Continuous time option pricing with scheduled jumps in the underlying asset

Dmitry Storcheus Sergey Gelman

LFE & ICEF, Higher School of Economics, Moscow

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Motivation

- Stock prices do jumps
- Jumps occur regularly
  - shareholders' and board meetings
  - earnings announcements
- Impact on option prices and hedging should be accounted for

Stylized facts

- scheduled macroeconomic announcements cause jumps in bond prices (Beber and Brandt 2006, Piazzesi 2005, Johannes 2004)
- evidence of jumps in stock prices on the dates of earnings announcements (Pattel and Wolfson 1981, 1984; Maheu and McCurdy 2004; Dubinsky and Johannes 2006)
- earnings announcements-caused jumps have an impact on the implied volatility (Pattel and Wolfson 1979, 1981; Dubinsky and Johannes 2006)

Literature review: Solutions for option pricing

- Combination of Merton's (1976) jump diffusion with scheduled jumps (Abraham and Taylor, 1991,1997; Boes et al. 2007 - for overnight jumps)
  - heavily parameterized, computationally intensive
- Combination of stochastic volatility with scheduled jumps (Dubinsky and Johannes 2006)
  - no analytical solution; heavily parameterized

Our Contribution

- Parsimonious model accounting for jumps on earnings announcement dates (Black-Scholes + a scheduled jump)
  - computational speed enhancement of order 2
- Provide some evidence of higher relevance of scheduled jumps compared to random
- Derive implications for hedging
  - larger position in stock
  - more intensive rebalancing out-of-the money, less intensive at-the-money

Outline

The model

- SDE for stock:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_{t-} d\xi_t$$

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$$\xi_t = \begin{cases} 1, & \text{if } t < t_1 \\ \Psi(1), & \text{if } t = t_1 \\ \xi_{t_1}, & \text{if } t > t_1, \end{cases}$$

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- $$\Psi(1) \sim \text{Uniform}(1 - a; 1 + a)$$

The model

- Solving for the price of the European call option

$$V(0) = E^Q\left(e^{-rT}\max\left(S_T - K;0\right)\right),$$

where K is the strike.

- We assume that jumps are diversifiable (jump distribution under risk-neutral measure = jump distribution under market measure)

$$S_T^Q = S_0\exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t^Q\right)\times\xi_{t_1}$$

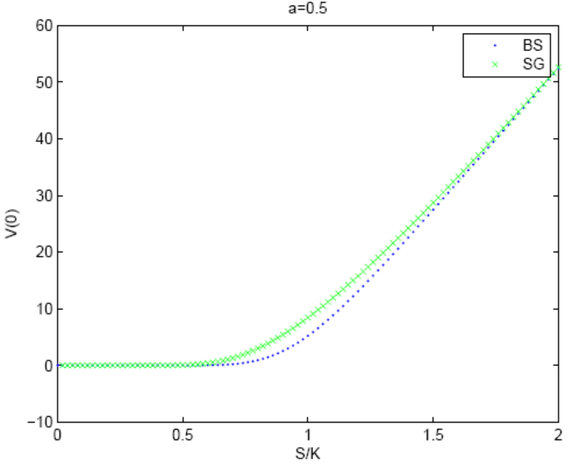
The model

$$V\left(S_0\right)=\frac{1}{2}\left[V_+^{BS}+V_-^{BS}\right]+\frac{1}{2a}\left[V_+^{BS}-V_-^{BS}\right]+C,$$

where V_+^{BS} and V_-^{BS} denote the BS-value of an option under maximum $(1+a)$ and minimum $(1-a)$ jump correspondingly. C is a correction term.

Model with jump vs. BS

Figure 1: Theoretical prices of SG and BS for $a = 0.5$. $\sigma = 0.2$, $K = 50$, $T = 1$, $r = 0.05$



Data

- call option prices on 5 low dividend paying US stocks: AAPL, MSFT, INTC, CSCO, AMD; from IvyDB
- Time range 1999-2008
- OTM $\left(\frac{S}{K} < 0.97\right)$ and ATM $\left(0.97 \leq \frac{S}{K} \leq 1.03\right)$ options
- Focus: only options with one earnings announcement before expiration
- 'Cleaning' filter:
  - 0.05$< IV_i < 1$
  - put-call-parity violations $< 1.5 \times$ *spread*
  - no theoretical bounds violations
  - best bid $>$0

Descriptive statistics

Sample Descriptive Statistics for INTC (figures in USD)

Moneyness		T (days) < 30	$[30; 60]$	> 60	Subtotal N
< 0.97	price	0.69	1.10	1.20	
	spread	0.07	0.08	0.08	
	N	(613)	(1746)	(74)	(2433)
$[0.97; 1.03]$	price	2.17	3.18	3.50	
	spread	0.10	0.13	0.14	
	N	(307)	(696)	(36)	(1039)
Subtotal N		(920)	(2442)	(110)	(3472)

Empirical set-up

- estimate parameter values over annual periods via minimizing squared pricing errors

$$\widehat{\theta} = \arg\min_{\theta} \left[\frac{1}{N} \sum_{i=1}^{N} \left(M_i - V_i(\theta) \right)^2 \right],$$

  where M_i is the observed market price of option i.
- Compare to benchmarks in-sample for each year-stock sub-sample
  - to Black/Scholes (1973) - F-test
  - to Merton (1976) - Diebold-Mariano (1995) test

$$DM = \frac{\overline{d}}{\sqrt{var\left(\overline{d}\right)}},$$

$$d_i = \left(M_i - V_i^{SG}(\theta) \right)^2 - \left(M_i - V_i^{JD}(\theta) \right)^2$$

- Out-of-sample: one year rolling window; Diebold-Mariano (1995) for both comparisons

Results for OTM options with one EA until expiry

		σ	a	λ	k	δ	RMSE	RE
AMD	BS	0.61					0.1705	0.2270
	SG	0.56	0.14				0.1563	0.2189
	JD	0.06		125.14	0.08	0.05	0.1626	0.2102
CSCO	BS	0.42					0.2428	0.2853
	SG	0.38	0.10				0.2293	0.2586
	JD	0.04		58.34	0.10	0.05	0.2267	0.2652
INTC	BS	0.38					0.2053	0.3009
	SG	0.34	0.08				0.1962	0.2759
	JD	0.02		41.48	0.06	0.01	0.1967	0.2683
AAPL	BS	0.54					0.6349	0.2821
	SG	0.48	0.11				0.6128	0.2795
	JD	0.09		26.03	0.02	0.15	0.6038	0.2873
MSFT	BS	0.32					0.2741	0.3372
	SG	0.28	0.06				0.2699	0.3191
	JD	0.13		92.22	0.01	0.06	0.2676	0.3156

Results for ATM options with one EA until expiry

		σ	a	λ	k	δ	RMSE	RE
AMD	BS	0.61					0.1915	0.0896
	SG	0.51	0.19				0.1699	0.0821
	JD	0.18		150.78	-0.04	0.01	0.1844	0.0854
CSCO	BS	0.43					0.2676	0.1053
	SG	0.39	0.10				0.2527	0.0960
	JD	0.00		120.48	0.01	0.00	0.2655	0.1040
INTC	BS	0.39					0.2585	0.1081
	SG	0.35	0.08				0.2446	0.1002
	JD	0.09		126.65	-0.02	0.00	0.2559	0.1064
AAPL	BS	0.54					0.6981	0.1113
	SG	0.46	0.14				0.6646	0.1059
	JD	0.02		45.33	-0.02	0.06	0.6885	0.1081
MSFT	BS	0.32					0.3759	0.1362
	SG	0.28	0.06				0.3711	0.1325
	JD	0.05		27.73	-0.06	0.06	0.3677	0.1272

In-sample results: summary

- SG model significantly (5%) outperforms Black-Scholes
  - in 38 out of 50 cases for OTM
  - in 35 out of 50 cases for ATM
- Merton's jump diffusion never does significantly outperform SG
- SG shows superior performance than JD
  - in 22 out of 50 for OTM
  - in 28 out of 50 for ATM

Out-of-sample results, ATM options (1)

	BS			SG			JD		
	MSE	MAE	MRE	MSE	MAE	MRE	MSE	MAE	MRE
AMD	0.14	1.27	0.19	0.10	0.22	0.17	0.14	0.34	0.19
CSCO	0.38	0.36	0.20	0.36	0.35	0.18	0.37	0.31	0.18
AAPL	1.32	0.62	0.10	1.25	0.62	0.11	1.30	0.66	0.10
MSFT	0.30	0.42	0.25	0.27	0.38	0.19	0.27	0.53	0.18
INTC	0.62	0.60	0.15	0.63	0.62	0.15	0.63	0.87	0.15

Out-of-sample results, ATM options (2)

Company	d_i	mean	st. error (N-W)	p-value
AAPL	$BSSE_i - SGSE_i$	0.066	0.101	0.255
	$SGSE_i - JDSE_i$	-0.051	0.165	0.620
AMD	$BSSE_i - SGSE_i$	0.039	0.011	0.000
	$SGSE_i - JDSE_i$	-0.039	0.011	0.999
CSCO	$BSSE_i - SGSE_i$	0.013	0.008	0.066
	$SGSE_i - JDSE_i$	-0.007	0.008	0.808
INTC	$BSSE_i - SGSE_i$	-0.006	0.043	0.558
	$SGSE_i - JDSE_i$	-0.001	0.049	0.502
MSFT	$BSSE_i - SGSE_i$	0.024	0.011	0.018
	$SGSE_i - JDSE_i$	0.001	0.008	0.471

Hedging portfolio

$$P_t = V(S_t) - \Delta S_t,$$

where $\Delta = \partial V(S_t) / \partial S_t$

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- Different properties if you can distinguish jump and diffusion components compared to aggregate volatility: $\sigma_{per}^{BS} = \sqrt{\sigma^2 + \frac{a^2}{3T}}$.

Delta

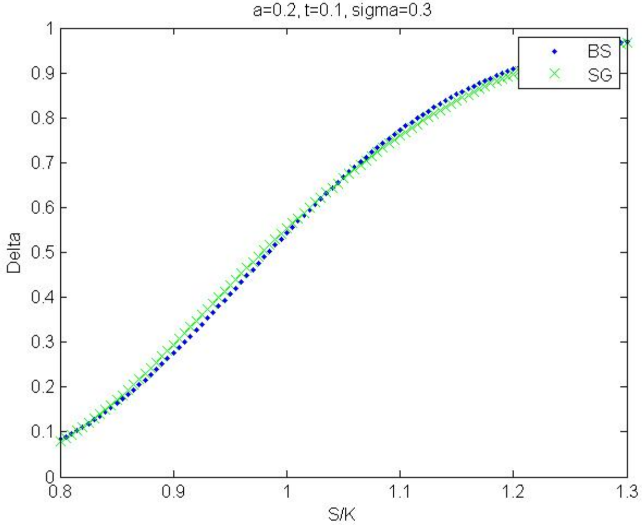


Figure: *Delta* as a function of moneyness S/K

Gamma

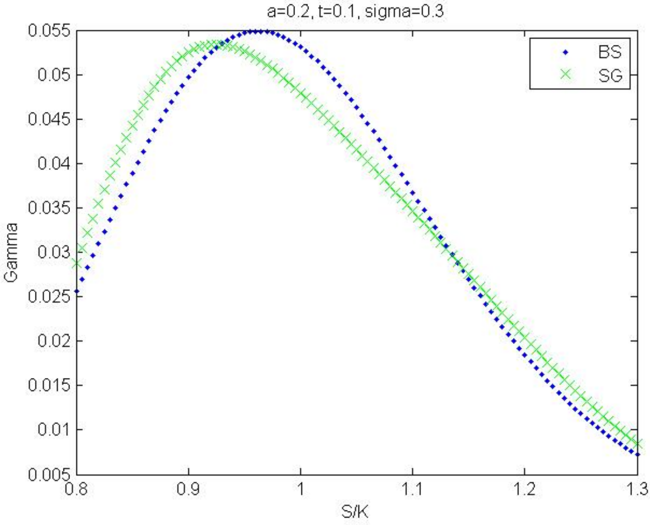


Figure: *Gamma* as a function of moneyness S/K

Concluding remarks

- We find supportive evidence that option prices incorporate scheduled jumps
- Our results suggest that jumps on earnings announcements dates are more price relevant than jumps at random time points
- Hedging strategies prior to earnings announcements should be altered

Future research steps

- extend the model to several jumps
- introduce a jump-diffusion process where scheduled jumps may alter the volatility

Thank you for your attention!

Correction term

$$C = \frac{K^2}{4aS_0} e^{\left(\sigma^2 - 2r\right)T} \left\{ \Phi\left(d_3^+\right) - \Phi\left(d_3^-\right) \right\} - \frac{(1+a)^2 S_0}{4a} \left\{ \Phi\left(d_1^+\right) - \Phi\left(d_1^-\right) \right\}.$$

$$d_1^+ = \frac{\ln\left(\frac{S_0(1+a)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_1^- = \frac{\ln\left(\frac{S_0(1-a)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_3^+, d_3^- = d_1^+, d_1^- - 2 \cdot \sigma\sqrt{T}.$$

Delta formula

$$\begin{aligned}\Delta &= \frac{\partial V\left(S_0\right)}{\partial S_0} = \frac{1}{2}\left[\Phi\left(d_1^+\right) + \Phi\left(d_1^-\right)\right] + \frac{1+a^2}{4a}\left[\Phi\left(d_1^+\right) - \Phi\left(d_1^-\right)\right] \\ &\quad - \frac{K^2}{4aS_0^2} e^{T\left(\sigma^2 - 2r\right)} \cdot \left[\Phi\left(d_3^+\right) - \Phi\left(d_3^-\right)\right].\end{aligned}$$

for BS:

$$\Delta^{BS} = \Phi\left(d_1\right),$$

where d_1 is standard BS notation for

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}.$$

if only aggregate volatility perceived

$$\sigma_{per}^{BS} = \sqrt{\frac{\sigma^2 T + \frac{a^2}{3}}{T}} = \sqrt{\sigma^2 + \frac{a^2}{3T}}.$$

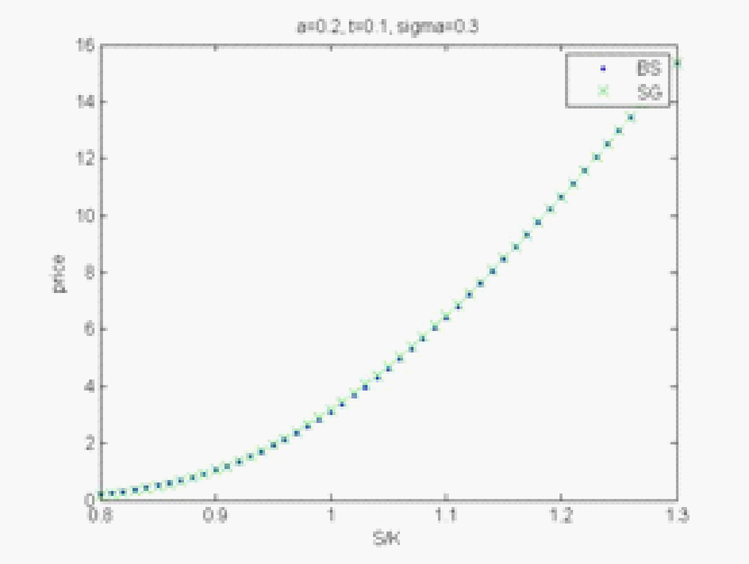
Gamma formula

$$\Gamma = \frac{\partial^2 V(S_0)}{\partial S_0^2} = \frac{\partial \Delta}{\partial S_0} = \frac{1}{2} \cdot \frac{\left[\phi\left(d_1^+\right) + \phi\left(d_1^-\right)\right]}{S_0 \sigma \sqrt{T}}$$
$$+ \frac{\frac{a}{4} \cdot \left[\phi\left(d_1^+\right) - \phi\left(d_1^-\right)\right]}{S_0 \sigma \sqrt{T}} - \frac{1+a}{2\sigma\sqrt{T}} \cdot \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \cdot \phi\left(d_1^-\right)$$
$$+ \frac{1}{2} \cdot \frac{e^{T\left(\sigma^2 - 2r\right)} K^2}{a S_0^3} \cdot \left[\Phi\left(d_3^+\right) - \Phi\left(d_3^-\right)\right]. \quad (1)$$

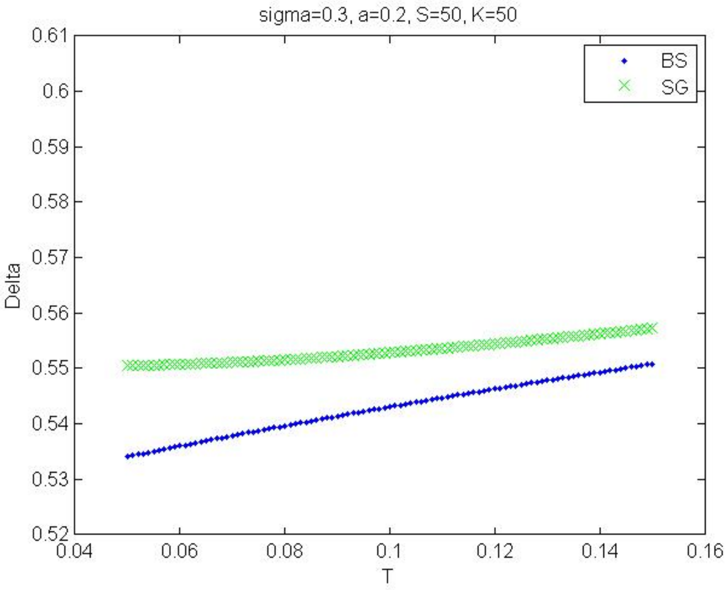
for BS:

$$\Gamma_{per}^{BS} = \frac{\phi(d_1)}{S_0 \sigma_{per}^{BS} \sqrt{T}}$$

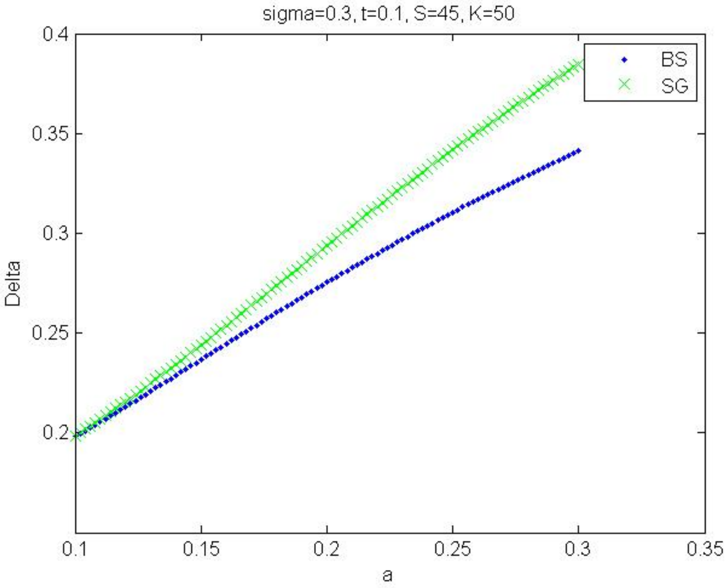
SG vs. perceived volatility BS



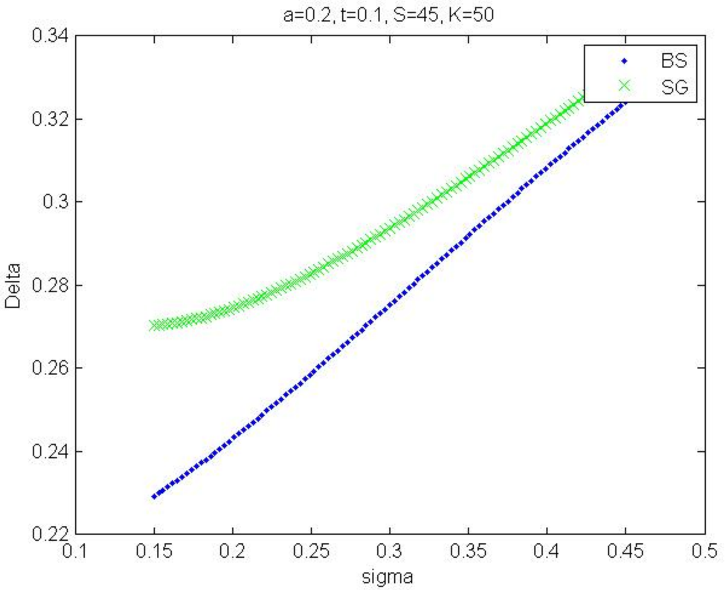
Delta as a function of T



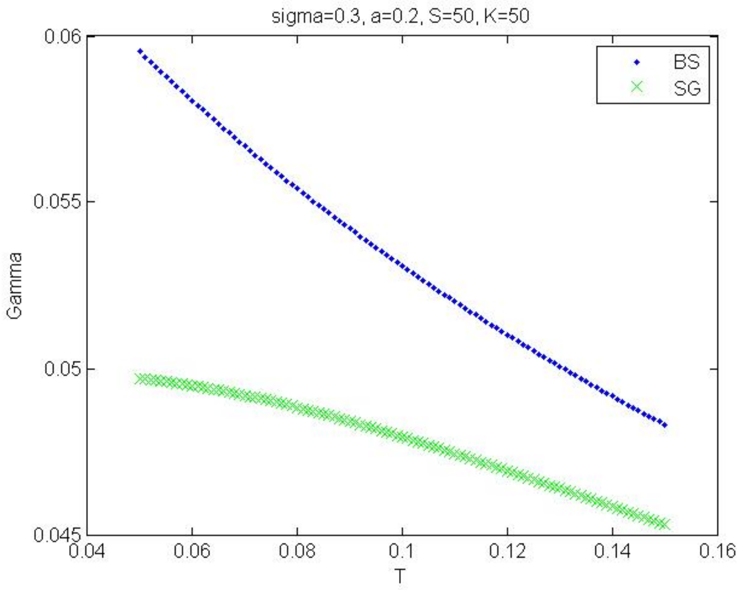
Delta as a function of a



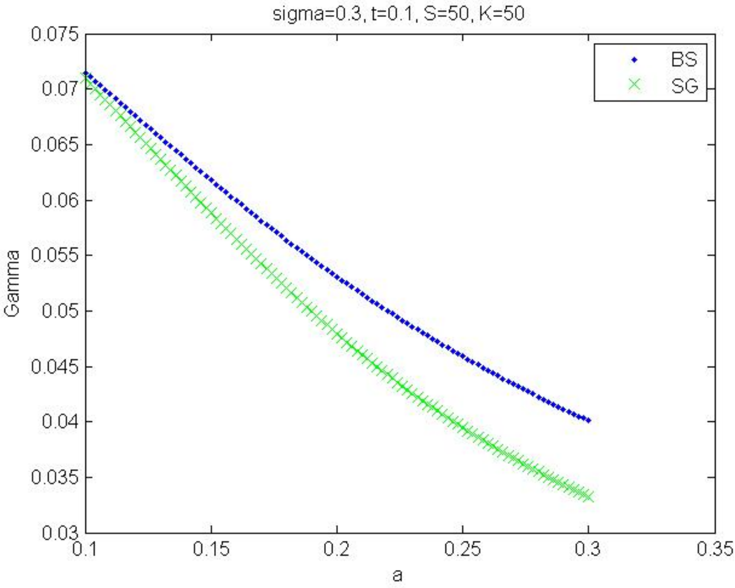
Delta as a function of σ



Gamma as a function of T



Gamma as a function of *a*



Gamma as a function of σ