# Discussion of “Robustness of equilibrium in the Kyle model of informed speculation” by Alex Boulatov and Dan Bernhardt

Discussant: Dmitry Makarov, NES

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# Summary

- Often, choice of equilibrium notion is driven by considerations of tractability
  - notion becomes widely used, while its properties are not well-understood
- It is important to know the properties (robustness, uniqueness) as they determine how reasonable/realistic the equilibrium outcome is
- Alexey's paper - investigates robustness of equilibrium in Kyle's (1983) model

# Summary

- Main finding: the linear Kyle equilibrium is robust, non-linear (if it exists) is not robust
- Paper is short, very well-written
- The message is very clear
- Analysis is sophisticated, but for most expressions the authors provide a clear economic intuition

# First reaction

- Standard calculus
  - We maximize objective function *f(x)*: solve *f(x)* -> max and get optimal $x^*$
  - Because $f'(x^*)=0$, small perturbations of $x^*$ have no first order effects on the value of *f()*
- Quick reaction to the paper: the paper's result is analogous to the above but when *x* is a functional
  - Question: Does it make any sense to think in this way?
- What's missing in the above: the above logic cannot explain non robustness of non-linear equilibria

# Main Comment

- What is the intuition behind such difference between linear and non-linear strategies?
- Appendix: a certain expectation equals

$$v - \overline{P}_e\left(X\left(v\right)\right) - X\left(v\right)\overline{P}'_e\left(X\left(v\right)\right) - \overline{P}''_e\left(X\left(v\right)\right)$$

- This is how second derivative appears
- It could be instructive to discuss each term, and particularly focus on the last one

# Expositional comments

- Definition of robustness: looks at variations in expected payoffs
  - Question: why not looking at variation in agent's best responses?
- Authors explain carefully how perturbing MM's conjecture about the insider affects the market clearing price. Effect on MM's estimate of efficient price is mentioned, but not explained - would be nice to provide some details

- Terminology: ''expected price'' and ''estimated price'' – same or not?
  - If not, comment in the paper on the difference

# Two ``naive'' comments

- What's the latest on the existence of non-linear equilibrium in Kyle-type models?
  - It would be good to briefly comment on this in the paper
  - Can your paper be used as an excuse for not looking for nonlinear equilibria in Kyle-type models?
- Analysis is mathematically sophisticated because *v* has a continuous support, leading to the use of functionals
  - What if *v* can take a finite set of values (e.g., 0 and 1)?