Discussion of "Belief Dispersion in the Stock Market" by Adem Atmaz and Suleyman Basak

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Stock markets and information

The paper starts with the large literature regarding information aggregation in the stock market

The model displays some empirically relevant elements

- Excess volatility

Two parts of the paper:

- Non-Bayesian part: risk-averse terminal-expected-wealth-maximizing individuals with heterogeneous beliefs about the fundamental value of a stock don't learn from aggregates (prices)
- Bayesian part: individuals learn from prices

Description of the model

There is one stock, with value D_t that fluctuates according to a lognormal diffusion process

- $$dD_t = \mu D_t dt + \sigma D_t d\omega_t$$
- mean μ can take a high or low value, but is unobservable
- ω_t is a standard Brownian motion

Preferences: CRRA with end of period wealth

- Thus, ongoing consumption is not part of the model (so consumption-smoothing is not part of the model)
- The evolution of the stock terminates at the horizon T
- Beliefs evolution comes to an end at T

Description of the model

- Beliefs are modeled as an additive constant θ that is added to the mean μ of the value process diffusion.
  - "Girsanov"-style measure change results in a perceived drift $\mu_{St} = \mu_{St} + \frac{\theta}{\sigma}\sigma_{St}$
  - (Notice that the subjective mean evolves intertemporally)
  - θ Gaussian initially distributed over the population with mean $\tilde{m}$ and variance $\tilde{\sigma}$, with $\tilde{m}$ also unknown
    - m_t and σ_t evolve intertemporally
- Price is observed with *subjective* dynamics

$$dS_t = \mu_{St} S_t + \sigma_{St} d\omega_t(\theta)$$

Two parts of the paper

The non-Bayesian part

- The economy converges to the correct posterior of the fundamental value of the firm because the individuals with the correct prior get richer, thus add greater weight to the price of the stock, so the price heads in the right direction
- The individuals with the wrong prior get poorer, and their opinion thus matters less in determining the price because they are a smaller part of the market
- Thus the price converges to the correct price through a "Darwinian" process
- Thus the model strongly predicts that the *distribution of wealth* will narrow, with most people becoming poor and a very few becoming rich!

Two parts of the paper

The Bayesian part

- In this section individuals are aware that they might have the wrong prior and attempt to glean information—feedback—from the price, and form a conditional forecast of the mean and variance of the initial distribution of beliefs
  - Thus, the equilibrium requires agents to track the evolution of *distributions* of their beliefs, which is equivalent here to tracking the mean and variance of beliefs, which dynamically change over time; this is an impressive technical achievement
  - Also impressive: the Bayesian learning model can piggyback on the non-Bayesian model; with the preference and stochastic structure there is a kind of certainty equivalence that allows them to solve in closed form, with the influence of the learning element somewhat separable from the heterogeneity of beliefs structure

Relationship to Kyle-type models

Similarities

- Private signal or information about the fundamental value
- Similar to Kyle (1989) rather than Kyle (1985) in that individuals are symmetric except for their information
- In Kyle (1985) information is explicitly transmitted through price to the uninformed market makers, and beliefs evolve in the correct direction over time

Differences

- Kyle (1989) is a static model, whereas this one is dynamic; Kyle (1985) is dynamic however, and shows intertemporal evolution of beliefs

Bayesian part: Relationship to my own (obscure and forgotten) work

Main idea: there is an information externality associated with nominal shocks

- Individuals want to know the aggregate nominal shock so that they can *avoid* reacting to it
- In my "information transmission" paper, for an equilibrium to exist, aggregate fluctuations are *necessary* to overcome incentives to not send a signal that can be usefully added to aggregates
- In my "optimal policy" paper, it is optimal to *induce* aggregate fluctuations using noisy feedback via aggregates for the same incentive reasons
- The Atmaz-Basak paper is similar: individuals would like to know the average bias in beliefs—equivalent in effect to a *nominal* shock

Information externality

In the Bayesian model there is an informational externality.

- It would be useful to have an explicit treatment of this externality: at what *rate* does information evolve
- Because the individuals are risk averse, how does the speed of evolution of the information affect welfare (improvement in aggregate risk?), and is there a trade-off?
- The market is inducing individuals to reveal their θs; is this incentive increased or decreased by the speed of the market?

Information externality

- More precisely, an individual's revelation of his θ via a purchase of the stock confers a positive externality on other traders: can you *quantify* this externality?
  - (Note that a correctly informed individual who knows he is correctly informed could make infinite profits; thus improved information improves profits if the individuals are aware that it is an improvement)
- In the non-Bayesian model the individuals with bad priors become poor; in the Bayesian model can the badly informed "survive" by learning?
- What is the implication of this for the distribution of wealth?

Origins and evolution of the heterogeneity of beliefs

The folklore of the Kyle model

- The privately informed "insiders" conduct research to become better informed
- They know they are informed
- Here, (almost) everyone is misinformed, and they know it

Where do the (wrong) beliefs come from?

- Nominal shocks? They why are they heterogeneous?

For the model to make sense, the heterogeneity of beliefs has to re-form *dynamically*, otherwise everything would converge

Excess volatility

Main comment: the volatility is induced by the belief dispersion but the belief dispersion is exogenously imposed

- Empirically, could we go backward from the empirically observed excess volatility and impute the mean and variance of the dispersion of beliefs?
  - No!—the econometrician has no more information than the agents in the model
  - Thus, the model is not an empirically testable model of excess volatility in the sense that it can't pin down the exact combination of $\tilde{m}$ and $\tilde{v}$ that lead to the observed excess volatility

Excess volatility

Notice that increased risk aversion *reduces* excess volatility, whilst in the mainstream excess volatility literature, increased risk aversion increases volatility (with the "puzzle" then being that the risk aversion you need is unrealistically high)

- Proposition 4, equation (24) and Proposition 8, equation (39)
- In the Bayesian section it is more ambiguous; be clearer about the impact of learning

Style and structure

There are two sections analyzing the situation in which there are only two belief types rather than a continuum

- These should go into an appendix

Conclusion

Very cool paper

- Technically significant accomplishment
- Useful and usable foundation for analyzing information externality