Discussion

Affine option pricing model in discrete time

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What's about?

- affine jump-diffusion models with stochastic volatility
  - nearly-closed form solutions for option pricing

    though... poor fit due to local Gaussian nature

- GARCH approach in discrete time
  - closed form option prices, even if nonhomogenous
  - additional degrees of freedom may eventually lead to better fit

    though... temporal aggregation and leverage effect issues

this paper... Heston-type affine model in discrete time

- compound autoregressive process (DGJ, JTSA 2006)
- closeness to temporal aggregation (MR, JoE 2003)
- leverage vs instantaneous causality vs volatility feedback

Ingredients

return does not Granger cause volatility

- disentangle leverage from volatility feedback due to risk premium
- consistent with an information flow view of stochastic vol as link between return and volume (Andersen, JF 1996)

volatility dynamics affine and same leverage in both measures

- characterization of how leverage affects the volatility smile
- exponential affine stochastic discount factor
- information content of options data (GGR, Ecta 2011)

Feounou and Tedongap (JBES 2012) is the closest paper

- design similar specification to handle conditional skewness

  though. . . pay no special attention to leverage effects
  stochastic discount factor not exponential affine

Restricting the dynamics...

$$\begin{aligned}\mathbb{E}\left[\exp(-u\sigma_{t+1}^2)\,\middle|\,I_t\right] &= \exp\left(-a(u)\sigma_t^2 - b(u)\right)\\ \mathbb{E}\left[\exp(-vr_{t+1}^2)\,\middle|\,I_t, \sigma_{t+1}^2\right] &= \exp\left(-\alpha(v)\sigma_{t+1}^2 - \beta(v)\sigma_t^2 - \gamma(v)\right)\end{aligned}$$

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- structure preserving across measures (SDF+leverage)
- statistical identification of the leverage effect
- exact discretization of Heston model

Empirical illustration

- it is not MLE that does not allow for long memory in volatility
- measurement error issue due to the use of realized measures
- instruments for returns are at best weak
- incongruent model, hence hard to interpret
- missing in-depth analysis of the leverage effect

(Bandi and Renò, JoE 2012)