Belief Dispersion in the Stock Market

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Motivation

- Empirical evidence on belief dispersion is vast and mixed
  - Negative relation between belief dispersion and stock return: Diether, Malloy, and Scherbina (2002), Chen, Hong, and Stein (2002), Goetzmann and Massa (2005), Park (2005), Berkman, Dimitrov, Jain, Koch, and Tice (2009), Yu (2011)
  - Positive or no significant relation: Qu, Starks, and Yan (2003), Doukas, Kim, and Pantzalis (2006), Avramov, Chordia, Jostova, and Philipov (2009)
- Existing theoretical works do not provide satisfactory answers

Our Work

- Provide a tractable model of belief dispersion which simultaneously supports empirical regularities in
  - stock price
  - mean return
  - volatility
  - trading volume
- Fully closed-form expressions for all quantities
- Key feature: summarize wide range of beliefs with
  - average bias in beliefs
  - dispersion in beliefs

Our Main Results

- Stock price is convex in cash-flow news
- Stock price increases, its mean return decreases in belief dispersion when the view on the stock is optimistic
- Stock price may increase in risk aversion in bad states
- Belief dispersion generates:
  - excess stock volatility
  - non-trivial trading volume
  - a positive relation between the two
- Belief dispersion reduces learning induced excess volatility
- Finitely-many-investor models do not necessarily generate our main results

Related Theoretical Literature

- Heterogeneous Beliefs
  - Abel (1989); Varian (1989); Shalen (1993); Harris and Raviv (1993); Kandel and Pearson (1995); Detemple and Murthy (1994); Zapatero (1998); Basak (2000, 2005); Scheinkman and Xiong (2003); Johnson (2004); Anderson, Ghysels, and Juergens (2005), Kogan, Ross, Wang and Westerfield (2006); Buraschi and Jiltsov (2006); Jouini and Napp (2007); David (2008); Yan (2008); Gallmeyer and Hollifield (2008); Cao and Ou-Yang (2008); Dumas, Kurshev, and Uppal (2009); Banerjee and Kremer (2010); Xiong and Yan (2010); Cvitanić and Malamud (2011); Bhamra and Uppal (2014); Chabakauri (2015)
- Parameter Uncertainty and Learning
  - Barsky and De Long (1993); Timmermann (1993, 1996); Veronesi (1999); Brennan and Xia (2001); Lewellen and Shanken (2002); Pastor and Veronesi (2003)

Model

- Pure-exchange economy with finite horizon $[0, T]$ and a single source of risk ω
- Two securities: a risky stock and a riskless bond
- Stock is in positive net supply and pays off D_T at horizon T, horizon value of the cash-flow news process D with

$$dD_t = D_t \left[\mu dt + \sigma d\omega_t\right]$$

- Stock price S has dynamics

$$dS_t = S_t \left[\mu_{St} dt + \sigma_{St} d\omega_t\right]$$

- Bond is in zero net supply

Investors' Beliefs

- Continuum of investors commonly observe cash-flow news D but have different beliefs
- Investors indexed by their type $\theta \in \Theta$
- Under θ-type investor's beliefs, cash-flow news follows

$$dD_t = D_t \left[(\mu + \theta)\, dt + \sigma d\omega_t (\theta) \right]$$

Investor Type Space

- Investor type space $\Theta = \mathbb{R}$
- Investor relative frequency is Gaussian
  - mean $\tilde{m}$
  - standard deviation $\tilde{v}$
- All investors endowed with equal stock shares
- Initial wealth of distinct θ-type investor

$$W_0(\theta) = S_0 \frac{1}{\sqrt{2\pi\tilde{v}^2}} e^{-\frac{1}{2}\frac{(\theta-\tilde{m})^2}{\tilde{v}^2}}$$

Investors' Preferences and Optimization

- Each θ-type investor chooses a portfolio process $\phi(\theta)$ to maximize:

$$\mathbb{E}^{\theta}\left[\frac{W_T(\theta)^{1-\gamma}}{1-\gamma}\right]$$

where θ-type investor's financial wealth $W_t(\theta)$ follows

$$dW_t(\theta) = \phi_t(\theta)\, W_t(\theta)\left[\mu_{St}(\theta)\, dt + \sigma_{St} d\omega_t(\theta)\right]$$

Average Bias and Dispersion in Beliefs

- Average bias in beliefs is the implied bias of the corresponding representative investor
  - weighted average of investors' biases

$$m_t = \int_{\Theta} \theta h_t(\theta)\, d\theta$$

with positive weights: $\int_{\Theta} h_t(\theta)\, d\theta = 1$

- Dispersion in beliefs is the standard deviation of investors' biases

$$v_t^2 \equiv \int_{\Theta} (\theta - m_t)^2\, h_t(\theta)\, d\theta$$

Equilibrium Average Bias and Dispersion in Beliefs

- Average bias in beliefs:

$$m_t = m + \left(\ln D_t - \left(m + \mu - \frac{1}{2}\sigma^2\right)t\right)\frac{v_t^2}{\gamma\sigma^2}$$

  - average bias is stochastic
  - dispersion amplifies the effects of cash-flow news
  - risk attitude influences average bias

- Dispersion in beliefs:

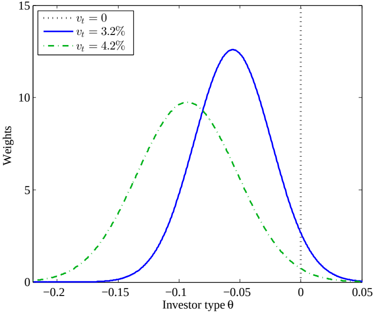
$$v_t^2 = \frac{v^2\sigma^2}{\sigma^2 + \frac{1}{\gamma}v^2 t}$$

  - dispersion also represents extra uncertainty

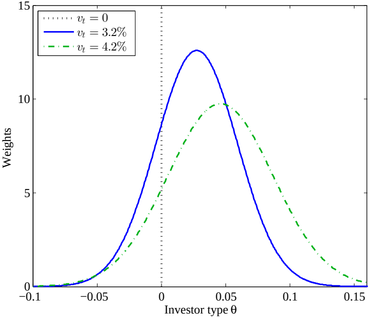
- Unique weights:

$$h_t(\theta) = \frac{1}{\sqrt{2\pi v_t^2}} e^{-\frac{1}{2}\frac{(\theta - m_t)^2}{v_t^2}}$$

Equilibrium Average Bias and Dispersion in Beliefs



(a) Relatively bad news



(b) Relatively good news

- Good news leads to optimism, bad news leads to pessimism
- Dispersion amplifies the average bias

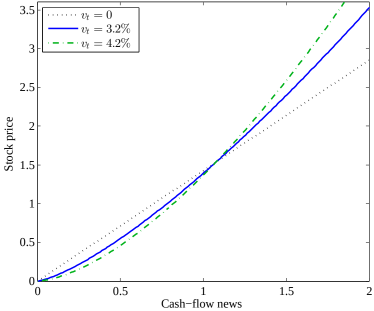
Equilibrium Stock Price

Benchmark no-dispersion economy stock price:

$$\overline{S}_t = D_t e^{\left(\mu - \gamma\sigma^2\right)(T-t)}$$

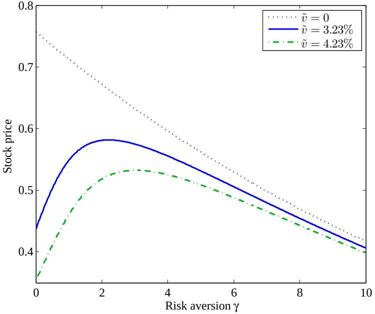
Equilibrium stock price:

$$\boxed{S_t = \overline{S}_t e^{m_t(T-t) - \frac{1}{2\gamma}(2\gamma-1)v_t^2(T-t)^2}}$$

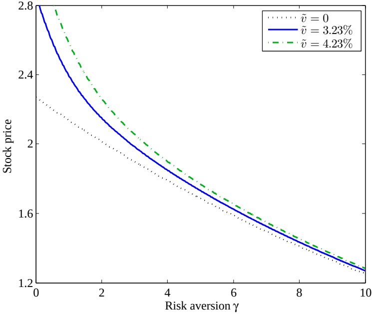


- Convex in cash-flow news (Basu (1997), Xu (2007), Conrad, Cornell, and Landsman (2002))
- Higher than in benchmark when the view on the stock is optimistic (Brown and Cliff (2005))
- Increasing in belief dispersion when the view on the stock is optimistic (Goetzmann and Massa (2005), Yu (2011))

Equilibrium Stock Price (cont'd)



(a) Relatively bad news



(b) Relatively good news

- May increase in risk aversion in bad states

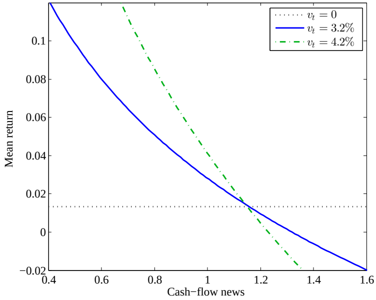
Equilibrium Mean Return

Benchmark no-dispersion economy mean return:

$$\overline{\mu}_{St} = \gamma \sigma^2$$

Equilibrium mean return:

$$\boxed{\mu_{St} = \overline{\mu}_{St} \frac{v_t^4}{v_T^4} - m_t \frac{v_t^2}{v_T^2}}$$



- Lower than in benchmark when the view on the stock is optimistic (La Porta (1996), Brown and Cliff (2005))
- Decreasing in belief dispersion when the view on the stock is optimistic (Diether, Malloy, and Scherbina (2002), Yu (2011))
- May decrease in risk aversion in bad states

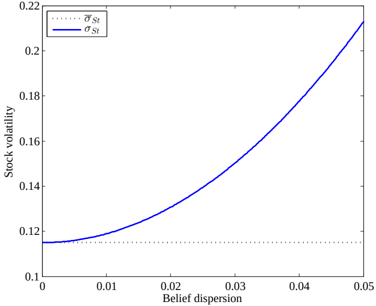
Equilibrium Stock Volatility

Benchmark no-dispersion economy stock volatility:

$$\overline{\sigma}_{St} = \sigma$$

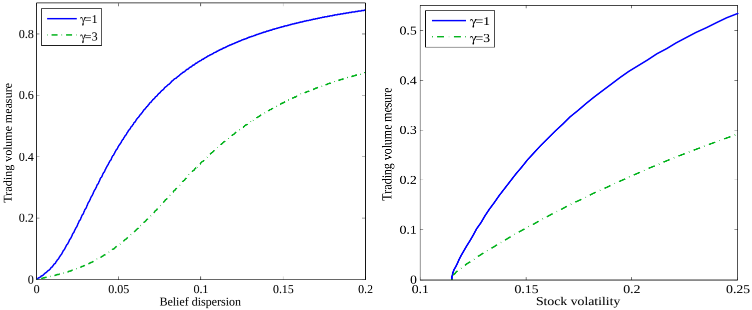
Equilibrium stock volatility:

$$\boxed{\sigma_{St} = \overline{\sigma}_{St} + \frac{v_t^2}{\gamma\sigma}(T - t)}$$



- Higher than in benchmark (Leroy and Porter (1981), Shiller (1981))
- Increasing in belief dispersion (Ajinkya and Gift (1985), Anderson, Ghysels, and Juergens (2005), Banarjee (2011))
- Decreasing in risk aversion

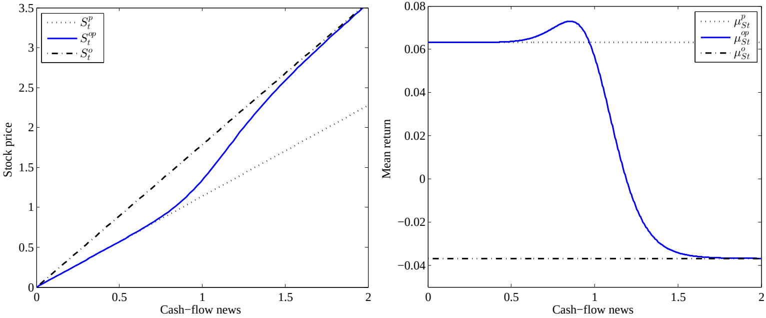
Equilibrium Trading Volume



Trading volume measure: $$\boxed{V_t \equiv \frac{1}{2}\int_{\Theta} \left|\sigma_{\psi t}(\theta)\right| d\theta}$$

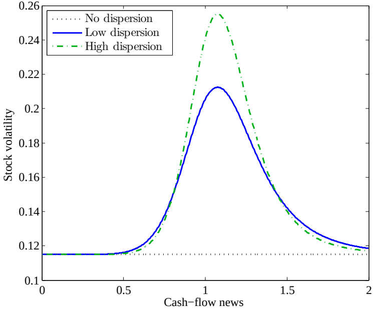
- Increasing in belief dispersion (Ajinkya et.al. (1991), Bessembinder et.al. (1996), Goetzmann and Massa (2005))
- Positively related to stock volatility (Gallant, Rossi, Tauchen (1992))

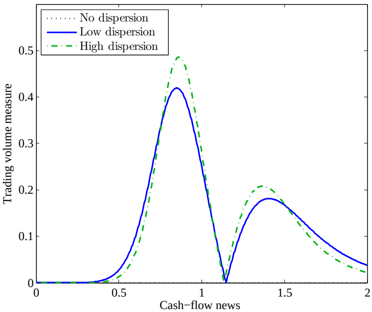
Comparisons with Two-Investor Economy



- Stock price no longer convex
- Mean return does not strictly decrease

Comparisons with Two-Investor Economy (cont'd)





- Stock volatility may decrease in belief dispersion
- Trading volume may decrease in belief dispersion

Bayesian Learning

- θ-type investor's prior: $\mathcal{N}\left(\mu+\theta, s^2\right)$
- θ-type investor's posterior: $\mathcal{N}(\mu+\widehat{\theta}_t, s_t^2)$
  - θ-type investor's time-t bias:

$$\widehat{\theta}_t = \frac{s_t^2}{s^2}\theta + \frac{s_t^2}{\sigma}\omega_t$$

  - parameter uncertainty:

$$s_t^2 = \frac{s^2\sigma^2}{\sigma^2 + s^2 t}$$

- Under θ-type investor's beliefs, cash-flow news follows

$$dD_t = D_t[(\mu + \widehat{\theta}_t)dt + \sigma d\omega_t(\theta)]$$

Equilibrium with Bayesian Learning

$$
\begin{aligned}
S_t &= \overline{S}_t e^{m_t(T-t) - \frac{1}{2}(2\gamma-1)\left(\frac{1}{\gamma}v^2+s^2\right)\frac{v_t^2}{v^2}\frac{s^2}{s_t^2}(T-t)^2} \\
\mu_{St} &= \overline{\mu}_{St}\frac{v_t^4}{v_T^4}\frac{s_T^4}{s_t^4} - m_t\frac{v_t^2}{v_T^2}\frac{s_T^2}{s_t^2} \\
\sigma_{St} &= \overline{\sigma}_{St} + \frac{1}{\sigma}\left(\frac{1}{\gamma}v^2 + s^2\right)\frac{v_t^2}{v^2}\frac{s^2}{s_t^2}(T-t)
\end{aligned}
$$

- Stock price is increasing, its mean return is decreasing in parameter uncertainty when the view on the stock is optimistic (Massa and Simonov (2005), Ozoguz (2009))
- Stock volatility is increasing in parameter uncertainty
- Learning induced excess volatility is decreasing in belief dispersion
- Trading volume is decreasing in parameter uncertainty when $\gamma \geq 1$

Conclusion

- Provide a tractable model of belief dispersion which simultaneously supports empirical regularities in stock price, mean return, volatility, trading volume

Key Results:

- Stock price is convex in cash-flow news
- Stock price increases, its mean return decreases in belief dispersion when the view on the stock is optimistic
- Belief dispersion generates:
  - excess stock volatility
  - non-trivial trading volume
  - a positive relation between the two
- Finitely-many-investor models do not necessarily generate our main results
- Above remain valid in a multi-stock economy