0,007128	1,103988
SNGS	Inf	0,007684	0,000121	8,80E-05	0,005613	0,892225
TATN	0,014369	0,006389	0,000382	0,000137	0,010687	1,192733
TRNF	Inf	0,011628	0,000319	0,000156	0,010028	1,003701

Table 15: MAE Performance Measure: 1-day ahead volatility forecasts

	GARCH(1,1)	ARCH(1)	HAR	ARFIMA	GARCH + Gaussian	EGARCH + Gaussian
AFLT	5623,497	1118,189	-14,2416	-13,1988	1740,438	20883517
CHMF	Inf	3761,459	-13,885	Inf	2918,805	29526725
GAZP	62605,57	6250,35	33,17732	-1,56E+01	4718,491	1,72E+08
GMKN	18030,49	3631,843	-1,55E+01	-1,52E+01	3631,843	79712948
HYDR	2811,843	6070,296	-11,2558	-1,52E+01	2811,843	1,51E+10
LKOH	Inf	4064,154	-11,8341	-1,51E+01	2407,336	1,39E+12
MAGN	Inf	937,9998	-13,6634	-13,4372	468,6256	24090198
MTSI	4932,759	6009,526	-1,11E+01	-1,58E+01	5162,871	3,04E+12
NLMK	Inf	924,2425	-13,6836	-13,258	618,065	15097085
NOTK	251,6589	620,4722	-8,14259	-13,338	278,6601	1,7E+08
OGKC	594,6578	1279,779	-14,016	-12,9163	594,6578	1,14E+09
OGKE	7021,533	763,4575	966,038	-12,7921	1899,403	68995639
PMTL	729,3565	745,6275	-8,87759	-11,1866	394,1691	64112593
RASP	1679,992	1627,859	-12,898	-15,6402	1679,992	17698880
ROSN	3224,659	6141,488	-1,54E+01	-1,52E+01	3351,907	88325513
RTKM	Inf	799,825	-9,15993	-13,71	571,4557	18032478
SBER	2734,628	6176,036	-1,53E+01	-17,0996	3516,019	2,38E+08
SIBN	1626,11	3393,282	-0,78244	-13,5852	2252,519	50052018
SNGS	Inf	3470,825	-12,6764	-1,45E+01	2227,026	52625486
TATN	7523,107	1451,199	0,72367	-13,6036	5088,856	57926513
TRNF	Inf	3463,272	0,104363	-13,0687	2992,72	27885352

Table 16: QLIKE Performance Measure: 1-day ahead volatility forecasts

	GARCH(1,1)	ARCH(1)	HAR	ARFIMA	GARCH + Gaussian	EGARCH + Gaussian
AFLT	15,78592	10,15357	1,088188	0,465937	11,43737	64,50049
CHMF	Inf	14,62919	0,262432	Inf	13,52106	68,30681
GAZP	26,72501	16,29613	2,484075	2,17E-01	14,76602	82,15204
GMKN	22,08581	15,29338	2,23E-01	2,20E-01	15,29338	78,53607
HYDR	14,16708	17,09741	0,715775	2,64E-01	14,16708	131,2801
LKOH	Inf	15,11687	0,565464	2,65E-01	12,93701	186,2975
MAGN	Inf	10,47516	0,192181	0,196794	8,391086	68,56263
MTSI	15,76857	16,55651	7,80E-01	3,18E-01	15,78356	197,8389
NLMK	Inf	10,33795	0,26727	0,224007	8,958072	64,10211
NOTK	6,497595	8,748967	0,706012	0,290639	6,678948	84,0654
OGKC	8,102006	10,29969	0,378884	0,386801	8,102006	100,0836
OGKE	15,83582	8,36101	5,1601	0,721615	11,18702	72,78332
PMTL	8,400109	8,462856	0,65071	0,473241	6,481825	71,52238
RASP	11,39116	11,25469	0,387518	Inf	11,39116	63,67995
ROSN	14,00195	16,4272	2,17E-01	2,16E-01	13,95	77,3937
RTKM	Inf	9,265784	0,779922	0,402891	8,02673	62,92568
SBER	13,38943	16,66539	2,56E-01	0,143224	14,04899	86,18994
SIBN	11,27225	13,7393	1,268264	0,401325	12,20011	71,48183
SNGS	Inf	14,22563	0,493315	3,10E-01	12,34037	72,53629
TATN	17,14088	11,09833	1,431029	0,383493	14,35614	72,90577
TRNF	Inf	13,57225	0,952675	0,347443	12,72046	65,68282

Table 17: R2LOG Performance Measure: 1-day ahead volatility forecasts

In order to compare the predictive accuracy of our models, we perform the Hansen and Lunde's (2005) and Hansen's (2005) Superior Predictive Ability (SPA) test, which compares the performances of two or more forecasting models.

The forecasts are evaluated using a loss function (e.g. the RMSE). The best forecasting model is the model that produces the smallest expected loss.

The SPA test compares for the best standardized forecasting performance relative to a benchmark model, and the null hypothesis is that none of the competing models is better than the benchmark.

Let $L(Y_t; \hat{Y}_t)$ denote the loss if one had made the prediction, $\hat{Y}_t$, when the realized value turned out to be Y_t. The performance of model k relative to the benchmark model (at time t), can be defined as:

$$X_k(t) = L(Y_t, \hat{Y}_{0t}) - L(Y_t, \hat{Y}_{kt}), \quad k = 1, \ldots, l; \quad t = 1, \ldots, n.$$

The question of interest is whether any of the models $k = 1, \ldots, l$ is better than the benchmark model.

To analyze this question Hansen (2005) formulates the testable hypothesis that the benchmark model is the best forecasting model.

This hypothesis can be expressed parametrically as

$$\mu_k = \mathrm{E}\left[X_k(t)\right] \leq 0, \quad k = 1, \ldots, l.$$

For notational convenience, Hansen (2005) defines an l-dimensional vector μ by

$$\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_l \end{pmatrix} = E \begin{pmatrix} X_1(t) \\ \vdots \\ X_l(t) \end{pmatrix}$$

Since a positive value of μ_k corresponds to model k being better than the benchmark, Hansen (2005) wants to test the hypothesis $H_0 : \mu_k \leq 0$ for $k = 1, \ldots l$. Therefore, the equivalent vector formulation is

$$H_0 : \mu \leq \mathbf{0}$$

One way to test this hypothesis is to consider the test statistic

$$T_n^{sm} = \max_k \frac{n^{1/2} \bar{X}_k}{\hat{\sigma}_k}$$

where

$$\bar{X}_k = \frac{1}{n}\sum_{t=1}^{n} X_k(t), \quad \hat{\sigma}_k^2 = \widehat{\text{var}}(n^{1/2}\bar{X}_k).$$

The latter is estimated by using the bootstrap method. The superscript "sm" refers to standardized maximum. Under the regularity condition, Hansen (2005) shows that

$$T_n^{sm} = \max_k \frac{\bar{X}_k}{\hat{\sigma}_k} \xrightarrow{p} \max_k \frac{\mu_k}{\sigma_k}$$

which is greater than zero if and only if $\mu_k > 0$ for some k. So one can test H_0 using the test statistic T_n^{sm}. The only remaining problem is to derive the distribution of T_n^{sm} under the assumption of a true null hypothesis.

Testing multiple inequalities is more complicated than testing equalities (or a single inequality) because the distribution is not unique under the null hypothesis. Nevertheless, a consistent estimate of the p-value can be obtained by using a bootstrap procedure, as well as an upper and a lower bound

	GARCH(p,q)			ARCH(1)			GARCH(1,1)		
	p_c	p_u	p_l	p_c	p_u	p_l	p_c	p_u	p_l
AFLT	0.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	0.000
CHMF	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
GAZP	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
GMKN	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
HYDR	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
LKOH	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
MAGN	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
MTSI	0.034	0.110	0.034	0.000	0.000	0.000	1.000	1.000	1.000
NLMK	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
NOTK	0.332	0.470	0.332	0.000	0.000	0.000	1.000	1.000	1.000
OGKC	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
OGKE	0.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	0.000
PMTL	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
RASP	0.003	0.009	0.003	1.000	1.000	1.000	0.003	0.006	0.003
ROSN	1.000	1.000	1.000	0.000	0.000	0.000	0.064	0.206	0.064
RTKM	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
SBER	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
SIBN	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
SNGS	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN
TATN	0.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	0.000
TRNF	1.000	1.000	1.000	0.000	0.000	0.000	NaN	NaN	NaN

Table 18: SPA test p-values (only GARCH type models)

	HAR-RV			ARFIMA-RV		
	SPA_c	SPA_u	SPA_l	SPA_c	SPA_u	SPA_l
AFLT	0,001	0,004	0,001	1	1	1
CHMF	1	1	1	0	0	0
GAZP	0	0	0	1	1	1
GMKN	0,466	0,828	0,466	1	1	1
HYDR	0	0	0	1	1	1
LKOH	0	0	0	1	1	1
MAGN	1	1	1	0,223	0,552	0,223
MTSI	0,001	0,001	0,001	1	1	1
NLMK	0,143	0,458	0,143	1	1	1
NOTK	0,012	0,045	0,012	1	1	1
OGKC	1	1	1	0,168	0,406	0,168
OGKE	0,022	0,022	0,02	1	1	1
PMTL	0	0,004	0	1	1	1
RASP	1	1	1	0	0	0
ROSN	0,31	0,778	0,31	1	1	1
RTKM	0	0	0	1	1	1
SBER	1	1	1	0	0	0
SIBN	0,002	0,002	0,002	1	1	1
SNGS	0,006	0,008	0,006	1	1	1
TATN	0	0	0	1	1	1
TRNF	0,008	0,028	0,008	1	1	1

Table 19: SPA test p-values (all models)

What is Next ?

Clearly the two previous research approaches (*Portfolio allocation* and *Volatility modelling*) will unite. We therefore plan to:

- *Use FHS with all the previous discussed GARCH and RV models* (and not only with the GARCH(1,1) model), and compute and compare the Mean-Variance, Mean-CVaR, and Mean-SRM portfolios;
- Consider *Copula-GARCH* type models (and compute and compare Mean-Variance, Mean-CVaR, and Mean-SRM)
- Consider *Copula-RV-HAR* type models (and compute and compare Mean-Variance, Mean-CVaR, and Mean-SRM)
- Consider *Copula-RV-ARFIMA* type models (and compute and compare Mean-Variance, Mean-CVaR, and Mean-SRM)
- Consider *GARCH-DCC* models (and compute and compare Mean-Variance, Mean-CVaR, and Mean-SRM)
- Consider *Multivariate RV-VARFIMA* models (and compute and compare Mean-Variance, Mean-CVaR, and Mean-SRM)

Main contributions:

- First large portfolio analysis with Russian data
- First large comparison among Mean-Variance, Mean-CVaR, and Mean-SRM using data including also the current global financial crisis
- Comparison among Mean-Variance, Mean-CVaR, and Mean-SRM to verify the effect of the Russian short sale ban in 2008-2009 (TO BE DONE)
- Comparison among Mean-Variance, Mean-CVaR, and Mean-SRM and a simple equally-weighted portfolio (TO BE DONE)

Main findings: (working in progress)

- Mean-CVaR usually outperforms Mean-Variance (particularly during the crisis)
- Mean-VaR allocations suffer from heavy tail losses
- Mean-Variance weights closer to Mean-CVaR weights in good times (as expected)
- Mean-SRM is extremely sensitive to the change of the user's coefficient of absolute risk-aversion k and tends to corner solutions

- RV-ARFIMA models are the best in case of volatility forecasting for Russian stocks, statistically outperforming GARCH type models;
- However, RV-HAR models cannot be outperformed (in most cases) by ARFIMA models and have similar performances.
- Returns standardized with RVs are approximately normal.

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