Conditional alphas and realized betas

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motivation

unconditional version of the CAPM does not provide in general a good description of equity markets

- well-known market anomalies
- nonzero (unconditional) alpha

however... the absence of pricing error does not suffice to ensure alpha equal to zero because time variation in betas may correlate with market volatility and/or with risk premia

allowing for time-varying betas explains most of the unconditional value premium given that value stocks are riskiest in recession times (Petkova and Zhang, JFE 2005; Zhang, JF 2005)

usual fix

conditional (rather than static) version of the CAPM based on alphas and betas that are affine on stock characteristics, interest rates and spreads as well as other business-cycle indicators

- easy to estimate and to interpret
- fit is much better
- **though...** not entirely correct! (Gagliardini, Ossola and Scaillet, 2013)

stylized facts

- market betas indeed vary over time
- pricing errors are smaller on average
- *nonzero* pricing errors that also vary over time

Shanken (JoE 1990), Jagannathan and Wang (JF 1996), Christopherson, Ferson and Glassman (RFS 1998), Lettau and Ludvigson (JF 2001), Wang (JF 2002), Adrian and Franzoni (JEF 2009)

questions we are interested in...

about methods...

- is the affine specification flexible enough?
- what is the optimal sampling frequency?

about pricing errors...

- what are the main drivers?
- how persistent/predictable are they?
- is it profitable to arbitrage them away?

remark focus is on pricing errors rather than on risk premium

- we make no attempt to model conditional betas, instead taking a realized approach Barndorff-Nielsen and Shephard (Ecta 2004)

Andersen, Bollerslev, Diebold and Wu (2006)

paper in one slide

ingredients

- stock prices evolve on continuous time with, whereas the state variables are in discrete time

- drift and diffusion components are measurable functions of the state variables and hence constant over shorter intervals of time

identification

- estimate realized beta and then adjust returns for risk

risk-adjusted return = conditional alpha + innovation

- innovations are independent of the conditioning state variables
- back out pricing errors by estimating conditional expectations

related literature in financial econometrics

integrating observations at different sampling frequencies
not exactly new in financial econometrics, especially as what concerns volatility estimation (Merton, JFE 1980; French, Schwert and Staumbaugh, JFE 1987; Ghysels, Santa Clara and Valkanov, JFE 2005)

more recently, combining low- and high-frequency observations to estimate continuous-time factor pricing models with constant factor loadings (Chang, Kim and Park, 2009)

nonparametric alphas and betas
local averages in time (Lewellen and Nagel, JFE 2006; Li and Yang, JEF 2011; Ang and Kristensen, JFE 2012) are purely descriptive, not allowing us to examine the main drivers of pricing errors and/or alpha portability

roadmap

1. from continuous to discrete time
2. conditional alphas
3. asymptotic theory
4. mispricings in the S&P 100 index constituents
5. trading pricing errors
6. concluding remarks

from continuous

to discrete time

CAPM and exact discretization

- estimation procedure relies on high-frequency data (as well as on infill asymptotics) and hence one must think carefully about the underlying continuous-time process governing asset prices

main issues that we must deal with. . .

- exact discretization of a continuous-time CAPM gives way to a multifactor model (Longstaff, JF 1989)

- asymptotic theory requires semimartingales in continuous time (Barndorff-Nielsen and Shephard, Ecta 2004)

starting point

for any $t \leq s < t+1$ and $i \in \{1, \ldots, N\}$,

$$\begin{aligned} \mathrm{d}P_i(s) &= \mu_{i,t}\,\mathrm{d}s + \boldsymbol{\Sigma}'_{i,t}\,\mathrm{d}\boldsymbol{W}_F(s) + \sigma_{i,t}\,\mathrm{d}W_i(s) && (1) \\ \mathrm{d}\boldsymbol{F}(s) &= \boldsymbol{\mu}_{F,t}\,\mathrm{d}s + \boldsymbol{\Sigma}_{F,t}\,\mathrm{d}\boldsymbol{W}_F(s), && (2) \end{aligned}$$

drift parameters	$\mu_{i,t} \equiv \mu_i(\boldsymbol{C}_t)$ $\boldsymbol{\mu}_{F,t} \equiv \boldsymbol{\mu}_F(\boldsymbol{C}_t)$
factor loadings	$\boldsymbol{\Sigma}_{i,t} \equiv \boldsymbol{\Sigma}_i(\boldsymbol{C}_t)$
factor covariance matrix	$\boldsymbol{\Sigma}_{F,t} \equiv \boldsymbol{\Sigma}_F(\boldsymbol{C}_t)$
Brownian motions	$\boldsymbol{W}_F(s) \perp\!\!\!\perp W_i(s)$

conditional semimartingale processes

Lemma 1: Let $\boldsymbol{X}_i(s) = (P_i(s), \boldsymbol{F}(s))$ evolve as in (1) and (2). Let also $\boldsymbol{C}(s) = \boldsymbol{C}_t$ for any $s \in [t, t+1)$ and define the filtration $\mathcal{F}_C(s) = \sigma(\boldsymbol{C}(\tau),\ \tau \leq s)$ for $s > 0$. If $\boldsymbol{C}(s)$ is independent of both $W_i(s)$ and $\boldsymbol{W}_F(s)$, then $\boldsymbol{X}_i(s)$ is a conditional semimartingale with independent increments given $\mathcal{F}_C(s)$.

- common risk factors $\boldsymbol{F}(s)$ depend on the conditioning factors $\boldsymbol{C}_t$ for any $s \in [t, t+1)$ through the drift and diffusion parameters
- $\boldsymbol{X}_i(s)$ has independent increments for any $s \in [t, t+1)$, so market microstructure effects are responsible for intraday autocorrelation
- genuine autocorrelation in the daily increments $\boldsymbol{x}_{i,t} = \int_t^{t+1} \mathrm{d}\boldsymbol{X}_i(s)$ may arise due to $\boldsymbol{C}_t-$dependence

exact discretization

Letting $r_{i,t+1} \equiv \int_t^{t+1} \mathrm{d}P_i(s)$ and $\boldsymbol{f}_{t+1} \equiv \int_t^{t+1} \mathrm{d}\boldsymbol{F}(s)$ denote continuously-compounded returns over the time interval $[t, t+1)$ yields

$$
\begin{aligned}
r_{i,t+1} &= \mu_{i,t} + \sigma_{i,t} \int_t^{t+1} \mathrm{d}W_i(s) + \boldsymbol{\Sigma}'_{i,t} \int_t^{t+1} \mathrm{d}\boldsymbol{W}_F(s) \\
\boldsymbol{f}_{t+1} &= \boldsymbol{\mu}_{F,t} + \boldsymbol{\Sigma}_{F,t} \int_t^{t+1} \mathrm{d}\boldsymbol{W}_F(s), \qquad t = 1, \ldots, T
\end{aligned}
$$

$r_{i,t+1} \,\Big|\, (\boldsymbol{f}_{t+1}, \boldsymbol{C}_t)$ is Gaussian with mean $\mu_{i,t} + (\boldsymbol{f}_t - \boldsymbol{\mu}_{F,t})'\boldsymbol{\Sigma}_{FF,t}^{-1}\boldsymbol{\Sigma}_{F,t}\,\boldsymbol{\Sigma}_{i,t}$ and variance $\sigma_{i,t}^2 + \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}_{i,t} - \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}_{FF,t}^{-1}\boldsymbol{\Sigma}_{i,t}$, where $\boldsymbol{\Sigma}_{FF,t} = \boldsymbol{\Sigma}_{F,t}\boldsymbol{\Sigma}'_{F,t}$

discrete-time multifactor model

$r_{i,t+1} = \alpha_{i,t} + \boldsymbol{f}'_{t+1}\boldsymbol{\beta}_{i,t} + \epsilon_{i,t+1}$ with

$$
\begin{aligned}
\alpha_{i,t} &= \mu_{i,t} - \boldsymbol{\mu}'_{F,t}\boldsymbol{\Sigma}_{FF,t}^{-1}\boldsymbol{\Sigma}_{F,t}\boldsymbol{\Sigma}_{i,t} \\
\beta_{i,t} &= \boldsymbol{\Sigma}_{FF,t}^{-1}\,\boldsymbol{\Sigma}_{F,t}\,\boldsymbol{\Sigma}_{i,t}
\end{aligned}
$$

realized beta estimation

Under very mild regularity conditions, the realized beta estimator

$$\widehat{\boldsymbol{\beta}}_{i,t}^{(M)} = \left[\sum_{j=0}^{M-1}\left(\boldsymbol{F}_{t+\frac{j+1}{M}} - \boldsymbol{F}_{t+\frac{j}{M}}\right)\left(\boldsymbol{F}_{t+\frac{j+1}{M}} - \boldsymbol{F}_{t+\frac{j}{M}}\right)'\right]^{-1} \sum_{j=0}^{M-1}\left(\boldsymbol{F}_{t+\frac{j+1}{M}} - \boldsymbol{F}_{t+\frac{j}{M}}\right)\left(P_{i,t+\frac{j+1}{M}} - P_{i,t+\frac{j}{M}}\right)$$

converges as $M \to \infty$ to $\boldsymbol{\beta}_{i,t+1} \equiv \boldsymbol{\beta}_i(\boldsymbol{C}_t)$

(Barndorff-Nielsen and Shephard, Ecta 2004)

risk-adjusted return

$$\begin{aligned}
\widehat{Z}_{i,t+1}^{(M)} &= r_{i,t+1} - \boldsymbol{f}'_{t+1}\widehat{\boldsymbol{\beta}}_{i,t}^{(M)} \\
&= \alpha_{i,t} + \epsilon_{i,t+1} - \boldsymbol{f}'_{t+1}\left(\widehat{\boldsymbol{\beta}}_{i,t}^{(M)} - \boldsymbol{\beta}_{i,t}\right)
\end{aligned}$$

conditional alphas

retrieving pricing errors from realized betas

$$Z_{i,t+1} = r_{i,t+1} - \boldsymbol{f}'_{t+1}\boldsymbol{\beta}_{i,t} = \alpha_{i,t} + \epsilon_{i,t+1}$$

- conditional alpha is the conditional expectation of the risk-adjusted return given $\boldsymbol{C}_t$ $$\alpha_{i,t} = \mathbb{E}\left(Z_{i,t+1} \,\middle|\, \boldsymbol{C}_t\right)$$

specification issues...

- parametric, typically affine on $\boldsymbol{C}_t$
  - easy to estimate, though... high misspecification risk!

- nonparametric approach
  - more flexible and robust, though... curse of dimensionality and nontrivial measurement error

nonparametric estimation issues

$$\widehat{\alpha}_{i,t}^{(M)} = \frac{\frac{1}{Th_T^k}\sum_{\tau=1}^{T-1}\widehat{Z}_{i,t+1}^{(M)}\boldsymbol{K}\left(\frac{\boldsymbol{C}_\tau-\boldsymbol{C}_t}{h_T}\right)}{\frac{1}{Th_T^k}\sum_{\tau=1}^{T-1}\boldsymbol{K}\left(\frac{\boldsymbol{C}_\tau-\boldsymbol{C}_t}{h_T}\right)}$$

(1) curse of dimensionality

most dimension reduction methods (e.g., single index, sliced inverse regression, variable-selection methods, additive models) require non-constant pricing errors $\longrightarrow$ too strong!

solution kernel-based estimator using the principal components of the conditioning state variable

(2) measurement error

solution we provide the conditions on the rate of growth of the number of intraday observations under which the contribution of the realized beta estimation error is asymptotically negligible

asymptotic theory

assumptions

A1 moment conditions for the drift and diffusion terms

A2 moment conditions for the risk-adjusted return
strict stationarity and $\alpha-$mixingness of $\left(Z_{i,t+1}, \boldsymbol{C}_t\right)$

A3 usual kernel conditions

A4-A6 smoothness and boundness of the conditional expectation of $Z_{i,t}$ and of the joint density of $\boldsymbol{C}_t$

remark C_t is a random variable and we must establish a uniform result over its support $\longrightarrow$ trimming à la Andrews (ET 1995)

asymptotic theory

conditional alpha

Proposition 1 trimming the standard kernel-based estimator of the conditional expectation has no asymptotic impact in the estimation as long as $d_T^{-1}\left(M^{-1}h_T^{-k} + M^{-1/2}T^{-1/2}h_T^{-k}\right) \to 0$

Proposition 2 standard kernel-based estimator of the conditional expectation is consistent as long as $d_T^{-1}M^{-1/2}h_T^{-k}$, $d_T^{-2}T^{-1/2}h_T^{-k}$, $d_T^{a/Q}$, and $h_T^2 d_T^{-2}$ converge to zero

appraisal ratio

Proposition 3 both trimmed and standard kernel estimators of the appraisal ratio are consistent if $d_T^{-1}M^{-1/2}h_T^{-k}$, $T^{-1/2}h_T^{-k}d_T^{-2}$, $d_T^{a/Q}$, and $h_T^2 d_T^{-2}$ converge to zero

mispricings in the S&P 100 index constituents

data description

sample period	January 2001 to December 2008 (1,915 observations)
stocks	90 constituents of the S&P 100 index
market portfolio	S&P 500 index
instruments	changes in VIX, volatility risk premium Fama-French factors + momentum + reversals changes in the Fed rate, credit spread, term spread
realized betas	multivariate realized kernel with refresh time (M and T are approximately of the same order)

realized betas

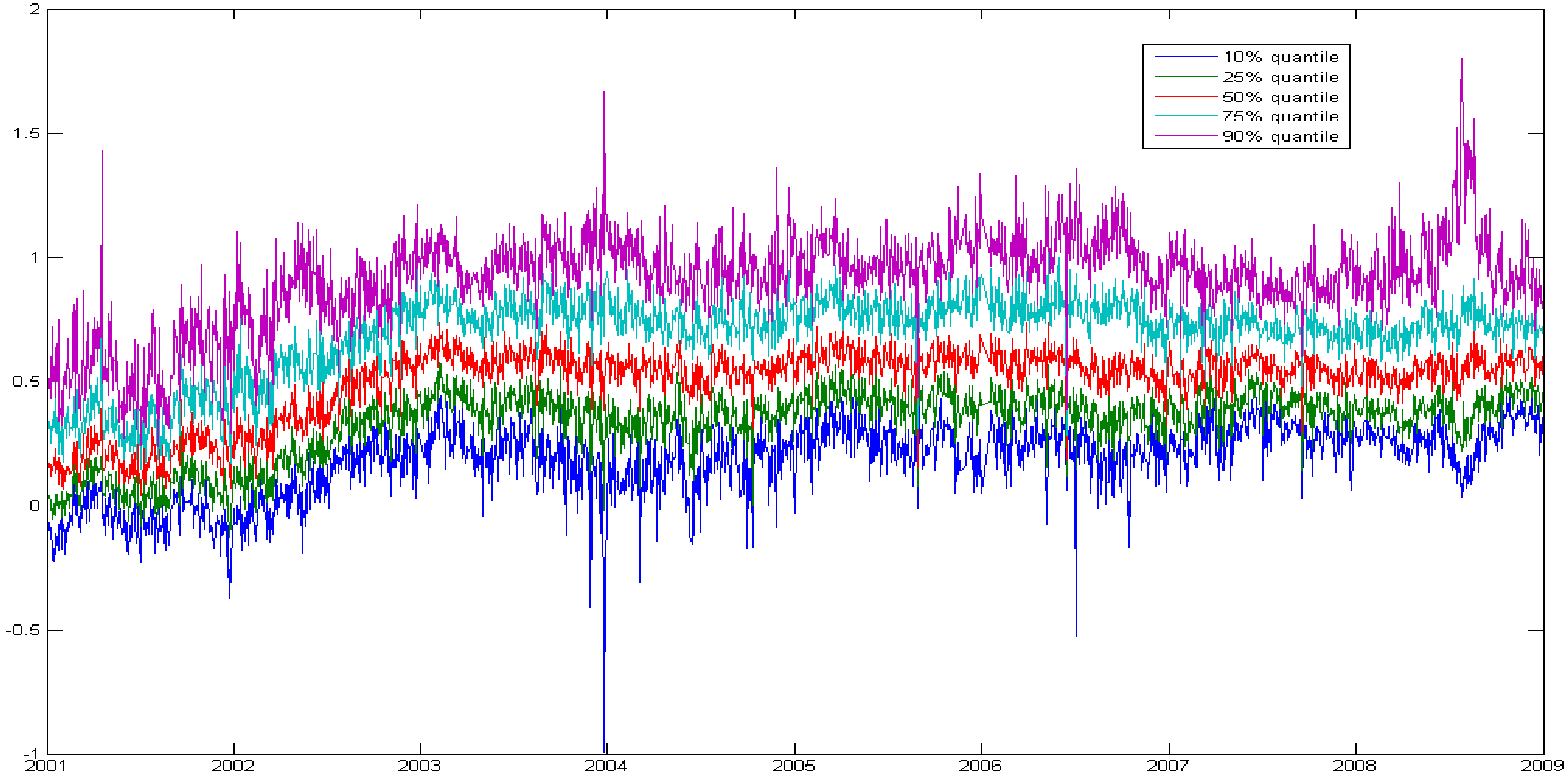
- lots of daily variation in the exposure to market risk
  this contradicts not only Lewellen and Nagel's (JFE 2006) claim that there is not enough monthly variation in the market betas to justify value premium as well as their low-frequency assumption

- cross-sectional dispersion is fairly constant over time
  though... median beta increases significantly throughout 2002

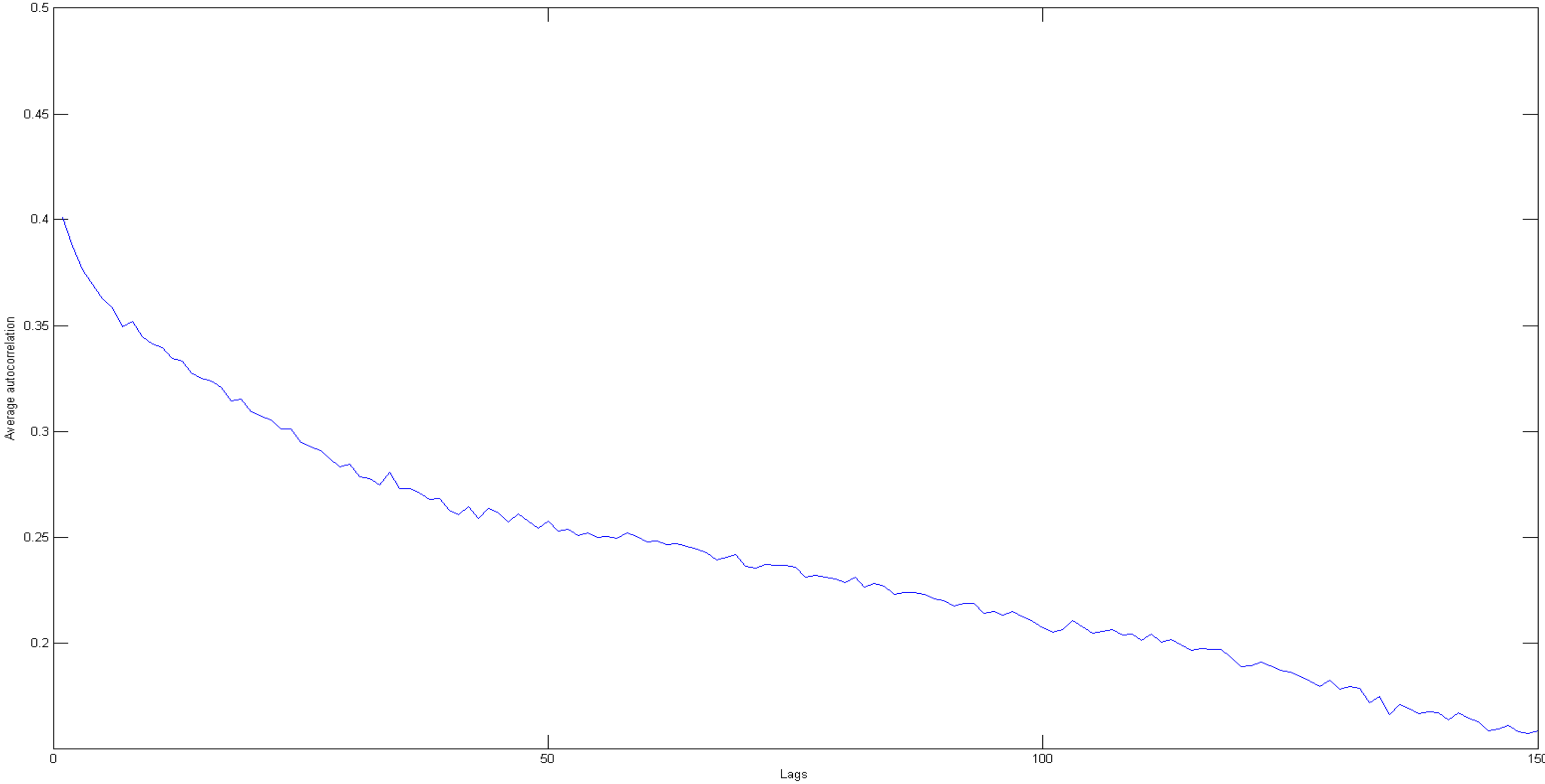
- weak co-movement among daily realized betas
  first three principal components explain slightly over a third of the overall variation

- persistent behavior over time
  evidence to some extent of long-range dependence

quantiles of the daily realized betas



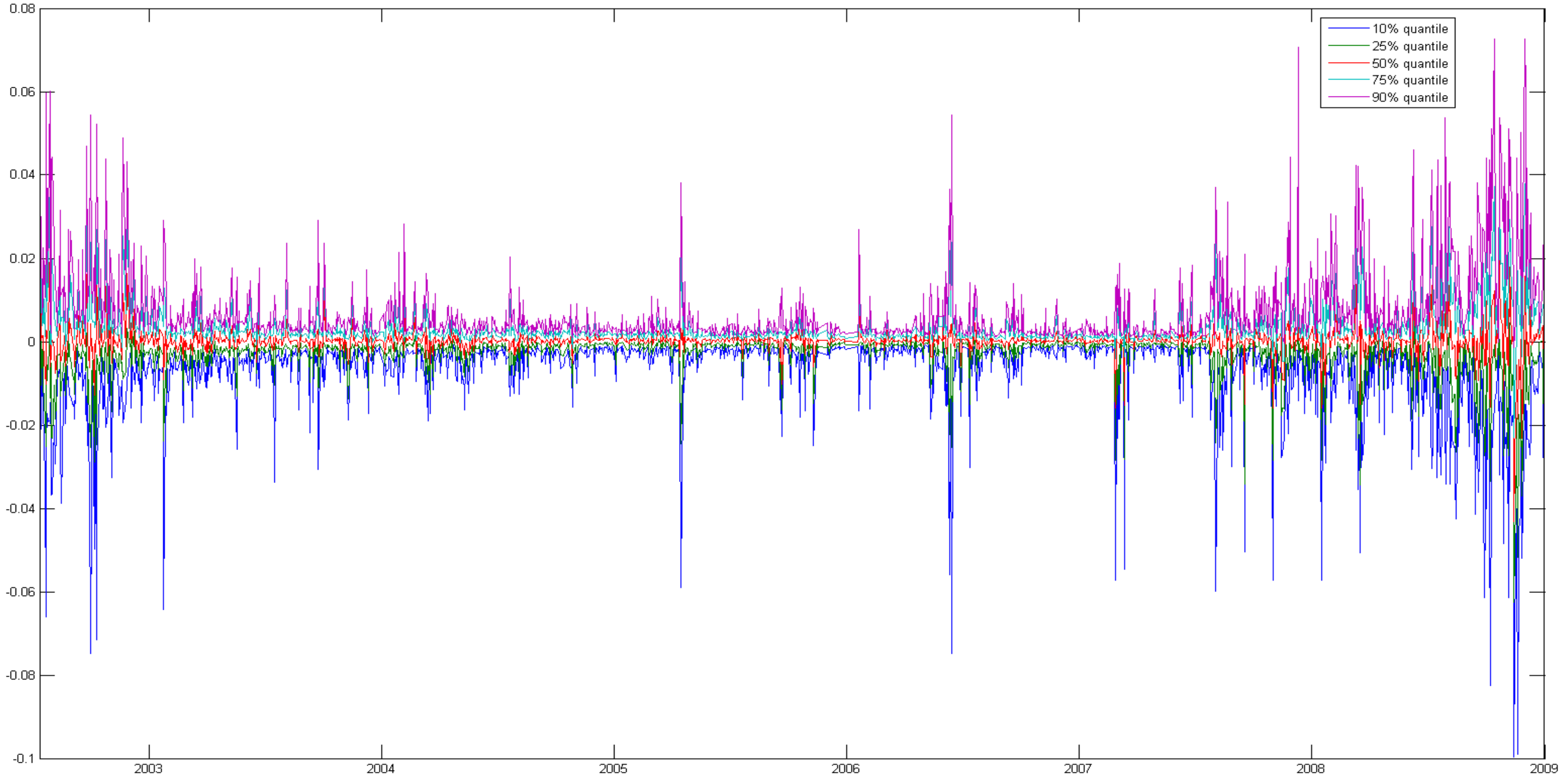
autocorrelation function of the daily realized betas



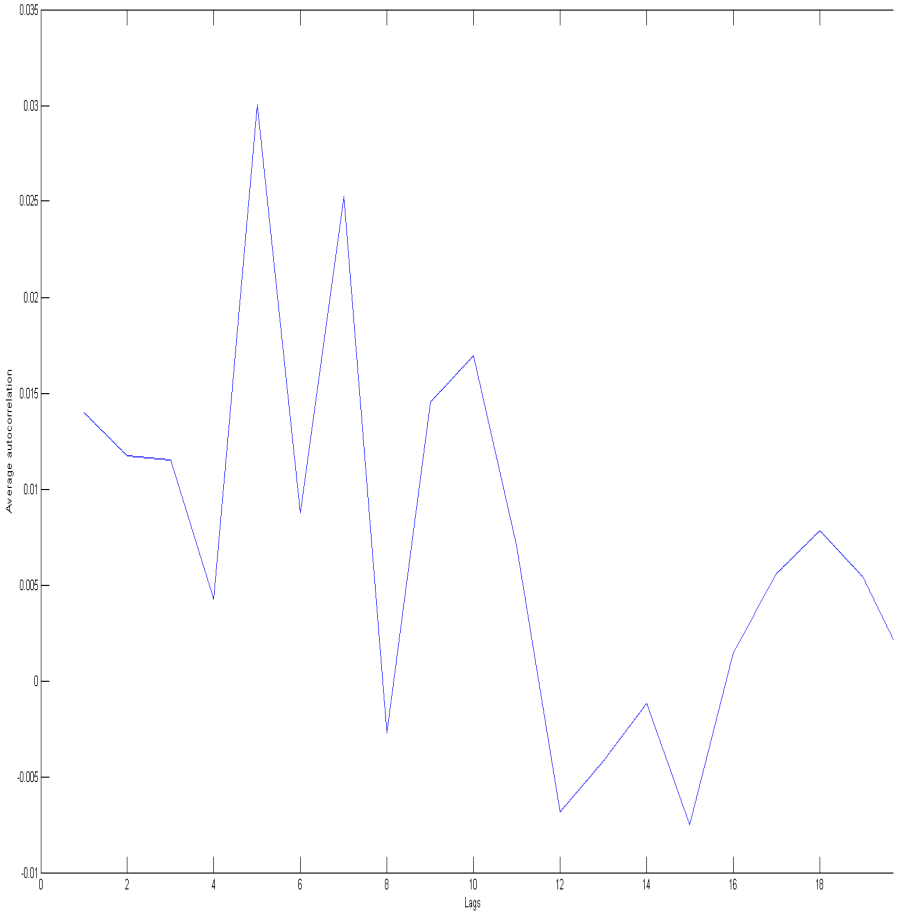
conditional alpha estimates

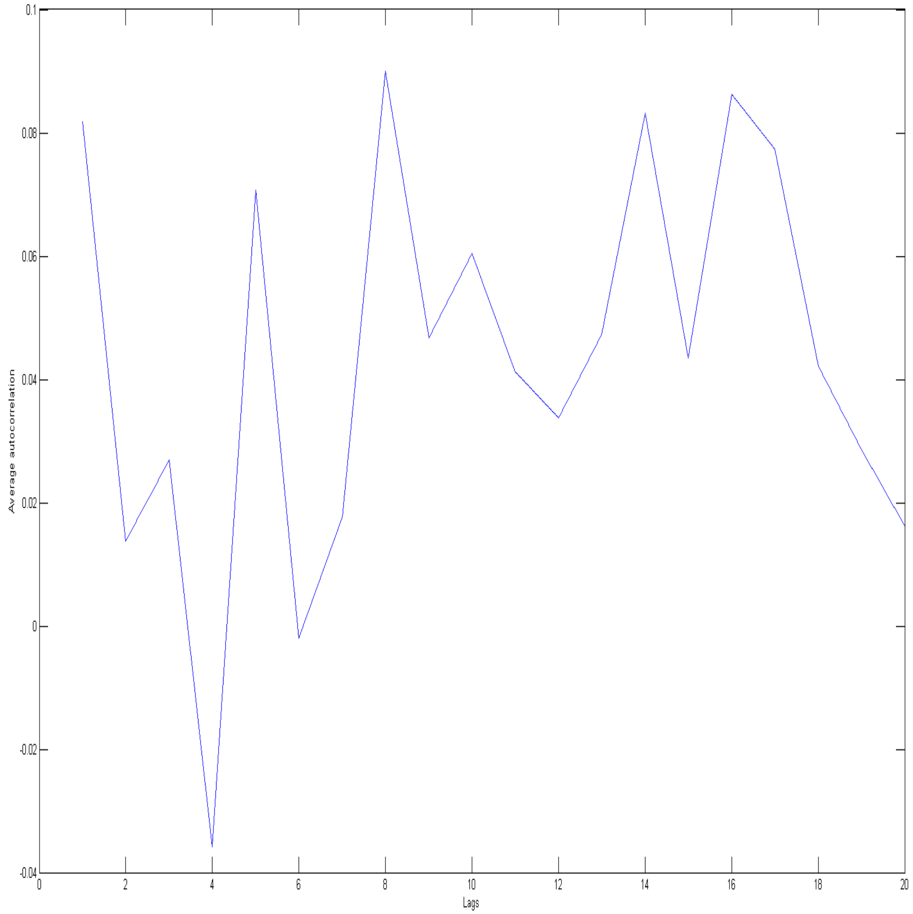
- daily variation is much more erratic
  though. . . there is a great deal of volatility clustering
- cross-sectional dispersion changes significantly over time
  relatively much higher before 2003 and after mid-2007
- stronger co-movement among conditional alphas
  first three principal components explain over 45% of variation in the nonparametric alphas, whereas they respond for over 87% of the overall variation in the affine alphas
- not much persistence
  average autocorrelation functions are quite low for both affine and nonparametric alpha estimates

quantiles of the conditional alpha estimates

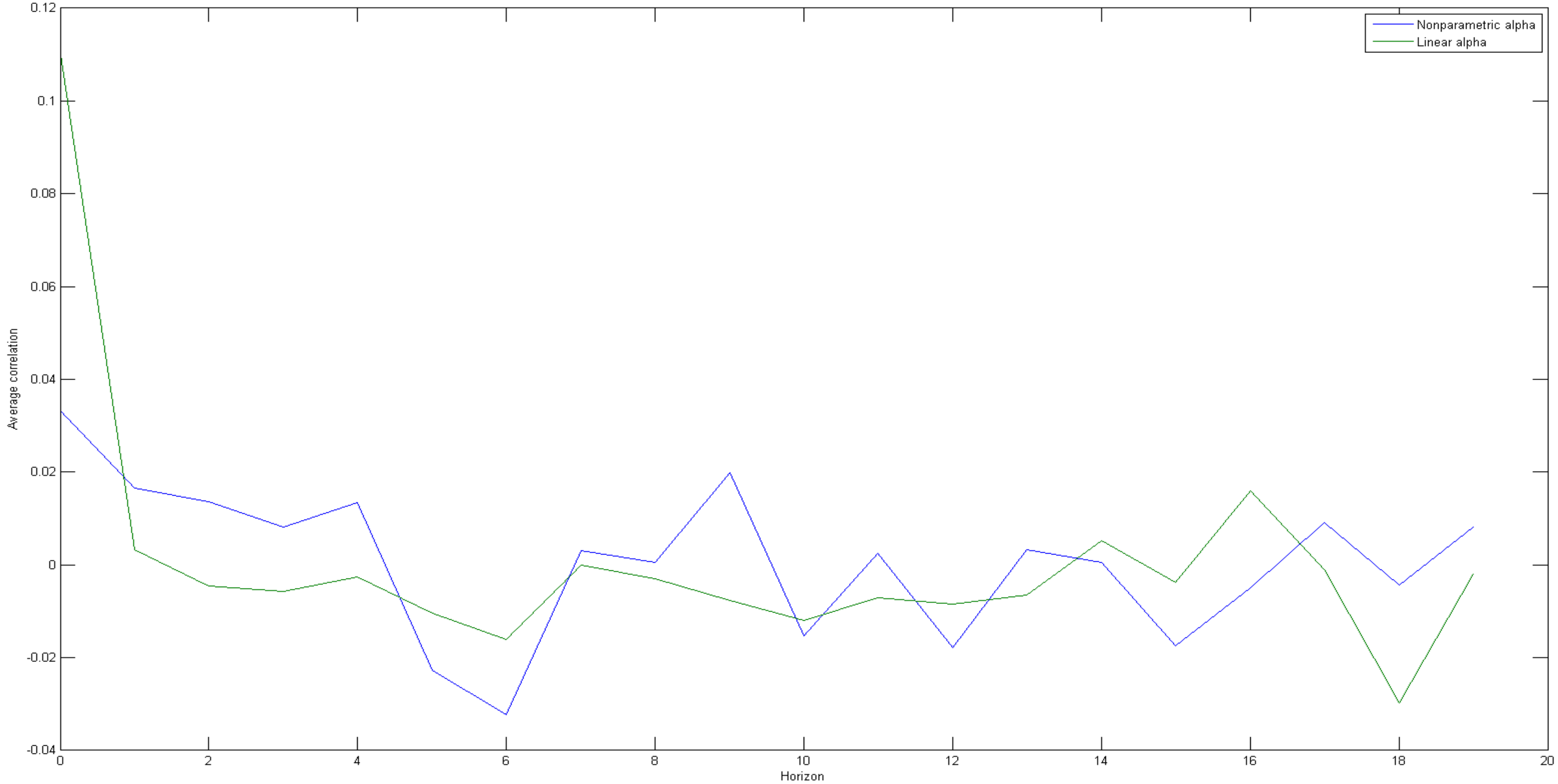


autocorrelation functions of the pricing errors





conditional alphas as leading indicators



trading pricing errors

alpha portability

long-short trading strategy

- long position on very undervalued stocks
- short position on very overvalued stocks

quantile approach

- time series versus cross-sectional information
- affine alpha versus nonparametric alpha versus appraisal ratio

economic significance of mispricings

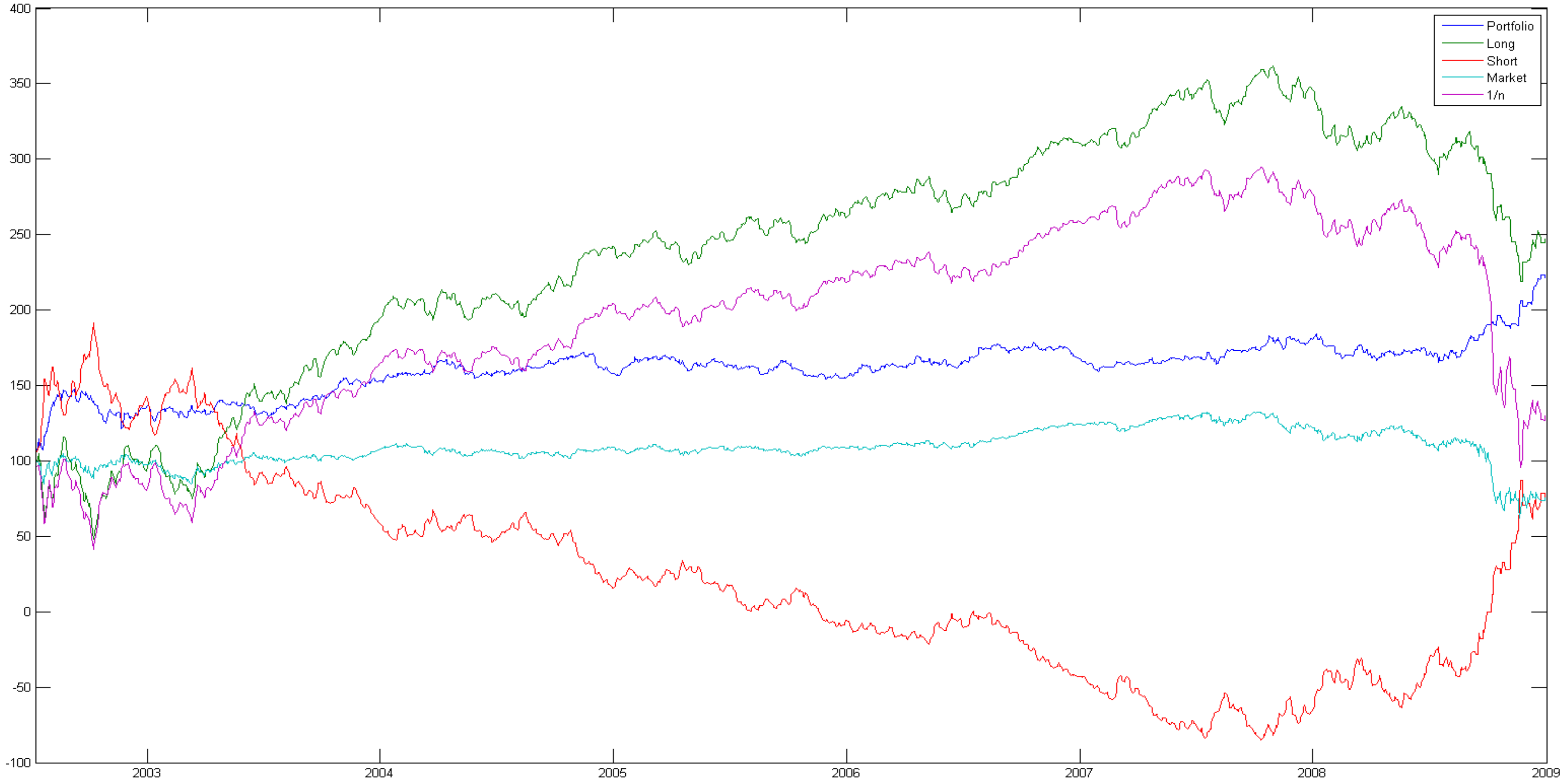
- slightly conservative trading costs of 4bps

trading strategy details

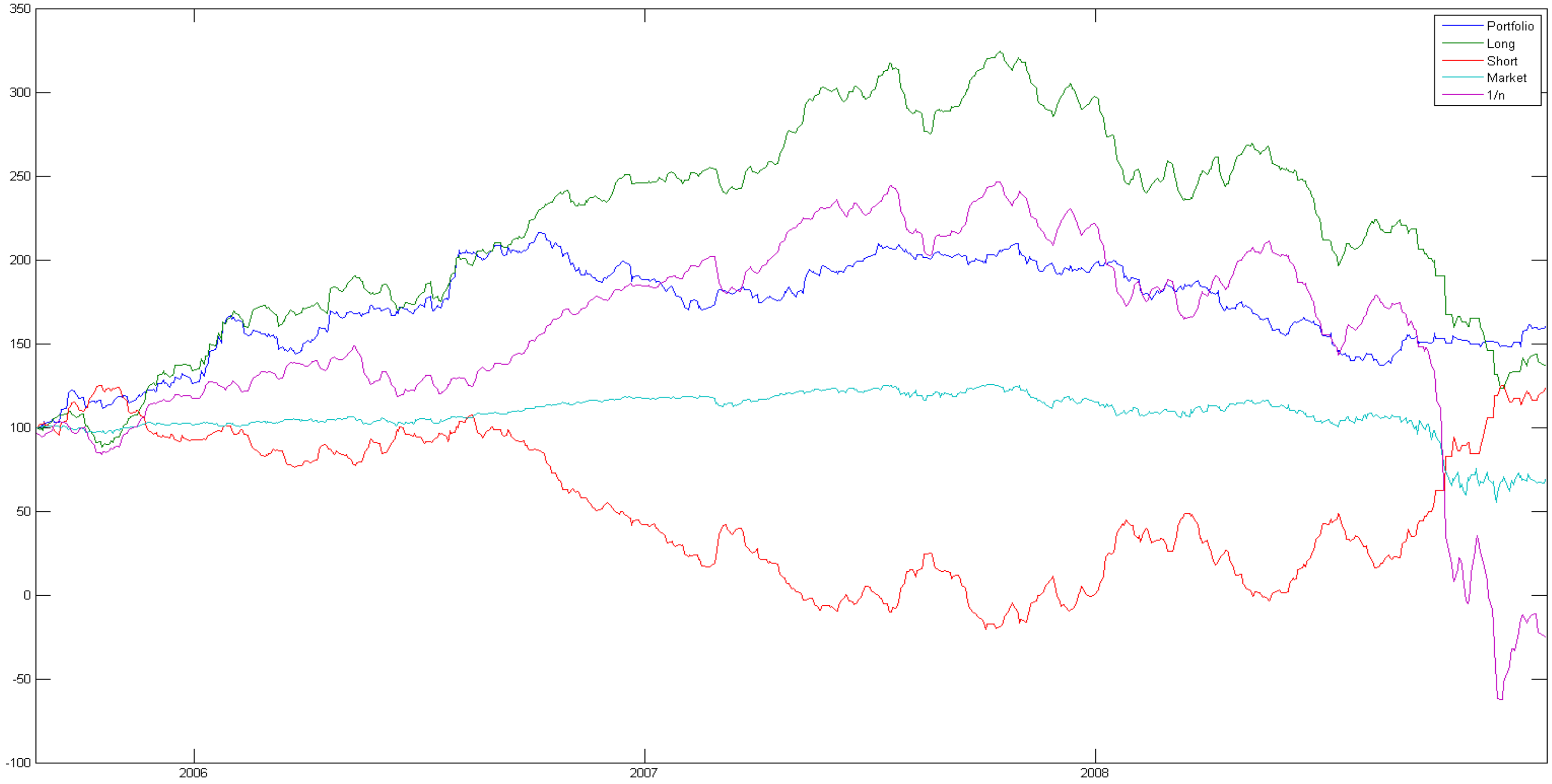
information	long portfolio	short portfolio	holding period
cross-section	top quintile	bottom quintile	1 to 5 days
time-series	top quartile	bottom quartile	1/5/10/22 days

- self-financing, with equal-weight long and short portfolios
- expanding window or rolling window of 750 trading days
- with and without risk-free adjustment
- different amounts of trimming (from none up to 0.20)

cross-sectional momentum strategy based on alphas



time-series momentum strategy based on alphas



quick performance measures

strategy	return	vol	Sharpe	skewness	appraisal	alpha	loadings
$cs(\alpha_n, 3)$	18%	20%	0.90	1.24	0.86	17.2	MOM
$ts(\alpha_n, 10)$	37%	45%	0.82	0.40	0.61	34.9	
$ts(\alpha_n, 22)$	59%	64%	0.92	0.19	0.80	60.9	(SMB)

- relative performance of affine alphas is much poorer
- risk-free adjustment only works for time-series momentum (↓vol)
- trimming puts enough discipline in the alphas
- appraisal ratio reduces both returns and vol, compensating only for the time-series momentum strategy

concluding remarks

Summary

novel approach to estimate pricing errors

- integrates high- and low-frequency data
- flexible and robust due to nonparametric character
- predictable alpha, allowing for portability

asymptotic theory

- curse of dimensionality
- nontrivial measurement error
- trimming is paramount

mispricings within the NYSE

- persistent alphas $\longrightarrow$ portability?
- nonlinear dependence on conditioning state variables