Belief Dispersion in the Stock Market

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Motivation

- Empirical evidence on belief dispersion is vast and mixed
  - Positive or no significant relation: Qu, Starks, and Yan (2003), Doukas, Kim, and Pantzalis (2006), Avramov, Chordia, Jostova, and Philipov (2009)
- Existing theoretical works do not provide satisfactory answers
Our Work

• Provide a tractable model of belief dispersion which simultaneously supports empirical regularities in
  ▶ stock price
  ▶ mean return
  ▶ volatility
  ▶ trading volume

• Fully closed-form expressions for all quantities

• Key feature: summarize wide range of beliefs with
  ▶ average bias in beliefs
  ▶ dispersion in beliefs
Our Main Results

- Stock price is convex in cash-flow news
- Stock price increases, its mean return decreases in belief dispersion when the view on the stock is optimistic
- Stock price may increase in risk aversion in bad states
- Belief dispersion generates:
  - excess stock volatility
  - non-trivial trading volume
  - a positive relation between the two
- Belief dispersion reduces learning induced excess volatility
- Finitely-many-investor models do not necessarily generate our main results
Related Theoretical Literature

- **Heterogeneous Beliefs**
  - Abel (1989); Varian (1989); Shalen (1993); Harris and Raviv (1993); Kandel and Pearson (1995); Detemple and Murthy (1994); Zapatero (1998); Basak (2000, 2005); Scheinkman and Xiong (2003); Johnson (2004); Anderson, Ghysels, and Juergens (2005), Kogan, Ross, Wang and Westerfield (2006); Buraschi and Jiltsov (2006); Jouini and Napp (2007); David (2008); Yan (2008); Gallmeyer and Hollifield (2008); Cao and Ou-Yang (2008); Dumas, Kurshev, and Uppal (2009); Banerjee and Kremer (2010); Xiong and Yan (2010); Cvitanić and Malamud (2011); Bhamra and Uppal (2014); Chabakauri (2015)

- **Parameter Uncertainty and Learning**
  - Barsky and De Long (1993); Timmermann (1993, 1996); Veronesi (1999); Brennan and Xia (2001); Lewellen and Shanken (2002); Pastor and Veronesi (2003)
Model

- Pure-exchange economy with finite horizon $[0, T]$ and a single source of risk $\omega$
- Two securities: a risky stock and a riskless bond
- Stock is in positive net supply and pays off $D_T$ at horizon $T$, horizon value of the cash-flow news process $D$ with

$$dD_t = D_t [\mu dt + \sigma d\omega_t]$$

- Stock price $S$ has dynamics

$$dS_t = S_t [\mu_{S_t} dt + \sigma_{S_t} d\omega_t]$$

- Bond is in zero net supply
Investors’ Beliefs

- Continuum of investors commonly observe cash-flow news $D$ but have different beliefs.
- Investors indexed by their type $\theta \in \Theta$.
- Under $\theta$-type investor’s beliefs, cash-flow news follows:

$$dD_t = D_t \left[ (\mu + \theta) \, dt + \sigma \, d\omega_t (\theta) \right]$$
Investor Type Space

- Investor type space $\Theta = \mathbb{R}$

- Investor relative frequency is Gaussian
  - mean $\tilde{m}$
  - standard deviation $\tilde{v}$

- All investors endowed with equal stock shares

- Initial wealth of distinct $\theta$-type investor

$$W_0(\theta) = S_0 \frac{1}{\sqrt{2\pi\tilde{v}^2}} e^{-\frac{1}{2} \frac{(\theta - \tilde{m})^2}{\tilde{v}^2}}$$
• Each $\theta$-type investor chooses a portfolio process $\phi(\theta)$ to maximize:

$$E^\theta \left[ \frac{W_T(\theta)^{1-\gamma}}{1-\gamma} \right]$$

where $\theta$-type investor’s financial wealth $W_t(\theta)$ follows

$$dW_t(\theta) = \phi_t(\theta) W_t(\theta) [\mu_{St}(\theta) dt + \sigma_{St} d\omega_t(\theta)]$$
Average Bias and Dispersion in Beliefs

- **Average bias in beliefs** is the implied bias of the corresponding representative investor
  
  - weighted average of investors’ biases
    
    \[
    m_t = \int_{\Theta} \theta h_t (\theta) \, d\theta
    \]
    
    with positive weights: \( \int_{\Theta} h_t (\theta) \, d\theta = 1 \)

- **Dispersion in beliefs** is the standard deviation of investors’ biases
  
  \[
  v_t^2 \equiv \int_{\Theta} (\theta - m_t)^2 h_t (\theta) \, d\theta
  \]
Equilibrium Average Bias and Dispersion in Beliefs

- Average bias in beliefs:

\[ m_t = m + \left( \ln D_t - \left( m + \mu - \frac{1}{2} \sigma^2 \right) t \right) \frac{v_t^2}{\gamma \sigma^2} \]

- average bias is stochastic
- dispersion amplifies the effects of cash-flow news
- risk attitude influences average bias

- Dispersion in beliefs:

\[ v_t^2 = \frac{v^2 \sigma^2}{\sigma^2 + \frac{1}{\gamma} v^2 t} \]

- dispersion also represents extra uncertainty

- Unique weights:

\[ h_t (\theta) = \frac{1}{\sqrt{2\pi v_t^2}} e^{-\frac{1}{2} \frac{(\theta - m_t)^2}{v_t^2}} \]
Equilibrium Average Bias and Dispersion in Beliefs

(a) Relatively bad news
(b) Relatively good news

- Good news leads to optimism, bad news leads to pessimism
- Dispersion amplifies the average bias
Equilibrium Stock Price

Benchmark no-dispersion economy stock price:

$$S_t = D_t e^{(\mu - \gamma \sigma^2)(T-t)}$$

Equilibrium stock price:

$$S_t = \overline{S}_t e^{m_t(T-t) - \frac{1}{2\gamma} (2\gamma - 1)v_t^2(T-t)^2}$$

- Higher than in benchmark when the view on the stock is optimistic (Brown and Cliff (2005))
- Increasing in belief dispersion when the view on the stock is optimistic (Goetzmann and Massa (2005), Yu (2011))
(a) Relatively bad news

(b) Relatively good news

- May increase in risk aversion in bad states
Equilibrium Mean Return

Benchmark no-dispersion economy mean return:

$$\bar{\mu}_{St} = \gamma \sigma^2$$

Equilibrium mean return:

$$\mu_{St} = \bar{\mu}_{St} \frac{v_t^4}{v_T^4} - m_t \frac{v_t^2}{v_T^2}$$

- Lower than in benchmark when the view on the stock is optimistic (La Porta (1996), Brown and Cliff (2005))
- Decreasing in belief dispersion when the view on the stock is optimistic (Diether, Malloy, and Scherbina (2002), Yu (2011))
- May decrease in risk aversion in bad states
Equilibrium Stock Volatility

Benchmark no-dispersion economy stock volatility:

$$\bar{\sigma}_S = \sigma$$

Equilibrium stock volatility:

$$\sigma_s = \bar{\sigma}_S + \frac{v_t^2}{\gamma}\left(T - t\right)$$

- Decreasing in risk aversion
Equilibrium Trading Volume

Trading volume measure:

\[ V_t \equiv \frac{1}{2} \int_\Theta |\sigma_{\psi_t}(\theta)| \, d\theta \]

- Increasing in belief dispersion (Ajinkya et al. (1991), Bessembinder et al. (1996), Goetzmann and Massa (2005))
- Positively related to stock volatility (Gallant, Rossi, Tauchen (1992))
Comparisons with Two-Investor Economy

- Stock price no longer convex
- Mean return does not strictly decrease
- Stock volatility may decrease in belief dispersion
- Trading volume may decrease in belief dispersion
Bayesian Learning

- $\theta$-type investor’s prior: $\mathcal{N}(\mu + \theta, s^2)$

- $\theta$-type investor’s posterior: $\mathcal{N}(\mu + \hat{\theta}_t, s^t_t)$

  - $\theta$-type investor’s time-$t$ bias:
    \[ \hat{\theta}_t = \frac{s^2_t}{s^2} \theta + \frac{s^2_t}{\sigma} \omega_t \]

  - Parameter uncertainty:
    \[ s^2_t = \frac{s^2 \sigma^2}{\sigma^2 + s^2 t} \]

- Under $\theta$-type investor’s beliefs, cash-flow news follows
  \[ dD_t = D_t[(\mu + \hat{\theta}_t)dt + \sigma d\omega_t(\theta)] \]
Equilibrium with Bayesian Learning

\[ S_t = S_t e^{m_t(T-t)-\frac{1}{2}(2\gamma-1)\left(\frac{1}{\gamma}v^2+s^2\right)\frac{v_t^2 s_t^2}{v^2 s^2_T} (T-t)^2} \]

\[ \mu_{S_t} = \mu_{S_t} \frac{v_t^4 s_T^4}{v_T^4 s_t^4} - m_t \frac{v_t^2 s_T^2}{v_T^2 s_t^2} \]

\[ \sigma_{S_t} = \sigma_{S_t} + \frac{1}{\sigma} \left(\frac{1}{\gamma}v^2+s^2\right) \frac{v_t^2 s_t^2}{v^2 s^2_T} (T-t) \]

- Stock price is increasing, its mean return is decreasing in parameter uncertainty when the view on the stock is optimistic (Massa and Simonov (2005), Ozoguz (2009))

- Stock volatility is increasing in parameter uncertainty

- Learning induced excess volatility is decreasing in belief dispersion

- Trading volume is decreasing in parameter uncertainty when \( \gamma \geq 1 \)
Conclusion

• Provide a tractable model of belief dispersion which simultaneously supports empirical regularities in stock price, mean return, volatility, trading volume

Key Results:

• Stock price is convex in cash-flow news
• Stock price increases, its mean return decreases in belief dispersion when the view on the stock is optimistic
• Belief dispersion generates:
  ▶ excess stock volatility
  ▶ non-trivial trading volume
  ▶ a positive relation between the two
• Finitely-many-investor models do not necessarily generate our main results
• Above remain valid in a multi-stock economy