WHO'S AFRAID OF SELECTION BIAS?

ROBUST INFERENCE IN THE PRESENCE OF COMPETITIVE SELECTION

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Thomas Noe

SBS/Balliol

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OUTLINE

WHEN DOES INFORMATION ABOUT A THE UNCONDITIONAL VALUE OF A CHOICE TELL YOU ABOUT ITS SELECTION CONDITIONED VALUE

- Consider the problem of a trader who believes that, on average, outside CEOs generate more value than inside CEOs
- The trader has private information that the CEO selected by a specific firm is an outsider. That's all he knows about the CEO candidate
- He also knows that firm knows CEO quality and is selecting the CEO candidate who will maximize value
- Will the trader profit from buying shares in the firm?

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WHAT DOES THE CONDITIONAL VALUE TELL YOU ABOUT THE UNCONDITIONAL VALUE

- Consider a problem of a real-estate economist
- The economist observes that first-time home buyers on average pay more than real estate professionals for houses
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GENERAL PROBLEM: SAMPLE SELECTION BIAS

- Making an inference from a selected sample about the underlying population, or vice versa
- Standard approaches to sample-selection bias when making inferences from selected sample to population
  - *Quantitative*: Attempt through IV, two-stage least squares (*inter alia*, Heckman, 1979) to derive point estimates of for population parameters from selected sub-population
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This paper's approach

- *Qualitative*: When does selection effect the direction of inference: can a better population generate a worse subsample?
- *Non-parametric*: Better and worse not defined based on any parameter of the distribution but rather by stochastic dominance
- *Bi-directional*: Conditions were both
  - Sample distribution's dominance implies the population distribution's dominance
  - Population distribution's dominance implies sample distribution's dominance
- Selection into the sample is competitive: selection is made by a value maximizing agent.
- Unusual approach to say the least. Distant relative
  - Manski (AER, 1990) distribution-free bounds the maximal size of selection effects when the treatment variable is dichotomous.

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ASSUMPTIONS

- GS hires only from Oxbridge
- For each position, it interviews one Oxford candidate and one Cambridge Candidate
- It selects the best candidate and pays compensation based on ability
- Ability for Oxbridge candidates ranges from 0 to 1.
- The distribution of ability is different at Oxford and Cambridge

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SPECIFICS

- *Ability distributions:*
  - *Oxford:* Simple and elegant

$$F_{\text{Oxford}}(a) := a, \; a \in [0,1] \tag{1}$$

  - *Cambridge:* Obtuse and complex

$$F_{\text{Cambridge}}(a) := \sqrt{3}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{(2-a)a}}{1-a}\right)\right) - \cos\left(\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{(2-a)a}}{1-a}\right)\right) + 1, \; a \in [0,1] \tag{2}$$

- Value of new hires (in thousands) as a function of ability, a

$$V(a) := 50 + 150a \tag{3}$$

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QUESTION

- Salary survey results show that the mean salary of Oxbridge graduates hired by GS are as follows:
  - Oxford: £141,500
  - Cambridge:£144,500
- Ignoring issues of sample size (or assuming that GS hires and infinite number of associates!)
  - Can we infer from this evidence that Cambridge students are more able than Oxford students?
  - Should GS hire more Cambridge grads?

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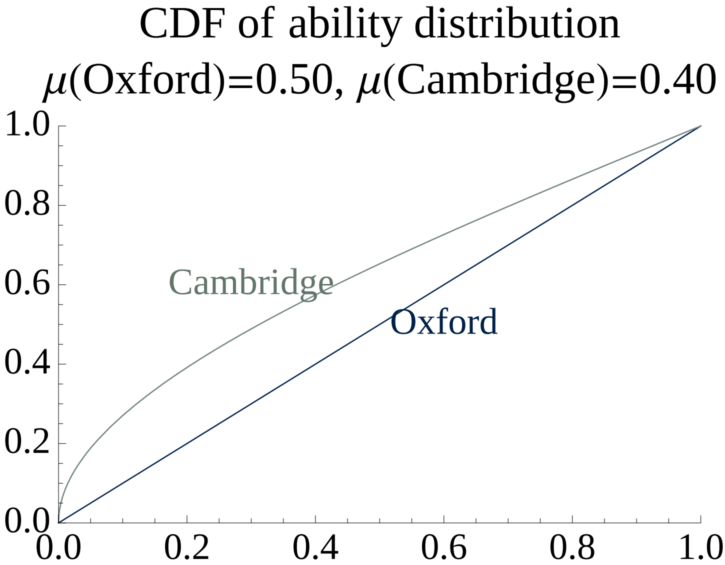
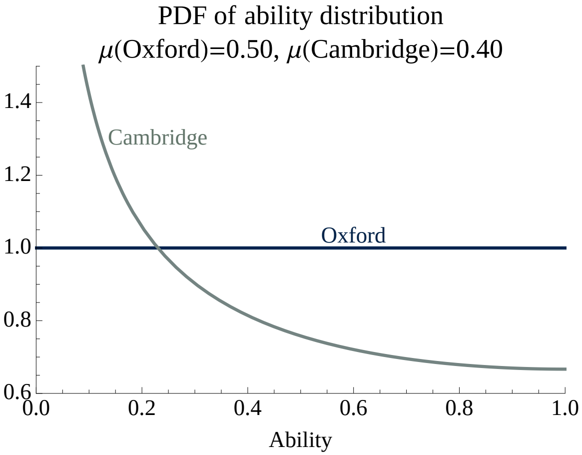
OXBRIDGE CDF OF ABILITY

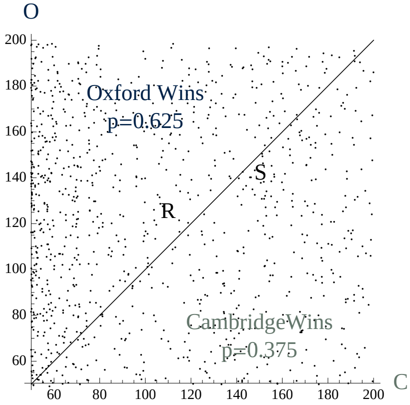


OXBRIDGE PDF OF ABILITY



Simulated competition: Blue vs. Blue

Value of Random (R) and selected (S)
Oxbridge Graduates



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SUMMARY OF EXAMPLE

- Oxford student quality stochastically dominates Cambridge student quality
- In fact, Oxford dominates Cambridge in the MLRP ordering
- But, conditioned on selection by GS, Cambridge student earn more.
- In fact, given *any* increasing value function, the selection conditioned sample of Cambridge students will have a higher expected value!
- i.e., conditioned on selection, Cambridge students stochastically dominate Oxford students

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- Determine the conditions under which one distribution dominates another after conditioning on competitive selection
- Determine when dominance relations are reversed (as in the Oxbridge example) and preserved
- Relate the conditions for selection dominance to standard statistical orderings and textbook statistical distributions

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TOOLS

- Two new "order relations" over distributions which characterize selection dominance
- Supermuiplicativity on average
  - Necessary and sufficient condition for selection dominance
  - But a rather ill-behaved relation over distributions: Not even transitive
  - Difficult to relate to shape of CDF/PDF
- Geometric dominance
  - Sufficient condition for selection dominance
  - "MLRP" plotted on log-log paper
  - For distributions with densities, can be defined based on the ratio the densities of the log CDF
  - Neither implied nor is implied by any standard statistical ordering of distributions (e.g. stochastic dominance, MLRP, hazard rate ordering)

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OUTLINE

DEFINITIONS

SELECTION DOMINANCE

We say that $\tilde{X}$ (or its distribution function F) *selection dominates* $\tilde{Y}$ (or its distribution function G) if, for all increasing functions v,

$$\mathbb{E}[v(\tilde{X})|\tilde{X} > \tilde{Y}] \geq \mathbb{E}[v(\tilde{Y})|\tilde{Y} > \tilde{X}].$$

DEFINITIONS

THE u TRANSFORM

- Let $\tilde{X}$ be a r.v. with distribution function F; Let $\tilde{Y}$ be a r.v. with distribution function G
- Let G^{-1} be the generalized inverse of G, i.e., G's quantile function.
- Then the transform function, u, associated with F and G is defined by

$$u(t) = F(G^{-1}(t)),\ t \in [0,1]$$

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Definitions

Supermultiplicative on average

Let $u = F \circ G^{-1}$; if

$$\text{For all } t \in (0,1),\ \int_0^1 \Big(u(t\,s) - u(s)\,u(t)\Big)\,ds \geq 0. \tag{4}$$

then u is *supermultiplicative on average*.

Geometric convexity

Let $u(t) = F \circ G^{-1}(t)$; if $\log u(t)$ is a convex function of $\log(t)$, then u is *geometrically convex*

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GEOMETRIC CONVEXITY

Let $u(t) = F \circ G^{-1}(t)$; if $\log u(t)$ is a convex function of $\log(t)$, then u is *geometrically convex*

OBSERVATIONS

- u being geometrically convex does not imply that u is convex
- u being convex does not imply that u is geometrically convex
- u being log convex implies that u is both convex and geometrically convex
- Geometric convexity is equivalent to the function, $\hat{u}$ defined by

$$\hat{u}(y) = \log \circ u \circ \exp(y), y \leq 0$$

  being convex, i.e., u being convex when plotted on log-log graph paper
- Geometric convexity implies that u is supermultiplicative and thus *a fortiori* supermultiplicative on average.

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For an admissible pair of distributions functions, F and G, the following statements are equivalent:

(I) F and G are geometrically equivalent, i.e., $u = F \circ G^{-1}$ is geometrically linear

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OUTLINE

CHARACTERIZING THE u TRANSFORM UNDER GEOMETRIC DOMINANCE

CROSSINGS OF F AND G

Suppose that F and G are an admissible pair of distributions and let $u = F \circ G^{-1}$. Suppose that F strictly geometrically dominates G, i.e, that u is strictly geometrically convex.

(I) If, on some neighborhood of $\underline{x}$, $F(x) < G(x)$, then for all $x \in (\underline{x}, \bar{x})$, $F(x) < G(x)$, and thus F strictly stochastically dominates G

(II) If, on some neighborhood of $\underline{x}$, $F(x) > G(x)$, then either

(A) $F(x) > G(x)$ for all $x \in (\underline{x}, \bar{x})$ and thus G strictly stochastically dominates F, or

(B) There exists a point $x^o \in (\underline{x}, \bar{x})$ such that for all $x \in (\underline{x}, x^o)$, $F(x) > G(x)$ and for all $x \in (x^o, \bar{x})$, $F(x) < G(x)$.

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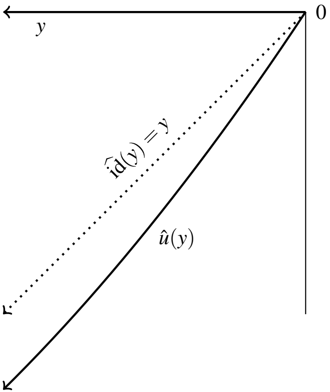

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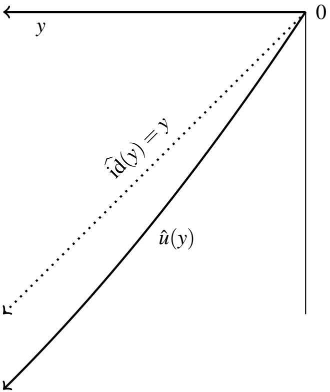

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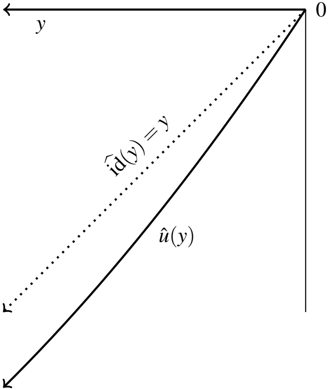

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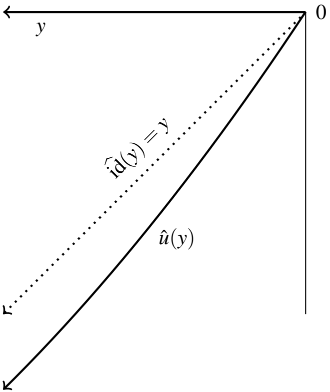

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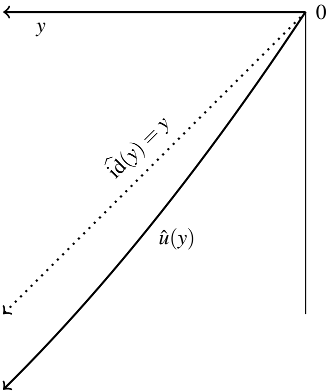

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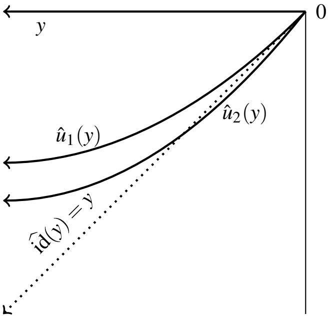

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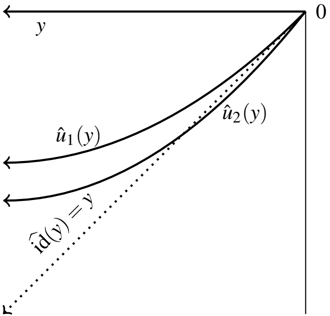

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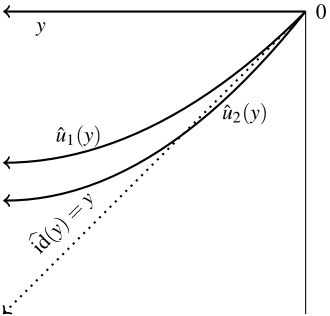

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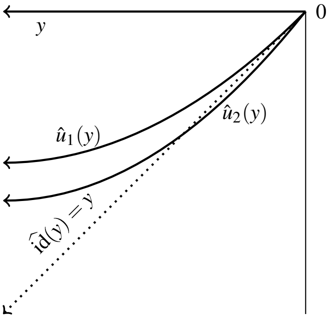

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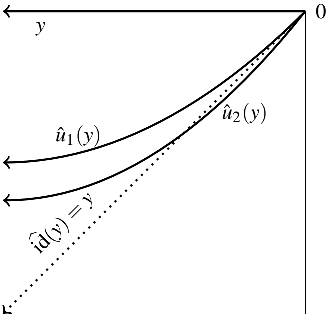

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SHAPE OF THE u TRANSFORM

Assume that F strictly geometrically dominates G. Then, one of the following three mutually exclusive characterizations of the distributions and transform function must hold.

1. $F(x) < G(x) x \in (\underline{x}, \bar{x})$, and u is strictly convex.
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SAID BUSINESS SCHOOL UNIVERSITY OF OXFORD

SUMMARY

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$$\hat{v}(y) = \hat{u}(y) - y, \quad y \leq 0.$$

- Note that

$$u(t) = t \exp[\hat{v}(\log(t))]$$

- Therefore,

$$u'(t) = \exp[\hat{v}(\log(t))] + \exp[\hat{v}(\log(t))]\,\hat{v}'(t) \tag{5}$$

- $\hat{v}'(t)$ is $\uparrow$ because $\hat{v}$ is strictly convex
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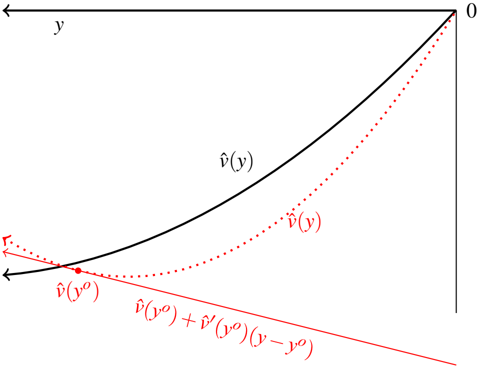
- If $\hat{v}$ is $\uparrow \Rightarrow u'$ is $\uparrow$, i.e. u is convex
- If $\hat{v}$ is $\downarrow \Rightarrow u(t)/t$ is $\downarrow$

$\hat{v}(t) < 0 \Rightarrow \hat{v}$ IS $\uparrow$



- $\hat{v} < 0$, $y < 0$ and
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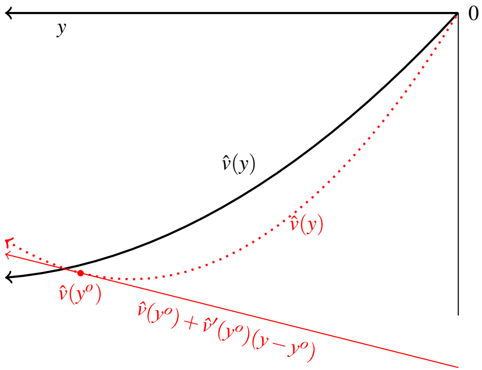

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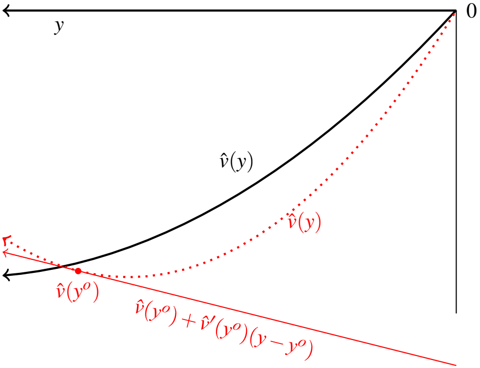

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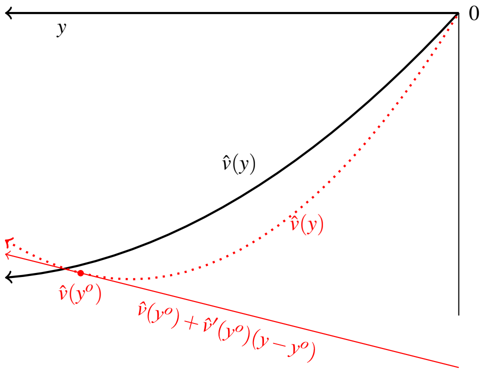

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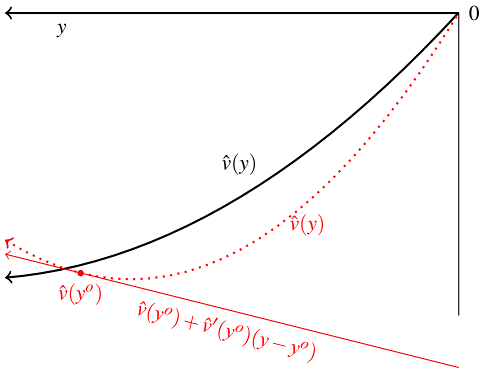

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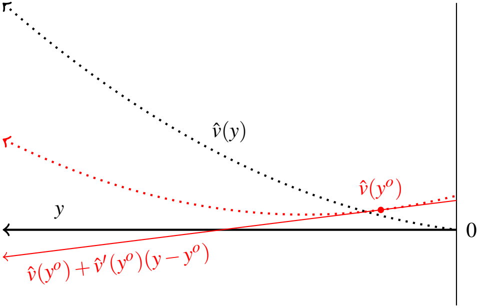

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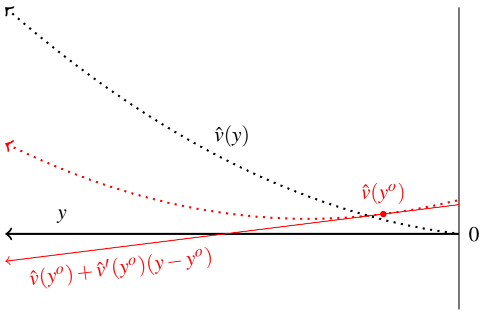

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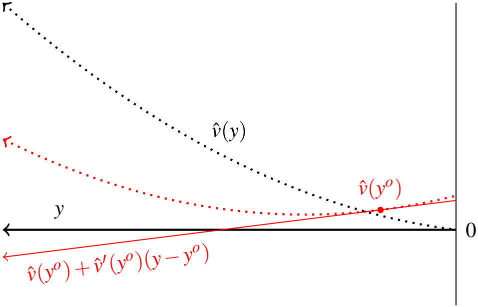

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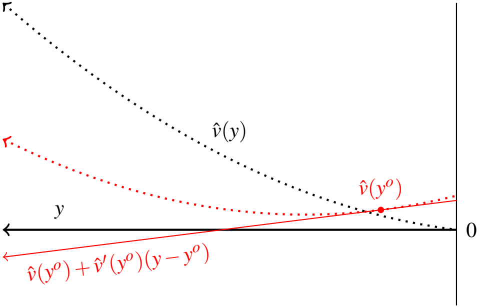

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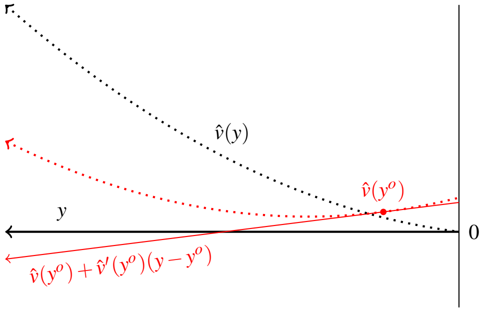

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1. Problem
2. Example: Oxford vs. Cambridge
3. Motivation
4. Selection dominance
5. Three "flavors" of geometric dominance
   - Sketch of proof
6. PDFs and Geometric Dominance
7. "Good" geometric dominance: Positive geometric dominance
8. "Bad" geometric dominance: Negative geometric dominance
9. "Ugly" geometric dominance

PDF characterizations

Geometric convexity in terms of PDFs

Suppose that F and G are regularly related and $u = F \circ G^{-1}$.

1. u is (strictly) convex if and only if $x \to f(x)/g(x)$ is (increasing) nondecreasing over $(\underline{x}, \bar{x})$, i.e. F dominates G in the MLRP order.
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- Most (all?) textbook statistical distributions when the two distributions being compared come from the same family and differ only with respect to a parameter which represents and additive or multiplicative shift, e.g.,
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EFFECT OF THE LEFT-TAIL EXPLOSION

- In a competitive selection context, the left tail explosion of the geometrically dominant distribution increases its selection-conditioned value for two reasons:
  1. *Censorship effect:* Left tail realizations are very unlikely to be selected and thus will be censored out of the selection-conditioned distribution, raising its conditional value
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- We know it can happen: See the Oxford–Cambridge example
- Occurs sometimes with textbook statistical distributions when the scale parameter is not multiplicative or additive shift
- *Example*: Kumaraswamy distribution, with common shape parameter $b < 1$.
- *Specific case*: Kumaraswamy distribution, where the for both F and G the shape parameter is $b = 1/2$, and the scale parameters = are $\alpha_F = 1$, and $\alpha_G = 2$. The two distributions corresponding to α_F and α_B, F and G, are

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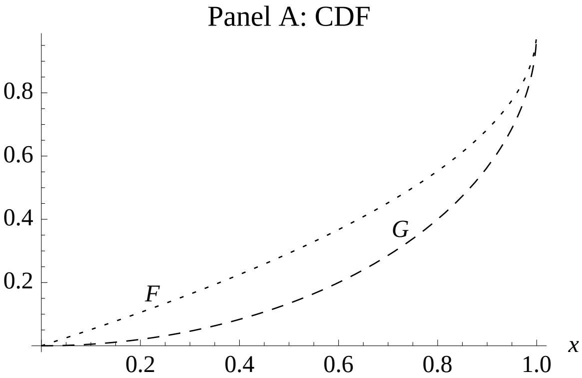
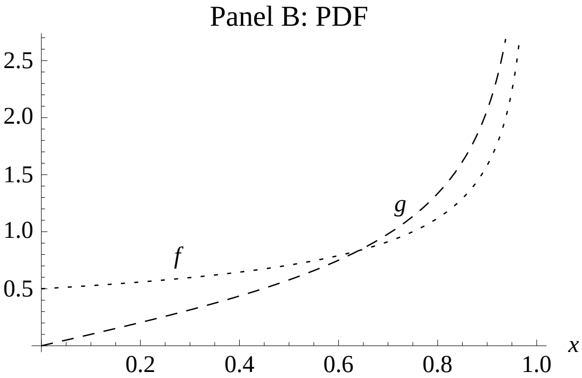
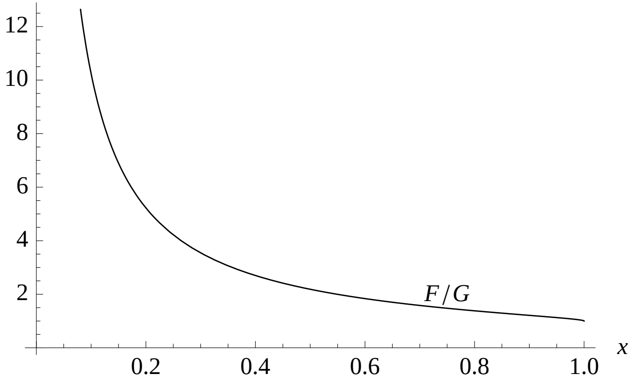
CDFs of for the Kumaraswamy example



PDFs OF FOR THE KUMARASWAMY EXAMPLE

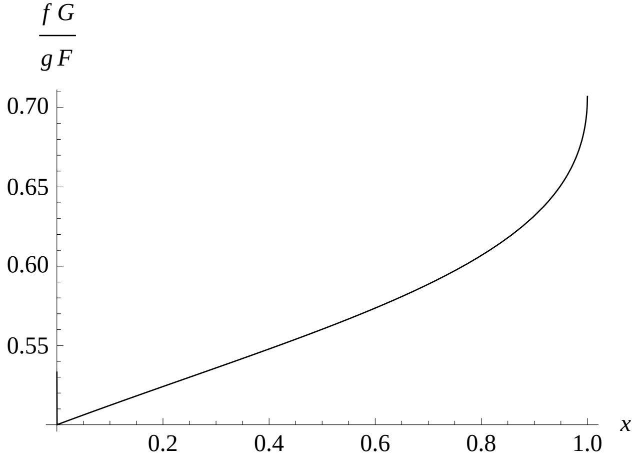


LOWER-TAIL RATIO EXPLOSION

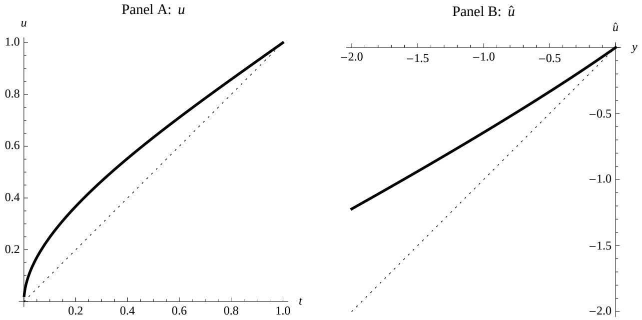


KUMARASWAMY: NEGATIVE GEOMETRIC DOMINANCE

Panel C: Geometric dominance



KUMARASWAMY: u AND $\hat{u}$ FUNCTION



OUTLINE

SELECTION DOMINANCE INDUCED BY DISPERSION

- When F is selection dominant with respect to G but neither stochastically dominant nor stochastically dominant, then
- F crosses G once from above
- F behaves like a negatively dominant distributions in its lower tail, (i.e., lower-tail ratio (F/G explosion) and like a positively dominant distribution in its upper tail (f/g increasing)
- Both the left tail censoring generated by the left tail ratio explosion and MLRP dominance at the upper tail favor selection dominance
- Therefore, the selection conditioned value of geometrically dominant distribution can be much higher than the dominated distribution's value
- Example:

$$F(x) = \begin{cases} e^{-c\sqrt{\frac{1}{x}}} & x \in (0,1] \\ 0 & x = 0 \end{cases}, \quad c > 0, \quad G(x) = x, \quad x \in [0,1].$$

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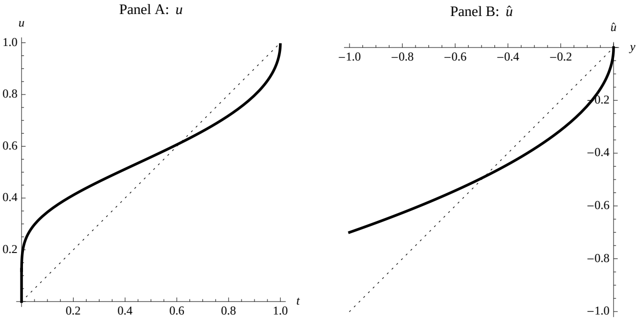
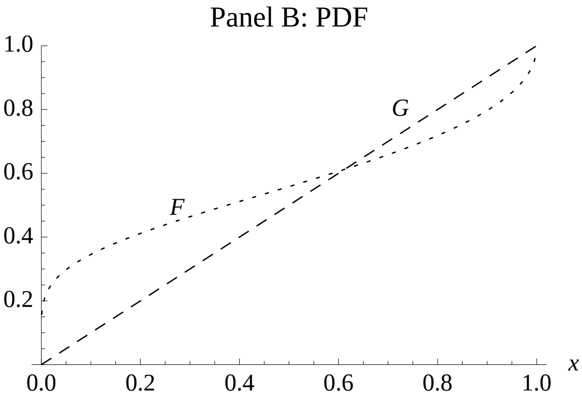
u AND $\hat{u}$ FUNCTIONS

$$u(t) = \begin{cases} e^{-c\sqrt{\frac{1}{x}}} & x \in (0,1] \\ 0 & x = 0 \end{cases}; \quad \hat{u}(t) = -c\sqrt{-y},\ y \in (-\infty, 0].$$

u AND $\hat{u}$ FUNCTIONS WHEN $c = 0.70$



CDFS WHEN $c = 0.70$



PDFs when $c = 0.70$

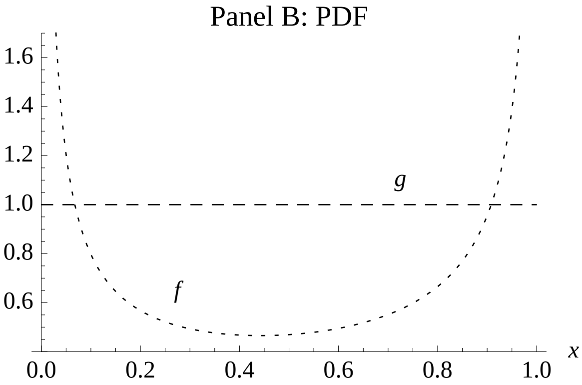


ILLUSTRATION OF SELECTION DOMINANCE RESULTING FROM DISPERSION

$$\begin{aligned} \mathbb{E}[\tilde{X}|\tilde{X} > \tilde{Y}] &= 0.7784 & \mathbb{E}[\tilde{Y}|\tilde{Y} > \tilde{X}] &= 0.5854 \\ \mathbb{P}[\tilde{X} > \tilde{Y}] &= 0.4352 & \mathbb{P}[\tilde{Y} > \tilde{X}] &= 0.5648 \\ \mathbb{E}[\tilde{X}] &= 0.4352 & \mathbb{E}[\tilde{Y}] &= 0.5000 \end{aligned}$$

TABLE : Expected payoffs under selection when geometrically dominant distribution is dispersive. In the table, $\tilde{X} \sim F$ and $\tilde{Y} \sim G$, $c = 0.70$