Accounting Rules, Equity Valuation, and Growth Options

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Motivation

FASB* Conceptual Framework for Financial Reporting (FASB 2010)

"The objective of general purpose financial reporting is to provide financial information about the reporting entity that is useful to existing and potential investors, lenders, and other creditors in making decisions about providing resources to the entity."

**Financial Accounting Standards Board*

Continuous-Time Model
Representative Firm - Technology

- Operating cash flow

$$CF_t = X_t K_t^{\alpha},\ 0 < \alpha < 1$$

- Demand shock X_t follows

$$\frac{dX_t}{X_t} = \mu_X dt + \sigma_X dz_X,\ X_0 > 0$$

- Capital stock K_t

$$dK_t = -\delta K_t dt + \sigma_K K_t dz_K + dI_t$$

- Investment is irreversible $dI_t \geq 0$
- K_t is purchased instantaneously and frictionlessly at a price P_t

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P dz_P,\ P_0 > 0$$

Firm Solves

$$\max_{\{I_t\}_0^\infty} \mathrm{E}_0 \left[\int_0^\infty e^{-rt} CF_t dt - \int_0^\infty e^{-rt} P_t dI_t \right]$$

$$s.t.\ dK_t = -\delta K_t dt + \sigma_K K_t dz_K + dI_t,$$

$$\frac{dX_t}{X_t} = \mu_X dt + \sigma_X dz_X,$$

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P dz_P,$$

$$I_{0-} = 0,\ dI_t \geq 0\ \forall t \geq 0,$$

- Innovations are mutually correlated

$$\rho_{XP} = \frac{dz_X dz_P}{dt},\ \rho_{KP} = \frac{dz_K dz_P}{dt},\ \rho_{XK} = \frac{dz_X dz_K}{dt}$$

- The information set of investors at time t includes the current and all past financial statements of the firm

$$\mathcal{I}_t = \left[\underbrace{\{B_\tau\}_{\tau \le t}}_{\text{Book Value of Assets}}, \{CF_\tau\}_{\tau \le t}, \underbrace{\{P_\tau \cdot dI_\tau\}_{\tau \le t}}_{\text{Investment Cash Outflow}} \right]$$

- Firm's net income from time $\tau - dt$ to τ

$$CF_\tau dt - P_\tau dI_\tau - B_{\tau-dt} + B_\tau$$

- Gross investment

$$GI_t \equiv \int_0^t P_\tau \cdot dI_\tau$$

Continuous-Time Model

Alternative Accounting Rules I

Cash Accounting

- $B_\tau^{cash} = 0 \Rightarrow$ investors only observe the firm's CFs.
- $CF_t - P_t dI_t$ is disbursed to shareholders immediately $\Rightarrow$ firm's cash balance is zero at all times.

Replacement Cost Accounting

- $B_t^{rc} \equiv P_t \cdot K_t \Rightarrow$ investors observe BVoA.
- New assets are recorded at their cost $P_t \cdot dI_t$.
- Consistent with International Financial Reporting Standards 13: BVoA reflects the "amount that the firm would have to pay today for new capital goods to replicate the current capacity of its assets purchased in the past."
- Imperfect information $\Rightarrow$ observing $CF_t = X_t \cdot K_t^\alpha$ and $B_t^{rc} = P_t \cdot K_t$ is not enough to to solve for (X_t, P_t, K_t)

Continuous-Time Model

Alternative Accounting Rules II

Value in Use Accounting

- BVoA is set equal to the PV(CFs) that these assets are expected to generate in the future

$$B_t^{viu} = \frac{CF_t}{\bar{r}},\ I_{t+\tau} = 0\ \vee \tau > 0$$

$$\bar{r} \equiv r - \mu_X + \alpha\delta - \alpha\rho_{KX}\sigma_K\sigma_X - \frac{\alpha(\alpha - 1)}{2}\sigma_K^2$$

- Financial statements prepared under ViU accounting do not provide information useful to investors over and above the firm's OCF.
- Information set under ViU is the same as under Cash Accounting.

Historical Cost Accounting

- Assets are capitalized at cost when they are acquired $\Rightarrow P_t$ is not timely reflected in BVoA evolving as

$$dB_t^{hc} = -\delta B_t^{hc} dt + \sigma_K B_t^{hc} dz_K + P_t dI_t$$

- Requires fixed depreciation schedules.

Conditionally Conservative Accounting

- Replacement cost accounting combined with asymmetric recognition of gains and losses
- All capital goods are initially recognized at acquisition cost $P_t dI_t$
- Accumulated depreciation

$$\sup_{\tau \leq t} \left(GI_\tau - P_\tau K_\tau\right)^+$$

- BVoA is given by

$$B_t^{cc} = GI_t - \sup_{\tau \leq t} \left(GI_\tau - P_\tau K_\tau\right)^+$$

- Important property

$$B_t^{cc} \leq P_t K_t$$

Continuous-Time Model

Firm Value $V(X, P, K)$

- Inaction region $dI_t = 0$ and $P > V_K$
  - Instantaneous expected return on $[t, t + dt]$

$$\mathrm{E}\left[CFdt + dV\right] = rV \cdot dt$$

  - Instantaneous change in firm's value $dV = \mathcal{L}V$
  - In inaction region V satisfies

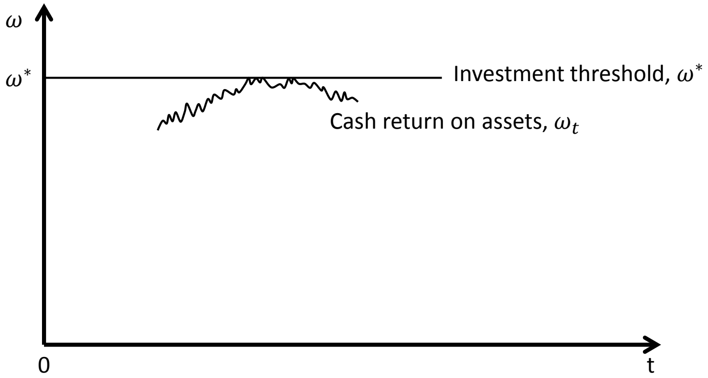
$$rV = \mathcal{L}V$$

- Investment region $P = V_K$ and $rV > \mathcal{L}V$
- Firm value satisfies variational inequality

$$\min\left(rV - \mathcal{L}V, P - V_K\right) = 0$$

- Define Cash Return to Economic Assets as

$$\omega_t \equiv \frac{CF_t}{P_t K_t} = \frac{CF_t}{B_t^{rc}}$$



PROPOSITION 1

The firm's equity value at time t is equal to

$$V_t = \frac{CF_t}{\bar{r}} \left(1 + A \cdot \left[\frac{CF_t}{B_t^{rc} \cdot \omega^*} \right]^{\lambda} \right),$$

where ω^* is the optimal threshold for investment, given by:

$$\omega^* = \frac{\bar{r} \cdot (1 + \lambda)}{\alpha \cdot \lambda}.$$

The firm makes its first investment so that

$$I_0^* = \left(\frac{X_0}{P_0} \omega^* \right)^{\frac{1}{1-\alpha}},$$

and then invests only when its cash return on assets ratio, ω_t, is equal to ω^*.

COROLLARY 1

Given historical cost accounting, value in use accounting, or cash accounting, the tightest bounds on the firm's equity value that hold almost surely conditional on $\mathcal{I}_t$ are:

If $P_t dI_t > 0$,

$$V_t = \frac{CF_t}{\bar{r}} \left(1 + A\right);$$

if $P_t dI_t = 0$,

$$\frac{CF_t}{\bar{r}} \leq V_t \leq \frac{CF_t}{\bar{r}} \left(1 + A\right).$$

COROLLARY 2

Given conditionally conservative accounting, the tightest bounds on the firm's equity value that hold almost surely conditional on $\mathcal{I}_t$ are:

If $P_t dI_t > 0$,

$$V_t = \frac{CF_t}{\bar{r}} \left(1 + A\right);$$

if $dB_t^{cc} < 0$,

$$V_t = \frac{CF_t}{\bar{r}} \left(1 + A \cdot \left[\frac{CF_t}{B_t^{cc} \cdot \omega^*}\right]^{\lambda}\right);$$

if $P_t dI_t = 0$ and $dB_t^{cc} \geq 0$,

$$\frac{CF_t}{\bar{r}} \leq V_t \leq \frac{CF_t}{\bar{r}} \left(1 + A \cdot \left[\frac{CF_t}{B_t^{cc} \cdot \omega^*}\right]^{\lambda}\right).$$

- Why discrete time?
  - Reversible investment $\Rightarrow K_t$ follows GMB $\Rightarrow I_t$ has an unbounded variation
  - SSC requires bounded variation
- Operating cash flow $CF_t = X_t K_t^{\alpha}$
- Demand $X_t = g_{X,t} \cdot X_{t-1}$
- Capital stock $K_t = g_{K,t} \cdot \hat{K}_t$

$$\hat{K}_t = (1-\delta) \cdot K_{t-1} + I_{t-1}$$

- Investment I_t

$$I_t = \hat{K}_{t+1} - (1-\delta)\,\hat{K}_t g_{K,t}$$

- Replacement costs $RC_t = P_t \cdot \hat{K}_{t+1}$ with price $P_t = g_{P,t} \cdot P_{t-1}$
- Shocks $\theta_t \equiv (g_{X,t}, g_{P,t}, g_{K,t})$ are i.i.d

Discrete-Time Model

Firm Value

- Firm value

$$V_t = CF_t - P_t I_t^* + \sum_{\tau=1}^{\infty} \frac{1}{(1+r)^\tau} \cdot \mathrm{E}_t \left[CF_{t+\tau} - P_{t+\tau} I_{t+\tau}^* \right]$$

Bellman equation

$$V_t\left(\hat{K}_t, X_t, P_t, g_{K,t}\right) = X_t g_{K,t}^{\alpha} \hat{K}_t^{\alpha} + P_t (1-\delta) \hat{K}_t g_{K,t} +$$
$$+ \max_{\hat{K}_{t+1}} \left\{ \frac{1}{1+r} \cdot \mathrm{E}_t \left[V_{t+1}\left(\hat{K}_{t+1}, X_{t+1}, P_{t+1}, g_{K,t+1}\right)\right] - P_t \cdot \hat{K}_{t+1} \right\}$$

- Firm value is finite if $1 + r > \mathrm{E}\left[g_X^{\frac{1}{1-\alpha}} g_P^{-\frac{\alpha}{1-\alpha}} \right]$
- No-arbitrage for $P(t) \Rightarrow 1 + r > (1-\delta)\,\mathrm{E}\left[g_K g_P\right]$

PROPOSITION 2

The firm's cum-dividend equity value at time t is equal to

$$V_t = CF_t - P_t I_t + (1 + C_1) \cdot RC_t,$$

and the firm's optimal investment policy is characterized by:

$$\hat{K}_{t+1} = C_2 \cdot \left(\frac{X_t}{P_t}\right)^{\frac{1}{1-\alpha}},$$

where C_1 and C_2 are two non-negative constants that depend only on the parameters of the stochastic processes X_t,P_t,K_t and not on their realizations.

- Ex-dividend firm value is equal to $(1 + C_1) \cdot RC_t$ and increases in RC_t
- Equity value is known as long as the replacement costs are known.
- RC_t at optimum

$$P_t \hat{K}_{t+1} = C_2 \cdot X_r^{\frac{1}{1-\alpha}} P_t^{-\frac{\alpha}{1-\alpha}}$$

- $P_t \uparrow \Rightarrow RC_t \downarrow$
- With irreversible investment firm value decreases in RC_t

Empirical Implications

- Empiricists need to control for the investment type, i.e. reversible vs irreversible
- In 1980's, under Statement of Financial Accounting Standards (SFAS) 33, large firms were required to disclose the current RCs of their assets such as plant, property,equipment, and inventories.
- No incremental value of SFAS 33 disclosures over the historical cost earnings (Beaver and Landsman (1983) and Beaver and Ryan (1985)).
- Revsine (1973) suggested that positive holding gains under replacement cost accounting can be good news for some firms and bad news for others, depending on how well each firm is able to react to price changes in its input markets.
- Dechow et al. (1999) and Hao et al. (2011) find small or negative coefficients on the book value of assets in equity valuation regressions.

Conclusion

- We studies the problem of equity valuation based on accounting information, not on generic *public* info used in AP.
- We run a horse race between several accounting reporting procedures.
- Replacement cost accounting wins $\Rightarrow$ outside investors will have sufficient information to value the firm's equity.
- The demand for replacement cost disclosures comes from investors' need to value the firm's growth options.