Social structure and propagation of depositors' panic

Roman Chuhay

Higher School of Economics,
ICEF, CAS

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Motivation

Seminal paper by Diamond Dybvig JPE (1983)

- Assumptions
  - 1st period depositors invest money to the bank.
  - 2nd period nature reveals their type:
    - Proportion s of them are impatient and withdraw money immediately.
    - Proportion $1 - s$ are patient and play coordination game.
- Conclusions
  - Deposits can provide allocation superior to those of exchange market.
  - There is a multiple equilibria, one of which is always a bank run.
  - Government provision of insurance may produce superior outcome

Motivation

- Depositors decisions are partially sequential
  - Descriptions of bank runs Sprague (1910), Wicker (2001)
  - Statistical data Starr and Yilmaz (2007)
- Many depositors make decision observing actions of others:
  - Kelly and Grada (2000) - bank run of Turkey's Islamic financial houses in 2001.
  - Iyer and Puri (2008) consider depositor level data for a bank that faced a run in India in 2001.
- Main contribution: we introduce social network as a coordination mechanism that depositors may use to make their decision.

Model

- There is a continuum of agents.
- Agents are embedded into the network of personal contacts, represented by a random graph with degree distribution $p(k)$.
- At period 0 each agent invests 1 unit into a bank account.
- At period 1 nature reveals agents type in 2 steps:
  - Nature draws proportion of impatient agents s in the society from distribution with CDF $Q(s)$.
  - According to realized s nature assigns types to depositors.

Model

- Impatient depositor withdraws money regardless of prevailing conditions.
- Patient depositor with k links withdraws according to the strategy $P_w(m, k) \in [0, 1]$.
- If a depositor withdraws money from the bank she gets pay-off $a(w(s))$
- If depositor waits till 2nd period she gets pay-off $b(w(s))$
- We assume single crossing property of $b(w) - a(w)$: $b(w) > a(w)$ for $w < \bar{w}$ and $b(w) > a(w)$ for $w > \bar{w}$

Model

- Probability that a randomly chosen neighbor of depositor withdraws:

$$\hat{w} = s + (1-s)\sum_{k=1}^{\infty}\xi(k)\sum_{m=0}^{k-1}P_w(m,k)\frac{(k-1)!}{m!(k-1-m)!}\hat{w}^m(1-\hat{w})^{k-1-m},$$

where $\xi(k)$ is the degree distribution of depositor's neighbor.

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- Proportion of agents that withdraw is:

$$w(s,\hat{w}) = s + (1-s)\sum_{k=0}^{\infty}p(k)\sum_{m=0}^{k}P_w(m,k)\frac{k!}{m!(k-m)!}\hat{w}^m(1-\hat{w})^{k-m}$$

Maximization problem

- Depositor solves the following maximization problem:

$$\sum_{m=0}^{k} P(M = m|k) \int_0^1 \left[(1 - P_w(m,k))b(w(s)) + P_w(m,k)a(w(s))\right] P(S = s|m,k)ds$$

- $P(M = m|k)$ is the probability to observe m out of k neighbors withdrawing
- $P(S = s|m,k)$ Bayesian updating of belief about true state s.

Optimal decision

Proposition

Optimal decision strategy of agent $P_w(m, k)$ is a cut-off rule, such that agent withdraws if $m \geq m_k$ and waits otherwise. Moreover, cut-off value $m_{k+1} \in \{m_k, m_{k+1}\}$.

Maximization problem

- Knowing that optimal strategy is cut-off rule maximization problem becomes:

$$\int_0^{\bar{w}} b(w) dQ(w^{-1}(s)) + \int_{\bar{w}}^1 a(w) dQ(w^{-1}(s)) -$$

$$- \int_0^{\bar{w}} [b(w) - a(w)] I_w(m_k, k+1-m_k) dQ(w^{-1}(s)) -$$

$$- \int_{\bar{w}}^1 [a(w) - b(w)][1 - I_w(m_k, k+1-m_k)] dQ(w^{-1}(s))$$

- The second term is loss due to the 1st type error (false positive)
- The third term is 2nd type error (false negative)

Examples

- States: $s_l = 0, s_h = \frac{1}{2}, q = \frac{1}{2}$ Pay-off: $a(w) = 1$, $b(w)$ equals 2 for $w < \bar{w}$ and 0 otherwise.

k=1

m=0, U=1, w^l=1, w^h=1

m=1, U=1,25, w^l=0, w^h=3/4

m=2, U=2, wl=0, wh=1/2

k=2

m=0, U=1, wl=1, wh=1

m=1, U=1, wl=1, wh=1

m=2, U=2, wl=0, wh=5/8

m=3, U=2, wl=0, wh=1/2

BR, U=1

no BR, U=1 | **no BR, U=2**

Optimal decision

Proposition

Depositor's utility is increasing function in the number of links.

Optimal decision

Proposition

Depositor's utility is increasing function in the number of links.

- Assume that it is optimal for the agent to have $\frac{m_k}{k} = \frac{1}{2}$.
- Depositor with 2 links by setting $m_k = 1$ has exactly the optimal cut-off value.
- Depositor with 3 links can approximate optimal cut-off strategy only by $\frac{1}{3}$ or $\frac{2}{3}$.

Continuous Case

Proposition

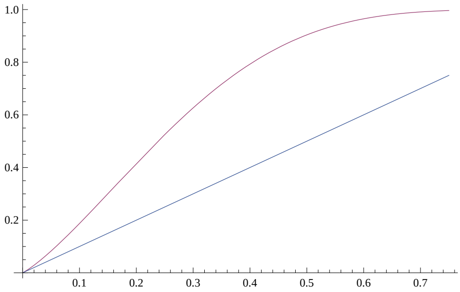
For an arbitrary degree distribution and exogenously given cut-off rule $m_k = \alpha k$, if mean degree converges to infinity then the following holds:

$$\hat{w} = \begin{cases} s, & s < \alpha \\ 1, & \textit{otherwise} \end{cases}$$

Continuous Case

Proposition

For any two degree distributions $F(k)$ and $\tilde{F}(k)$ and for their corresponding neighboring node degree distributions $G(k)$ and $\tilde{G}(k)$, if the following holds: $\tilde{F}(k)$ **FOSD** *$F(k)$ and $\tilde{G}(k)$* **FOSD** *$G(k)$ and for any k, $0 < m(k+1) - m(k) < 1$, then there are $\underline{s}$ and $\bar{s}$, such that $w^*_{\tilde{F}}(s) < w^*_F(s)$ for $s < \underline{s}$ and $w^*_{\tilde{F}}(s) > w^*_F(s)$ for $s > \bar{s}$.*



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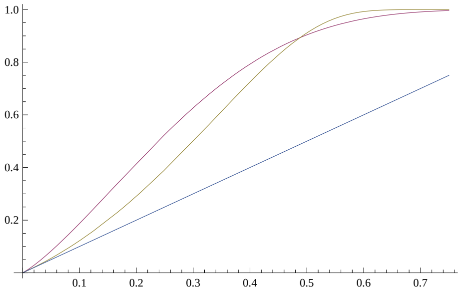

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